MASTER OF SCIENCE EXAMINATION, 2024

(2nd Year, 1st Semester)

PHYSICS

PAPER : PG/SC/CBS/PHY/TH/204

(General Relativity & Cosmology—I)

Time : *Two Hours* *Full Marks* : 40

*Answer any **four** questions.*

(*Each question carries* 10 *marks*)

1. *(a)* State the Cosmological principle and Hubble's law and show that the cosmological principle requires an expanding Universe to obey Hubble's law.

(b) Calculate the present day mass density of matter, where $\Omega_{m0} = 0 \cdot 3$. The value of $G = 6 \cdot 67 \times 10^{-11}$, in SI unit is given. The Big Bang model does not occur in only λ dominated Universe. Explain it.

(c) Observations seem to suggest $\Omega_{m0} = 0 \cdot 3$, $\Omega_{\lambda 0} = 0 \cdot 7$ and $h = 0 \cdot 7$ for our Universe. Calculate the value of de-acceleration parameter (q) for this model at present.

(2+2)+(2+2)+2

2. *(a)* Show the age of the Universe for free expansion is greater than the case where there is gravity. Present the general expression for the age of the Universe using the Friedmann equation.

(b) Using the Einstein–de Sitter model, calculate the comoving distance and bolometric luminosity distance for a given redshift z.

(c) Calculate the angular diameter distance (d_A) at redshift $z = 4$ for a flat radiation dominated Friedmann model. The value of $h = 0{\cdot}7$ is given for your calculation.

(3+2)+(2+1)+2

3. *(a)* Using the variational principle, show that the geodesic is the curve of extremum length between two points in a manifold enddowed with a metric $g_{\mu\nu}$. Is it a curve of maximum or minimum length for a space time with $g_{\mu\nu} = \eta_{\mu\nu}$? Explain.

(b) State and explain Einstein's principle of equivalence.

(6+1)+3

4. *(a)* Write down the expressions for Riemann curvature tensor, geodesic equations in locally inertial system. Does locally inertial system correspond to flat space? Explain.

(b) Using the idea of the weak field limit of gravity, state your mathematical justifications clearly in arriving the Einstein's field equations. Also state the nature of the equations. Write down the equation in vacuum. 4+6

5. Consider a general static spherically symmetric spacetime, with the metric written as

$$ds^2 = -e^{2\alpha|(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Compute the non-zero Christoffel symbols and the non-zero components of the Ricci tensor. 10

6. Consider the space-time defined by the line element :

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Except $r = 2M$, the coordinate t is always timelike and the coordinate r is spacelike. Find a transformation to a new coordinate system (τ, r, θ, ϕ), so that we get $g_{rr} = 0$. Further check if in this new coordinate system, the metric tensor is non-singular at $r = 2M$. 10

7. By considering the Schwarzschild solution (metric), discuss the symmetries of the vacuum solution. Discuss why the singularity at $r = 2M$ is said to be an apparent singularity. Show that the Schwarzschild solution is *asymptotically flat*. At $r = 2M$, discuss which of the following (considering the Schwarzschild solution mentioned in the previous question) geometrical quantities is singular — the metric tensor, Riemann curvature tensor, the Ricci tensor and the Ricci scalar? Find how many independent components does the Riemann curvature have in dimension 2, 3 and 4. 10

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