M. Sc. (Instrumentation) Examination, 2025

(1st Year, 1st Semester)

Paper : MI 101

(Advance Mathematics and Computer Programming)

Time : 4 *Hours* *Full Marks* : 80

Use separate answer scripts for each part.

PART—I

CO 1 : Answer *any* **three** questions.

1. What are the advantages of Fourier series? Obtain a Fourier series of the function $f(x) = x + \frac{x^2}{4}$ for $-\pi \le x \le \pi$. 3+7

2. *(i)* Write Dirichlet's condition for a Fourier series.

(ii) Obtain a cosine series expansion of the function $f(x) = 1 + x$ for $0 \le x \le 2$. 3+7

3. Find the Fourier transform of x^{n-1} and $e^{-a^2x^2}$, $a > 0$. 5+5

4. *(i)* Find the Laplace transformation of the function

$$f(t) = \begin{matrix} t-1 & \text{for } 1 < t < 2 \\ 3-t & \text{for } 2 < t < 3 \end{matrix}$$

(ii) Find the inverse Laplace transformation of

$$F(s) = \log\left(1 + \frac{1}{s}\right)$$ 5+5

5. $f(t) = \begin{matrix} t & \text{for } 0 < t \le a \\ 2a - t & \text{for } a < t < 2a \end{matrix}$

(i) Draw the graph of the above periodic function.

(ii) Find its Laplace transformation. 3+7

CO 2 : Answer *any* **one** question.

6. Solve the Partial Differential equations:

(i) $\frac{\partial^2 y}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x-y}$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = x + y$ 5+5

7. *(i)* Find the complimentary function and particular integral of the equation

$$(D^2 + D'^2)z = x^2 y^2 \quad D = \frac{\partial}{\partial x},\ D' = \frac{\partial}{\partial y}$$

(ii) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$ 5+5

GROUP—B

Answer *any* **two** questions (**CO4** and **CO5**) 10×2=20

5. Write a C program to perform the following integration using Simpson's one-third rule.

$$\int_0^5 (x^2 - 5x)dx$$

6. Write a C program to estimate the following integration using Trapezoidal rule

$$\int_0^{\pi/2} (5\sin^2 x + \cos x)\,dx$$

7. Suppose there are 50 numbers given in a file "sample.dat". Write a C program to read the data from the file and calculate mean and standard deviation for the set of data. It is given that mean is $\overline{x} = \frac{1}{50}\sum_{i=1}^{i=50} x_i$ and the standard deviation is expressed as $\sigma = \sqrt{\frac{(x_i - \overline{x})^2}{50}}$.

8. Write a C program to solve the given differential equation using Euler's method

$$\frac{dy}{dx} + 2y = 0 \quad y(0) = 1$$

Take $h = 0{\cdot}1$ as step size and print the result at alternative steps up to $x = 1$.

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PART—II

GROUP—A

Answer *any* **two** questions (**CO3** and **CO5**)

1. *(a)* If a number be rounded to n correct significant figures, then show that the relative error is less than $\dfrac{1}{k \times 10^{n-1}}$, where k is the first significant figure in the number. Hence determine the number of correct digits in the number $376 \cdot 3$ with relative error $0 \cdot 3$.

(b) Determine the general formula for finding out roots of an algebraic or transcendental equation. Hence show that the above iterative method surely converges.

[(5+1)+(3+1)]

2. The Lagrange's Interpolation formula for a set of data points (x_i, y_i) where $i = 0,1,2,..., n$ is given by

$$L(x) = \sum_{i=0}^{n} \frac{\omega(x)}{(x - x_i)\omega'(x_i)} y_i$$

where $\omega(x) = (x - x_0)(x - x_1) ... (x - x_{n-1})(x - x_n)$.

(a) Show that the form of Lagrangian functions $\omega_i(x) = \dfrac{\omega(x)}{(x - x_i)\omega'(x_i)}$ remains invariant under linear transformation.

(b) Show that the sum of Lagrangian functions is unity i.e. $\sum_{i=0}^{n} \omega_i(x) = 1$.

3. *(a)* Find the relation between shift operator (E) and the difference operator (Δ).

(b) Using the above relation and following table, construct a polynomial $f(x)$ in x of degree three. Then find the value of $f(1 \cdot 5)$.

x	0	1	2	3
$f(x)$	1	2	11	34

(c) Using Newton's forward difference interpolation formula, check the value of $f(1 \cdot 5)$ with the value obtained in question (b) above. 2+4+4

4. *(a)* Solve the following system of simultaneous linear equations using Gauss-elimination method.

$$x + 3y + 2z = 5$$

$$2x - y + z = -1$$

$$x + 2y + 3z = 2$$

(b) Evaluate $\int_0^1 (4x - 3x^2) dx$, taking 10 intervals by both Trapezoidal and Simpson's one-third rule. Calculate the exact integration value. Then find absolute and relative errors in both cases. 4+6