BACHELOR OF SCIENCE EXAMINATION, 2024

(2^{nd} Year, 1^{st} Semester)

PHYSICS

PAPER : PHY/TH/05

(Mathematical Methods - II)

Time : *Two Hours* *Full Marks* : 40

(20 marks for each group)

Use separate Answer Scripts for Groups A and B

GROUP—A (Mathematical Methods)

Answer ***any two*** questions.

1. Consider the following differential equation for the function $\psi(x)$:

$$\frac{d^2\psi(x)}{dx^2} + A\psi(x) = f(x)$$

where $\psi(x)$ vanishes in the limit $|x| \to 0$. Define the Green's function for the above equation and express the general solution $\psi(x)$ in terms of it. Hence explain how the method of Green's function works for solving inhomogeneous PDEs. 10

2. Consider the wave equation in one-dimension :

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi(x, t) = 0$$

By choosing a new set of variables $(x, t) \to (\zeta, \eta)$, show how the general solution of the wave equation can be written down (where $\zeta = x + ct$ and $\eta = x - ct$). Explain the physical principles and intuitions behind the assumption (case-wise / example-wise) of expecting solutions (of PDEs) in the form of *separated variables* to exist. By choosing the explicit example of the Laplace's equation in 3d spherical polar coordinates, demonstrate the method. 10

3. Write down the heat equation in one-dimension. Find its Green's function and hence write down the most general form of its solution. Simplify your result for the special case when the external source is a 1-d spatial delta function. 10

GROUP—B (Mathematical Methods)

*Answer **any two** questions.*

1. *(a)* Determine the coefficient of x^6 in the expansion of $(8-x)^{-2/3}$ by using an appropriate differential equation.

(b) Use the method of variation of parameters to find a particular solution of the equation

$$\frac{d^2y}{dx^2} + 9x = 2\sin x\cos^2 x$$

5+5=10

2. *(a)* Given that $P_3(z) = \frac{5}{2}z^3 - \frac{3}{2}z$, locate the zeros of the function $P_3(x^2)$. Hence plot the function with respect to x.

(b) Use the generating function of Legendre polynomials to find the value of $P_n(0)$. 5+5=10

3. *(a)* Derive the orthogonality relation for two Hermite polynomials (of *unequal* orders) directly from the Hermite differential equation.

(b) Verify whether the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$ has two linearly independent series solutions, without attempting to solve the equation by series method. 6+4=10

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