9. *(a)* Find the moment of inertia (by first fiinding the moment of inertia tensor about appropriate principal set of axes) of a uniform rod of mass M and length L about an axis that passes through a point at distance l ($l < L$) from one end of the rod and makes an angle θ with the rod.

(b) A horizontal circular disc rotates about a vertical axis that passes through a point on the edge of the disc. An insect moves along the circumference of the disc with a uniform velocity with respect to the disc. What are the directions of the centrifugal and Coriolis forces as observed from an origin fixed with the axis of rotation? 7+3=10

10. *(a)* Consider a dumbbell consisting of two masses m_1 and m_2 connected by a massless rod of length l. Find the torque required to keep this setup rotating with a uniform angular velocity ω about an axis that passes through the centre of mass, making an angle β with the rod.

(b) A particle of mass m, located in the northern hemisphere at a latitude λ, moves towards south with a velocity v. The earth rotates from west to east with a uniform angular velocity Ω. Find the direction and magnitude of the Coriolis force acting on the particle. 6+4=10

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B. Sc. Physics Examination, 2025

(2nd Year, 1st Semester)

(Classical Mechanics)

Time : *Two hours* *Full Marks* : 40

Use separate script for each group.

GROUP—A

Answer **two** from 1, 2 and 3 **[CO1]** and **two** from 4, 5 and 6 **[CO4]**

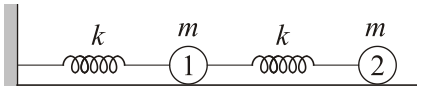
1. Consider a simple pendulum with its suport moving at a uniform velocity v vertically downward.

(a) Write down the constraint equation(s). Define and clearly identify allowed displacement and vertual displacement for the pendulum bob.

(b) State the principle of zero virtual work. Hence identify the direction of the constraint force for above system.

(c) For a coupled system, e.g. a double pendulum, will the zero virtual work apply independently for each bob? Explain. 1+2+2

2. Consider the mass-spring system given below :



(a) Find the Lagrangian of the system.

(b) Hence obtain the equations of motion.

(c) Find the normal mode frequencies and amplitudes for the system. 1+1+3

3. Consider a double pendulum.

(a) Write down the equations of constraint.

(b) Find the degrees of freedom and identify suitable generalized coordinates.

(c) Find Langrangain and Lagrange's equation for the system. 1+1+3

4. Consider a simple pendulum, motion governed by $\ddot{\theta} = -\sin(\theta)$.

(a) Identify the fixed points between $-2 < \theta < 5$.

(b) Use linear stability analysis and obtain the Jacobian at the fixed points.

(c) Hence find local phase portrait. 1+2+2

5. Consider the one-dimentional dynamical system, $\dot{x} = r\,x + x^3$. Obtain the fixed point(s) for relevant values of the parameter *r*, identify their stability and hence obtain the bifurcation diagram. Identify the bifurcation. 5

6. Equation describing two competing species may look like $\dot{x} = x(3 - x - 2y)$ and $\dot{y} = y(2 - x - y)$. Identify the fixed points. Find the phase portrait relevant for the system $(x > 0,\ y > 0)$. 5

GROUP—B

Answer **one** from 7 and 8 **[CO2]** and **one** from 9 and 10 **[CO3]**

7. *(a)* Compare the Lagrangian formalism and the Hamiltonian formalism. How does the Hamiltonian 'phase space' differ from Lagrangian 'coordinate space?

(b) Let the Lagrangian of a particle of mass *m* be

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\sin^2\theta\dot{\phi}^2 + r^2\dot{\theta}^2\right) - V(r,\theta)$$

Find out the Hamiltonian of the system in terms of phase space coordinates. Show that p_ϕ is conserved.

(c) A dynamical system has generalized coordinates q_i and generalized momenta p_i. Verify the following properties of the Poisson brackets : $[p_i, p_j] = [q_i, q_j] = 0$. (2+1)+(3+1)+3

8. *(a)* What is canonical transformation? What is the role of generating function in this context? Write four forms of generating functions.

(b) Show that for a canonical transformation, the Lagrangian in old and new coordinates differ only by an exact differential.

(c) Show that transformation $Q = 1/p,\ P = qp^2$ is canonical. Find the generating function. (1+1+1)+3+(3+1)