(b) Consider that you will perform a measurement on a system for a physical quantity *H*. What will be your probable results? When you have performed you get result *E*, then determine the state of the system immediately after your measurement. Will it change if you perform repeated measurements of the same quantity on the same system? Explain.

[**CO2+CO4 :** (3+3)+4]

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Ex/SC/PHY/PG/CORE/TH/103/2025

M. Sc. Examination, 2025

(1st Year, 1st Semester)

PHYSICS

Paper : PHY/PG/CORE/TH/103

(Quantum Mechanics — I)

Time : *Two Hours* *Full Marks* : 40

Make sure that questions from each CO are answered.

Use separate answer scripts for Group A and Group B.

GROUP—A

Answer Q. No. **1** or **2** (1×6=6) [**CO1**]

1. *(i)* Obtain an orthonormal basis set from a non-orthonormal basis set in case of 2-d LVS.

(ii) Establish the completeness condition taking a set of basis vector of a LVS.

(iii) What do you mean by $\langle X|P\rangle$ and $\langle P|X\rangle$? Calculate $\langle X|P\rangle$. (2+2+2) **(CO1)**

2. *(i)* Show that unitary operator can relate two different sets of orthonormal basis vectors of a LVS.

(ii) Prove that $e^{-i\theta S_z} S_x e^{i\theta S_z} = S_x \cos\theta + S_y \sin\theta$

(Symbols have their usual meanings)

(2+4) **(CO1)**

Answer Q. No. **3** or **4** (1×6=6) [**CO2**]

3. A particle is moving under a potential given by

$$V(x) = -\alpha[\delta(x+a) + \delta(x-a)]$$

How many bound state (s) does it have? Calculate one bound state and sketch the energy eigen state. (6) **(CO2)**

4. *(i)* Find out the normalized Eigen state $|\alpha\rangle$ of the lowering (a) operator.

(ii) Show that this state $|\alpha\rangle$ corresponds to the minimum uncertainty state. (2.5+3.5) **(CO2)**

Answer Q. No. **5** or **6** (1×8=8) [**CO4** + **CO5**]

(b) Determine and describe if any degeneracy appears in any of your state vector spaces.

[**CO1+CO3 :** (5+2)+3]

3. *(a)* Describe approximations used in WKB approximation method for finding solutions of Schrödinger equation. Give two examples of system where you can use this approximation.

(b) Find a general solution for wave function using this approximations.

(c) Derive quantization condition $\int_0^a p(x)\,dx = n\pi\hbar$ for 1-dimensional infinite potential well with boundaries at $x=0$ and $x=a$.

(d) Find the relation between state vector and wave function of a system. Is there any advantage of using Dirac notation? Explain. [**CO2+CO4 :** 2+3+3+2]

(OR)

4. *(a)* Show that the mean value of the Hamiltonian is stationary in the neighbourhood of its discrete eigenvalues. Apply this concept and method to find the energy of ground state for one dimensional harmonic oscillator.

5. *(i)* What is the effect of the following perturbation on the unperturbed states?

(a) Perturbation is time independent, unperturbed states are non-degenerate

(b) Perturbation is time independent, unperturbed states are degenerate

(c) Perturbation is time dependent

(ii) Consider perturbation of the form $\frac{1}{2}bx^2$ to the L.H.O. of spring constant k.

(i) Find the exact revised value of energy of this L.H.O. and expand it as a power series of $\frac{b}{k}$ upto 2nd order.

(ii) Calculate the first and second order correction to energy due to the perturbation. Compare your result with that of (i). (1.5+6.5) **(CO4, CO5)**

6. *(i)* Considering the first order correction, calculate the transition probability from one level to other when a two level atom experiences a time dependent perturbation of the form $H' = V_0 \cos \omega t$.

(ii) Graphically show how does the transition probability vary with time. What should be done in order to 'catch' the system in the upper state? (7+1)**(CO4, CO5)**

GROUP—B

1. *(a)* Explain how the state vectors and observables are described in Schrödinger picture, Heisenberg picture and interaction picture. Does prediction of quantum mechanics depend on the picture in which your system is described? Explain.

(b) Obtain time evolution equation of state vector in interaction picture and find its solution.

(c) If $j(j+1)\hbar$ and $m\hbar$ are the eigenvalues of J^2 and J_Z for the same eigen ket $|k, j, m\rangle$, prove that $-m \leq j \leq m$.

[CO1+CO3 : 3+5+2]

(OR)

2. *(a)* Consider a system of two spins of magnitude $\frac{1}{2}\hbar$. Construct the possible sets of basis vectors by which you can describe the system. Define Clebsch-Gordon coefficient and find it for this system.