B. Sc. PHYSICS EXAMINATION, 2024

(1st Year, 1st Semester)

PHYSICS

PAPER : Major 101

(Mechanics, Waves & Oscillations)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer Script for both Groups A and B

GROUP—A

Answer Question 1 (compulsory), **any one** from Questions 2 and 3, and **any one** from Questions 4 and 5.
(Total 3 questions need to be answered)

1. [CO-3] *(a)* A particle of unit mass moves according to the equation

$$\vec{r} = \hat{i}(2+3t^2) + \hat{j}5t^2 + \hat{k}t$$

Find the force $\vec{F}$ acting on it, torque $\vec{\tau}$ and angular momentum $\vec{L}$ about the origin. Hence verify $\vec{\tau} = \frac{d\vec{L}}{dt}$.

(b) Calculate the inertia tensor of a homogeneous cube of density ρ, mass *m* and side of length *b*. Let one corner be at the origin and three adjacent edges lie along the coordinate axes. [(2+1+1+1)+5]

2. [CO-1] A particle of mass m moves vertically downwards in a line from rest under gravity in air where the resistive force of air is proportional to $v^2 \hat{v}$; v is the instantaneous speed of the particle. Set up a differential equation describing the motion of the particle. Solve the equation to find the velocity of the particle as a function of time. Sketch a graph of velocity vs time. [1+3+1]

(OR)

3. [CO-1] Find expression for velocity and acceleration in cylindrical coordinate systems. [2+3]

4. [CO-3] A cylindrical metal bar of length 24 cm and diameter 4 cm is suspended by a wire 50 cm long such that the axis of the bar is horizontal. It is observed that the arrangement makes 100 torsional oscillations in 235.9 s. Find the rigidity modulus of the material of the wire. Given that the density of the material of the bar is 9 g/cc and radius of the wire is 0·1 cm. Derive the necessary formula to be used in the calculation. [2+3]

(OR)

5. [CO-3] Water flows through a horizontal tube of length 20 cm and internal radius 0·081 cm under 20 cm water pressure. If 864 cc of water is delivered in 12 minutes through the tube, calculate the coefficient of viscosity of water. Take density of water $\rho = 1$ g/cc. Derive the necessary formula to be used in the calculation. [2+3]

(OR)

10. [CO-1] An isotropic harmonic oscillator of mass m is subjected to a force law $F(r) = -kr$, where k is a constant. Its angular momentum is L. What is its centrifugal potential (V_{cent}) in terms of L, m and r? Calculate its effective potential (V_{eff}). Draw the energy-diagram of $V(r)$, $V_{cent}(r)$ and $V_{eff}(r)$ as function of r for the isotropic harmonic oscillator. In the diagram, show the two turning points in motion. How many force centers are there in the plane of the orbit? [1+1+2+1]

11. [CO-4] *(a)* Two trains A and B, each has proper length L and move in the same direction. A's speed is $\frac{4c}{5}$ and B's speed is $\frac{3c}{5}$. A starts behind B. How long, as viewed by person C on the ground, does it take for A to overtake B? By this we mean the time between the front-end of A is passing the back-end of B, and the back-end of A is passing the front-end of B.

(b) A train with proper length L moves with speed $\frac{5c}{13}$ w.r.t. the ground. A ball is thrown from the back-end of the train to the front-end. The speed of the ball w.r.t. the train is $\frac{c}{5}$. As viewed by someone on the ground, how much times does the ball spend in the air and how far does it travel? [(1+1+1)+(1+1)]

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GROUP—B

Answer any ***Two*** from questions (6, 7 and 8), any ***One*** from questions (9 and 10) and **question 11** (compulsory)
(Total 4 questions need to be answered)

6. [CO-2] A mass m is attached at one end of a spring that obeys Hooke's law and it excutes 1-D simple harmonic oscillations. The force equation is given by $F_x(x) = -k\,x$, where x is the displacement of the spring-mass from its equilibrium position and k is a positive number called the force constant.

(a) Suppose the system has a stable equilibrium at origin $x = 0$, explain the behavior of the potential energy $U(x)$ in the vicinity (small displacements) of the equilibrium position. Further, show $k = \dfrac{d^2U}{dx^2}$ at $x = 0$. State the condition for unstable equilibrium.

(b) A particle of mass m is oscillating under a potential $V(x) = \dfrac{1}{2}m\omega_0^2x^2 + \dfrac{a}{2mx^2}$, where ω_0 and a are positive constants. Calculate the angular frequency of a small oscillation for the simple harmonic motion of the particle about a stable equilibrium of the potential $V(x)$. [(2+1)+2]

7. [CO-2] Calculate the average kinetic, potential, and total energies of the weakly damped harmonic oscillator of mass m with equation of motion :

$x = x_0 \exp\left(-\dfrac{\gamma l}{2}\right)\cos(\omega_0 t + \phi)$, where damping frequency $\gamma = b/m$ and natural frequency $\omega_0 = \sqrt{k/m}$. Here b and k are damping and spring constants respectively. [(2+2+1)]

8. [CO-2] Consider an oscillator of mass m is immersed in a viscous fluid (damped harmonic oscillator). Suppose the oscillator is subject to a restoring force proportional to the distance from a fixed point on the x-axis and a damping force proportional to the velocity v (v is not too large). The damped harmonic oscillator is also acted by an external force $F(t) = F_0 \cos \omega t$. It acts along the x-axis.

(a) Set up the differential equation for the lightly damped forced harmonic oscillator. Write down the steady state solution (derivation is not required).

(b) The velocity amplitude of a particle is equal to the half the maximum (corresponds to velocity resonance) value at frequency ω_1 and ω_2 of the external harmonic force. Find the frequency corresponding to the velocity resonance. [(1+1)+3]

9. [CO-1] *(a)* A particle of mass m is subjected to a central force $F(r)$ only. Its angular momentum is L. Let r and θ be the polar coordinates in the plane of the motion of the particle. Derive the following differential equation for orbit of the system (the path of the particle in space)

$$\frac{d^2u}{d\theta^2} = -u - \frac{m}{L^2u^2}F\left(\frac{1}{u}\right)$$

where the parameter $u = \dfrac{1}{r}$.

(b) A particle of mass m is observed to move in a spiral orbit given by the equation $r = k\theta$, where k is a constant. Determine the form of the force function. [3+2]