BACHELOR OF SCIENCE EXAMINATION, 2025

(2^{nd} Year, 1^{st} Semester)

PHYSICS

PAPER : PHY/TH/21/204

(Mathematical Physics)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer Script for each Group.

GROUP—A

1. The Bessel's equation of order n, (n being an integer), is given by :

$$x^2 y'' + xy' + \left(x^2 - n^2\right) y = 0$$

If $G(x)$ and $H(x)$ are the two solutions of the above equation, then show that their Wronskian is of the form :

$$GH' - HG' = \left(\frac{\text{constant}}{x}\right)$$ **[CO1]** 5

(OR)

2. The Laplace's equation in a 3-space with spherical symmetry is given by :

$$\nabla^2 \Phi(r, \theta, \phi) = 0$$

Write it down explicitly in spherical polar coordinates and show how a solution can be obtained using the principle of symmetry (separation of variables). [You do not need to actually solve the individual equations. Simply indicate how the PDE can be re-written as individual ODEs, each involving only a single variable]. **[CO2]** 5

3. Consider the well-known Legendre differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\lambda y=0$$

which arises in a wide variaty of physical situations. Show that in such cases, physical considerations and mathematical requirements of convergence/finiteness forces λ to be of the form : $\lambda = n(n+1)$, where $n \in \mathbb{Z}$

[CO2] 9

(OR)

4. *(a)* Consider the general ODE of second order in the following standard form :

$$\frac{d^2Y(x)}{dx^2}+P(x)\frac{dY(x)}{dx}+Q(x)Y(x)=0,$$

where $P(x)$ and $Q(x)$ are rational functions of x. What do you mean by a regular singular point of the above equation? Obtain/derive the conditions for the point $x=\infty$ to be a regular singular point of the above equation.

(b) Discuss the nature of singularities of

$$f(z)=\frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$$

(c) Use Cauchy's integral formula to evaluate $\oint \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$, where the closed path is *(i)* $|z|=3/2$ and *(ii)* $|z|=3$. **[CO4]** 1.5+2.5+3

8. *(a)* State the Laurent's series expansion of a function $f(z)$ of complex variable about an isolated singularity z_0 and hence calculate the coefficient of negative power of z in that expansion.

(b) With proper explanation find out the residue of a function $f(z)$ at its pole $z=z_0$ of order m. **[CO4]** 4+3

9. *(a)* With proper explanation draw a contour for the evalaution of $\int_0^\infty \frac{\sqrt{x}}{1+x^2}dx$.

(b) Using indented contour evaluate $\int_{-\infty}^{\infty}\frac{\cos x}{a^2-x^2}dx$.

[CO4] 1+6

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(b) Consider the differential equation :

$$\left(x^2+1\right)y''-4xy'+6y=0$$

and its power series solution around $x=0$. Find explicitly, the first 4 terms of this power series solution.

[CO2] 3+6

GROUP—B

Answer **one** question :

5. *(a)* Show that the Fourier series coefficient b_n is zero for a periodic function $f(x)$ that is even and defined in the interval $-\pi$ to π.

   (b) Find the Fourier series of the following triangular waveform :

$$\begin{aligned} f(x) &= 1+2x/\pi \ \text{ for } -\pi \le x < 0 \\ &= 1-2x/\pi \ \text{ for } 0 \le x \le \pi \end{aligned}$$

   (c) Show that Laplace transform of $\dfrac{d^n f(t)}{dt^n}$ is

$$s^n F(s)-s^{n-1}f(0)-s^{n-2}\left.\frac{df(t)}{dt}\right|_{t=0}-s^{n-3}\left.\frac{d^2f(t)}{dt^2}\right|_{t=0},$$

$$\left|\ldots\frac{d^{n-1}f(t)}{dt^{n-1}}\right|_{t=0}$$

   where $F(s)$ is the Laplace transform of $f(x)$.

   (d) Solve the 1D heat equation $\dfrac{\partial^2 u(x,t)}{\partial x^2}=\dfrac{1}{c^2}\dfrac{\partial u(x,t)}{\partial t}$ along a road of length L with the conditions $u(x, 0)$ and $u(0, t) = u_0$, using Laplace transform.

[CO3] 2+4+2+4

(OR)

6. *(a)* Prove Perseval's formula

$$\int_{-l}^{l}[f(x)]^2\,dx = la_0^2/2+\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right),$$ where a_0, a_n and b_n are the Fourier coefficients and $f(x)$ is a periodic function.

   (b) Find the Fourier series of the following full wave rectifier signal : $f(t) = |\sin(t)|$.

   (c) Calculate the Laplace transform of $f(t)/t$ if $F(s)$ is the Laplace transform of $f(t)$.

   (d) Solve for the current in an L-R circuit with the conditions $E(t) = E$ in the time range $0 < t < a$ and $E(t) = 0$ when $t > a$, using Laplace transform. Here, E is the e.m.f. of the battery. **[CO3]** 2+4+2+4

GROUP—C

Answer *any* **two** questions : 7×2=14

7. *(a)* Graphically show the region in the Z-plane represented by $1 \le |Z-i| \le 3$ and state whether it is simply connected or multiply connected region.