(*b*) An unbiased coin is tossed thrice. Assuming the tosses to be independent, find the probability that there are more 'heads' than 'tails'.

(*c*) How many distinct numbers can be written taking the numerals of the number 700032 (Pin code of Jadavpur University). Any number of numerals can be taken at a time from the set $\{7, 0, 0, 0, 3, 2\}$. 3+3+4

3. (*a*) Using the properties of Levi-Civita tensor ε_{ijk} simplify $\vec{A} \times (\vec{B} \times \vec{C})$.

(*b*) The Cartesian coordinates x, y, z of a point P are expressed in terms of three curvilinear coordinates u_1, u_2, u_3, where $u_k = u_k(x, y, z)$, $k = 1, 2, 3$. Find the unit vectors along the u_1, u_2, u_3 curves, which represent the direction of increase of the respective generalized coordinates.

(*c*) Hence find an expression for the gradient in terms of these generalized curvilinear coordinates. 4+4+2

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Ex/SC/PHY/UG/CORE/TH/01/2025/OLD

BACHELOR OF SCIENCE EXAMINATION, 2025

(1st Year, 1st Semester)

PHYSICS

PAPER : CORE-01

(Mathematical Physics—I)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer Script for each Group.

GROUP—A

Answer *any* **four** questions. 5×4=20

1. Let $u = u(x, y)$ be a function of x and y. If $x = r\cos\theta$, $y = \sin\theta$ show that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta r^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

2. Using Taylor's theorem expand $f(x, y) = e^x \log(1 + y)$ in powers of x and y up to the term of third degree.

3. If $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$ then show that u, v and w are functionally related with each other. State the relevant property of Jacobian if used.

4. Define a homogeneous function of degree n in x and y. If z is a homogeneous function of degree n in x and y show that

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=n(n-1)z\ .$$

5. Reduce the Cauchy's homogeneous linear equation $\left(x^n\frac{d^n y}{d^n x}+k_1x^{n-1}\frac{d^{n-1}y}{d^{n-1}x}+.....+k_ny=X(x)\right)$ to a linear equation with constant coefficients.

6. Find the Fourier series of the function $f(x)$ defined in the range (–2, 2) as follows

$$f(x) = \{2, -2\le x\le 0$$
$$= \{x, 0<x<2$$

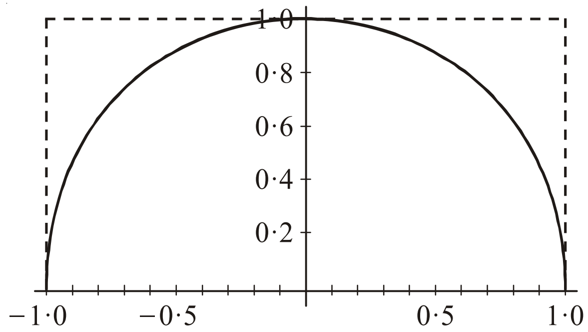
GROUP—B

1. *(a)* Verify whether the following matrices can represent rotation,

$$A=\begin{pmatrix}0 & 0 & -1\\ -1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix},\quad B=\begin{pmatrix}\frac{\sqrt{3}}{2} & 0 & \frac{1}{2}\\ 0 & 1 & 0\\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2}\end{pmatrix}$$

(b) Find the eigenvalues and the eigenvectors of the following matrix

$$C=\begin{pmatrix}2 & -1 & 1\\ 0 & 3 & 0\\ 0 & 2 & 1\end{pmatrix}$$



(c) A field is given by, $F(x,y)=(y\hat{i}-x\hat{j})/(x^2+y^2)$.

Find the line integral from (–1,0) to (1,0) moving along *(i)* semi-circular path (drawn in solid line); and *(ii)* the rectangular path from (–1,0) to (–1,1), then from (–1,1) to (1,1), and finally from (1,1) to (1,0) (drawn in dashed line). 3+4+3

2. *(a)* The probability density function of x is given by,

$$f(x)=Ae^{-\left(x-\frac{3}{2}\right)^2/2},\quad -\infty<x<\infty.$$

Find the value of A. Find the mean and standard deviation of x.