Ref. No. EX/SC/PHY/UG/CORE/TH/11/2024(S)

Bachelor of Science (Physics) Special supplementary Examinations 2024

(3rd Year, 1st Semester)

Quantum Mechanics and Applications

Time: Two hours Full Marks: 40

Group – A (20 marks)

Answer any two questions (2x10)

1. Consider that a particle subjected to $V = 0$ is confined within a three dimensional box. Find out the degenerate states having the lowest energy eigen value. Derivation is required. [CO1] (10)

2. (i) What do you mean by the quantum mechanical tunnelling of a particle? (ii) How can you calculate the probability of tunnelling of a particle using the time-independent Schrodinger equation? Schematic diagram and derivation are required. [CO1] (3 + 7)

3. Write down the Hamiltonian of a harmonic oscillator in one dimension. How many quantum numbers are required to express the energy eigenvalues of the same? Give the necessary derivation to justify it. How do you define raising and lowering operator? [CO2] (1+7+2)

4. (a) Write down the Hamiltonian operator for a hydrogen atom. How many quantum numbers are necessary to express the eigen state of the hydrogen atom. Write down the quantum numbers necessary to express ground state of the same. (b) Calculate (i) $[\mathbf{L_y}, \mathbf{L_z}]$ and $[\mathbf{L^2}, \mathbf{L_z}]$. Symbols have usual meaning. [CO3] (2+1+2)+5

[Turn over

Group B

Answer any two Questions. (2x10)

1. (i) Discuss the importance of Landé g-factor. Calculate Landé g-factor for the following states: $3^2D_{3/2}$, $3^2D_{5/2}$. (ii) Suppose the Schrödinger equation for some potential is $H^0\psi_n^0 = E_n^0\psi_n^0$, where ψ_n^0 is a complete set of orthonormal and non-degerate eigenfunctions. If the potential is perturbed slightly, show that the first order correction to the nth energy eigenvalue is $E_n^1 = \langle\psi_n^0|H'|\psi_n^0\rangle$, where H' is the perturbation.
[CO3] [(2+3)+5]

2. (i) Show that, the contribution to the energy levels due to spin-orbit interaction is $E_{so}^1 = \frac{E_n^2}{mc^2}\left[\frac{n\{j(j+1)-l(l+1)-3/4\}}{l(l+1/2)(l+1)}\right]$. (ii) Show the splitting of energy levels of Hydrogen atoms in a weak magnetic field is not uniformormly separated in energy
[CO1] [5+5]

3. (i) Show that the lowest order relativistic correction to Hydrogen Hamiltonian is $H_r' = -\frac{\hat{p}^4}{8m^3c^2}$, where $\hat{p}$ is the operator corresponding to the relativistic momentum. (ii) Further, show that the first order relativistic correction to the energy levels is $E_r^1 = -\frac{E_n^2}{2mc^2}\left[\frac{4n}{l+1/2} - 3\right]$. [5+5]
[CO4]