M. Sc. Physics (Day) Examination, 2024

(1st Year, 1st Semester)

Physics (Day)

Paper : PHY/PG/CORE/TH/103

(Quantum Mechanics—I)

Time : 2 *Hours* *Full Marks* : 40

Make sure that questions from ***each CO*** are answered.

Use separate Answer Scripts for **Group—A (Q. Nos. 1, 3, 6, 7, 8) and Group—B (Q. Nos. 2, 4, 5, 9)**.

Answer Q. Nos. 1 or 2 (8×1=8) [CO1]

1. *(a)* *(i)* Which factor determines the number of discrete basis vectors of a LVS?

(ii) How is one set of orthonormal basis vectors related to other set?

(iii) How does the basis operator change, if one goes from one set of basis vectors to other?

(iv) Does the Eigen value of an operator change due to the change of basis vectors?

(b) Write down the Schrödinger equation using the abstract notation of state vector and hence the Schrödinger equation of LHO in both position basis $\left(|X\rangle\right)$ as well as momentum basis $\left(|P\rangle\right)$ representation.

(*c*) Show that the expectation value of an operator $\hat{A}$ in the normalized state vector $|\psi(t)\rangle$ can be written as $\langle\psi(t)|\hat{A}|\psi(t)\rangle$. (3½+2½+2) **[CO1]**

(OR)

2. (*a*) Define $|R\rangle$ and $|P\rangle$ representations. Then express the state vector in $|R\rangle$ and $|P\rangle$ representations.

(*b*) Define Parity operator. Show that it is unitary.

(*c*) Prove that if two operators *A* and *B* commute, one can construct an orthonormal basis of the state space with eigenvectors common to *A* and *B*.

(*d*) State the quantization rules to construct the operator for a physical quantity that is already defined mechanics. (2+2+2½+1½) **[CO1]**

Answer Q. Nos. 3 or 4 (8×1=8) [CO2]

3. (*a*) Find out the action of the Displacement operator $D(\alpha)$ on $|0\rangle$, the ground state of LHO.

(*b*) Show that the $D(\alpha)$ is unitary operator.

(*c*) Prove that the time evolution of the coherent state $|\alpha\rangle$ is just like that of classical harmonic oscillator. (2+2+4) **[CO2]**

(OR)

9. Assume a system is perturbed as $H(t) = H_0(t) + W(t)$. Define the state vector as

$$|\psi_I(t)\rangle = U_0^\dagger(t,t_0)|\psi_s(t)\rangle$$

where $$i\hbar\frac{\partial U_0(t,t_0)}{\partial t} = H_0(t)U_0(t,t_0)$$

(*a*) Now show that

$$i\hbar\frac{d|\psi_I(t)\rangle}{dt} = W_I|\psi_I(t)\rangle, \text{ where}$$

$$W_I(t) = U_0^\dagger(t,t_0)W(t)U_0(t,t_0)$$

(*b*) Solve this equation to find $|\psi_I(t)\rangle$. 3+5 **[CO5]**

★★★

(OR)

4. (a) Show that the Hamiltonian of H-atom can be expressed as a function of only one variable r just as if a particle of mass μ subjected to effective potential

$$V_{\text{eff}} = V(r) + L^2/(2\mu r^2)$$

(b) Solve the radial part and find the energy eigenvalues and the r-dependence.

(c) Plot the ground state and first excited state.

(2+5+1) **[CO2]**

Answer Q. No. 5 (8×1=8) [CO3]

5. (a) If $j(j+1)\hbar^2$ and $m\hbar^2$ are the eigenvalues of J^2 and J_z associated with same eigenvector $|k, j, m\rangle$, then show that j and m satisfy $-j \le m \le j$.

(b) Consider a spin system of two spin 1/2 particles. Describe its state space. Then find the sets of C.S.C.O. of the system.

(c) Find the most general state of the system.

(d) Find the triplets and singlets states of this composite system. 2+2+2+2 **[CO3]**

Answer Q. Nos. 6 or 7 (8×1=8) [CO4]

6. Calculate the first order correction to the doubly degenerate states of unperturbed Hamiltonian due to time independent perturbation. Discuss the results when "good" quantum states are chosen as unperturbed degenerate states. 6+2 **[CO4]**

(OR)

7. Suppose the Hamiltonian, in matrix form is

$$H = H_0 + H' = V_0 \begin{pmatrix} 1-\varepsilon & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix}, \text{ where } \varepsilon \ll 1.$$

(a) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian $(\varepsilon = 0)$.

(b) Solve for the exact eigenvalues of $\boldsymbol{H}$. Expand each term as a power series in ε, up to second order.

(c) Use degenerate perturbation theory to find the first order correction to the two initially degenerate eigenvalues.

2+3+3 **[CO4]**

Answer Q. Nos. 8 or 9 (8×1=8) [CO5]

8. (a) A system of atoms in presence of electromagnetic wave undergoes stimulated emission. Discuss the cause of stimulated emission and calculate the term responsible for it. In this light explain why spontaneous emission occurs at all.

(b) How does the transition probability of stimulated absorption vary with **time and frequency** when a system of atoms is irradiated by a monochromatic electromagnetic wave? What should be done in order to 'catch' the system in the upper state?

(c) Explain why population inversion is required for getting LASER. 3+3+2 **[CO5]**