(c) Consider a linear triatomic molecule in which two atoms each of mass m_1 are placed at same distances on either side of mass m_2. The atoms are oscillating about their respective positions of stable equilibriums along the line joining the two atoms. The interaction force between the atoms is equal to the elastic force of spring constant 'k'. Obtain the kinetic energy and potential energy matrices of the system. Hence obtain the normal frequencies of oscillations of the system. Comment on the result you obtain. [2+2+6=10]**(CO4)**

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M. Sc. Examination, 2025

(1st Year, 1st Semester)

PHYSICS (DAY)

Paper : PHY/PG/CORE/ TH/101

(Classical Mechanics)

Time : *Two Hours* *Full Marks* : 40

Use separate answer scripts for each group.

GROUP—A

Answer questions 1 [**CO 3**], 2 [**CO 2**], 3 [**CO 5**] and 4 [**CO 6**] from Group A

1. Answer *(a)* and any one of *(b)* or *(c)*.

(a) For a rigid body moving with one point fixed, define Euler angles. Sketch necessary diagrams, clearly showing the angles and axes. 2

(b) Obtain explicitly the general rotation matrix $\mathcal{R} \doteq R_{(z,x,z)}(\phi,\theta,\psi)$, for Euler rotations (ϕ,θ,ψ), sucessively about axes (z,x,z). 3

(OR)

(c) Given the gereral Euler matrix , $\mathcal{R} = R_{(z,x,z)}(\phi,\theta,\psi)$, for rotations (ϕ,θ,ψ) , sucessively about axes (z,x,z) , show that $\mathcal{R}$ is orthogonal and Det ($\mathcal{R}$) = 1. 3

2. Answer *(a)* and any one of *(b)* or *(c)*.

(a) Consider an inertial laboratory frame and a second frame with common origin, rotating with angular velocity $\vec{\omega}(t)$. Find the equation connecting accelerations in the two frames. 2

(b) Show that Newton's equations of motion in rotating frame, require introduction of fictitious forces. Discuss physical situations where they appear. 4

(OR)

(c) Consider only the rotation of Earth about its axis and ignore the orbital motion around the Sun. Consider a point on Earth's surface at latitude 45º North and longitude 30° East. Two equal masses m each are launched, one along longitude and one along latitude. Compare the centripetal and Coriolis forces of the two masses. 4

3. *(a)* Consider a conservative spring-mass system in configuration space with n degrees of freedom which is undergoing small oscillations about their respective positions of stable equilibrium. Show that the potential energy (V) and the kinetic energy (T) of the system can be expressed as :

$V = \sum_{ij} V_{ij} q_i q_j$, and $T = \sum_{ij} T_{ij} \dot{q_i} \dot{q}_J$, where the symbols have their usual meanings.

(b) Obtain the Lagrangian and the Euler - Lagrange's equations of motion for the above system. Solve the Euler – Lagrange's equations by considering a trial oscillatory solution. Comment on the solution of the equation you achieved in matrix form in terms of eigen values and amplitude vectors of vibrations.

[(2+3)+1+2+2=10] **(CO4)**

(OR)

4. *(a)* Explain the terms stable and unstable equilibriums in terms of energy diagrams *v(q)* as a function of *q*. [Here the symbols have usual meanings.]

(b) A potential function is given by $V(x) = \frac{1}{4}bx^4 - \frac{1}{2}ax^2$, where a, b are positive constants. How many turning points this potential function has?

3. Answer any one of *(a)* or *(b)*.

(a) Find the fixed points for the two dimensional system,

$\dot{x} = x(3-x-2y),\ \dot{y} = y(2-x-y)$. Find the eigenvalues and eigenvectors for the Jacobian near the fixed points. Hence draw local phase plots near each fixed point, and finally complete the phase portrait. 1+3+2

(OR)

(b) Analyze the following two-dimensional equations and determine the bifurcation(s) that occur(s) with change in the parameter μ.

$\dot{x} = x^2 - \mu,\ \dot{y} = -y.$ 6

4. Answer any one of *(a)* or *(b)*.

(a) Write down the Lorentz transformation equation for relative velocity $\vec{v}$ along the *z* axis. Hence show the consequence "length contraction." 3

(OR)

(b) Define the 'Interval' Δs^2 between events in the Special Theory of Relativity. Show that Δs^2 remains invariant under Lorentz transformation. 3

GROUP—B

1. *(a)* What do you mean by the terms 'generalized coordinates' and 'virtual displacement'? Consider a particle moving in space. Using spherical polar coordinates (r, θ, φ) as the generalized coordinates, express the virtual displacement 'δz' in terms of the generalized coordinates (r, θ, φ).

(b) If the Lagrangian of a system does not depend on time explicitly, then show that its Hamiltonian is a constant of motion.

(c) A particle of mass '*m*' moves inside a bowl. If the surface of the bowl is given by the equation $z = \frac{1}{2}a(x^2 + y^2)$, where '*a*' is constant, find the Lagrangian of the system and hence its equation of motion. [(1+1+2)+2+4=10] **(CO1)**

(OR)

2. *(a)* Obtain Euler – Lagrange's equations of motion from Hamilton's Variational principle. Using Variational principle, show that the shortest path between two points in a circular cylinder is a helix.

(b) The Lagrangian (L) of a particle of mass '*m*' and coordinate 'q' is expressed as $L = \frac{1}{2}m\,\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$, where λ is constant.

Obtain the Hamiltonian of the system.

[(4+3)+3=10] **(CO1)**