Master of Science Examination, 2024

(2nd Year, 1st Semester)

Physics

Paper : Core 105

(Quantum Mechanics—II)

Time : *Two Hours* *Full Marks* : 40

*Answer **any four** questions.*

1. *(a)* Explain the idea of the exchange degeneracy of identical particles.

(b) By using the basis $\{|1 : u_i; 2 : u_j>\}$ of a two-particle system, prove that P_{21} is a Hermitian operator.

(c) Show that permutation operators are products of transposition operators for a three-particle system.

(d) Obtain the inverse of permutation operators, P_{213} and P_{312}. (2+2+3+3)

2. *(a)* Using the variational method, find out the ground state energy of the delta function potential.

(b) Find the best bound on ground state E_{gs} for one-dimensional harmonic oscillator using a trial wave function of the form $\psi(x) = A/(x^2 + b^2)$, where A is determined by normalization and b is the adjustable parameter. 4+6

3. *(a)* What is the metric tensor, $g_{\mu\nu}$? How do you calculate the elements of the inverse of a metric tensor?

(b) What is d'Alembertian operator?

(c) Express the Klein-Gordon equation in the non-relativistic limit.

(d) Obtain *(i)* the probability current and *(ii)* the continuity equation corresponding to the Klein-Gordon equation. 4+1+2+3

4. *(a)* How many transposition operators are there in a physical system consisting of three ($N = 3$) identical particles? Following the basic action of the permutation operator, explain whether any two transposition operators for $N = 3$ commute or not.

(b) Show that the decomposition of P_{321} is not unique.

(c) What is an antisymmetrizer (**A**)? How an antisymmetrizer can be used to understand the Pauli's exclusion principle? 4+3+3

5. *(a)* Write down the Lippmann-Schwinger (LS) equation used for the description of the scattering phenomenon.

(b) With the help of the LS equation and using the position basis, derive the following integral equation

$$<x|\psi^{\pm}> = <x|\phi> - 2m/\hbar^2 \int d^3x' [e^{\pm ik|q|}/4\pi|q|] V(x') <x'|\psi^{\pm}>$$

with $|q| = |x - x'|$. Here symbols have their usual meanings. 2+8

6. *(a)* Following the formulation by Ritz, prove that $H|\psi> = <H>|\psi>$.

(b) Suggest a suitable trial function which can be used to calculate the ground state energy of Helium atom. How many parameters are there in your suggested trial function required for the variational method? By using a suitable trial function and applying the same method, calculate the ground state energy of the Helium atom. 3+7

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