M. Sc. Examination, 2025

(1st Year, 1st Semester)

PHYSICS (DAY)

Paper : PHY/PG/CORE/ TH/104

(Statistical Mechanics)

Time : *Two Hours* *Full Marks* : 40

Use separate answer scripts for each group.

GROUP—A

1. *(a)* Consider a gas containing N indistinguishable classical non-interacting atoms in thermal equilibrium at a temperature T. The gas is held in a neutral atom trap by a potential of the form $V(r) = ar$ where $r = \sqrt{(x^2 + y^2 + z^2)}$. Find the particle partition function for the gas. Find the expression for the equation of state in terms of canonical partition function. 5

(b) Starting from the general expression for grand canonical partition function calculate the partition function for Bose Einstein and Fermi Dirac statistics. 5

[CO1 and **CO2]**

(OR)

(a) A crystalline solid contains N similar, immobile, statistically independent defects. Each defects has 5 possible states. The energies of the states are given by $E_1 = E_2 = 0$, $E_3 = E_4 = E_5 = \Delta$. Find the partition function for the defect as a function of their number and the temperature T and hence find the defect contribution to the entropy of the crystal as a function of Δ and T. 3

(b) Derive the quantum mechanical analog of the classical Liouville's equation. Hence state the condition for equilibrium (corresponding ensemble is stationary). 2+2

(c) Define the expectation value of a physical quantity in terms of density matrix. 3 **[CO1** and **CO2]**

2. *(a)* Why fluctuations in various thermodynamic quantities are important for statistical system? 3

(b) Show that mean square fluctuation of the number density of a statistical system $\langle \Delta n^2 \rangle = KT \cdot k_T N^2 / V^3$, where k_T is the isothermal compressibility and other symbol have their usual meaning. 7 **[CO5]**

(OR)

(a) Define pair distribution and correlation function in a fluid. 3

(b) Derive the relation between structure factor and pair distribution function. 3

6. Consider a one-dimensional Ising chain with periodic boundary condition.

(a) Argue clearly, how Kadanoff's decimation transformation, transforms the system to an equivalent Ising system with lesser number of lattice sites.

(b) Derive a recursion relation between the old and the transformed systems.

(c) Find the fixed point(s) for the recursion relation derived in *(b)*. Starting from a finite non-zero interaction parameter K, show how the relevant fixed point is approached by successive transformations.

(d) Prove that the free energy per lattice site is given by,

$$\tilde{f}^{(n)}(K) = -\sum_{j=1}^{n} \frac{1}{2^{j-1}} g(K^{(j)}) - \frac{1}{2^n} \log 2.$$ 2+2+2+4

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(c) Define Ursell function in the context of spatial correlations in a fluid. Calculate the structure function in terms of Fourier transform of Ursell function. 2+2

[CO5]

GROUP—B

Answer **one** question from Question No. 3 and 4 **[CO3]** and **one** question from Question No. 5 and 6 **[CO4]**

3. (a) Express the energy of a ferromagnetic Ising system in a magnetic field h, in terms of number of sites N, number of up spins N_+, number of down spins N_- and number of up-up pairs N_{++}, up-down pairs N_{+-} and down-down pairs N_{--}.

(b) Simplify the above expression in terms of independent amongst those numbers.

(c) Find the partition function under Bragg-Williams approximation, providing clear explanation to justify this approximation and pointing out its limitations.

(d) The long range order parameter $\bar{L}$, satisfies

$$\bar{L} = \tanh\left(\frac{\gamma\varepsilon\bar{L}}{k_B T}\right).$$

Show that this equation has degenerate ordered states for $T < T_c$. Find explicit expression for T_c.

(e) Find $\bar{L}(t)$ as function of $t = (T - T_c)/T_c$. 1+1+3+3+2

4. Consider the Ising Hamiltonian, $H = -\sum_{i,j} J_{ij} s_i s_j - h\sum_i s_i$, where the summation runs over all site indices i and j.

(a) Let $M = (1/N)\sum s_i$ be the average value of spin for a particular configuration of spins $\{s_i\}$. Compare the probability of states obtained by flipping all spins, that is by changing $s_i \to -s_i$ for all $i = 1, \ldots, N$.

(b) Argue why from the above result we expect zero magnetization $m = \langle M \rangle$ for $h = 0$.

(c) Discuss the concept of spontaneous symmetry breaking in this context. Show the importance of the order in which the limits $h \to 0$ and $N \to \infty$ are taken.

(d) Consider small deviation δs_i of s_i about some average value M. Obtain the free energy as function of $m = \langle M \rangle$. Hence show that the minimization of free energy requires m to satisfy $m = \tanh[Jm/(k_B T)]$.

2+2+2+4

5. Ising energy for a one-dimensional chain is given by, $E(\{s_k\}) = -J\sum s_i \cdot s_j$. Assume $J > 0$.

(a) For chain length 8, schematically show all the states with minimum and maximum energies.

(b) Find the ratio of probabilities $(P_{\min}/P_{\max})$ of finding the system at the minimum and maximum energy levels.

(c) Find the partition function for a periodic chain having N sites, in terms of the transfer matrix.

(d) Hence find the free energy and magnetization per site.

1+2+3+4