Bachelor of Science Examination, 2024

(3rd Year, 1st Semester)

Physics

Paper : DSE/01/A2

(Condensed Matter Physics)

Time : *Two Hours* *Full Marks* : 40

(20 *marks for each group*)
*Answer from **both** the Groups as directed.*
Use separate Answer Scripts for each group.
The figures in the margin indicate full marks.

GROUP—A

Answer ***any two*** of the following questions : 10×2=20

1. *(a)* What is Wigner-Seitz cell? Explain with proper drawing and steps how Wigner-Seitz cell can be constructed from a square lattice. Is Wigner-Seitz cell a primitive unit cell? Explain your answer. What is it called in reciprocal space? (CO1) (1+2+1+1)

   (b) How many rotational symmetry axis and mirror symmetry planes are possible in a cube? Draw all the possible mirror planes. (CO1) (1+2)

(c) Explain why electron microscopes are needed to observe nanomaterials instead of optical microscope. (CO4) 2

2. *(a)* For mathematical description of symmetry of crystals, Group Theory is used. In this context what are the elements of these groups and how one defines the order of each group? (CO1) (1+1)

(b) How many point groups are there in 3-dimensional crystals? Describe any one-point group for an orthorhombic system with the help of stereographic diagram. (CO1) (1+2)

(c) Explain with proper diagram and calculations why 5-fold symmetry is not permissible in symmetry of lattice system. (CO1) 3

(d) Nanoparticles are more reactive than their bulk counterpart. Give reason. (CO4) 2

3. *(a)* What is the importance of reciprocal lattice? Describe with proper diagram how one can geometrically construct reciprocal lattice from a real lattice. (CO1) (1+2)

(b) If $\vec{R}$ be the Bravais lattice vector and $\vec{G}$ be the reciprocal lattice vector, then show that $\vec{G}.\vec{R} = 2\pi N$, where N are integers. (CO1) 2

(c) How conceptually Laue formalism is different from Bragg's formalism? Derive the Laue condition for diffraction. (CO1) (1+2)

6. *(a)* The total potential energy of the whole crystal can be expressed as

$$V = \frac{1}{2}\sum_{R,R'}{}' \Phi(R - R' + u_R - u_{R'})$$

where the symbols have their usual meanings. Now employing the three-dimensional Taylor series expansion, show that

$$V \approx \frac{1}{2}\sum_{R,R'}{}' \Phi(R - R') + \frac{1}{4}\sum_{R,R'}{}' [(u_R - u_{R'})\cdot\nabla]^2 \Phi(R - R').$$

(b) Consider a one-dimensional Bravais lattice of monoatomic basis in which the mass of the atoms is M and the nearest neighbours interact through the spring constant K, such that the total potential of the system is

$$V_{\text{Harm}} = \frac{1}{2}K\sum_n (u_n - u_{n+1})^2$$

Now show that the dispersion relation for the travelling wave solution is

$$w(k) = 2\sqrt{\frac{K}{M}}\left|\sin\left(\frac{K_a}{2}\right)\right|$$

(c) What will be the expressions of group velocity and phase velocity for the above dispersion relation when $k \approx 0$? (CO3) [4+4+(1+1)=10]

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(d) Define density of states and plot it as a function of energy for both 1-dimensional and 2-dimensional systems.

(CO4) (1+1)

GROUP—B

Answer ***any two*** of the following questions : 10×2=20

4. *(a)* State the assumptions of the Drude model.

(b) Using the expression of current density $\vec{j} = -ne\vec{v}$, show that according to Drude model, DC electrical conductivity can be expressed as

$$\sigma_0 = \frac{ne^2\tau}{m}$$

(c) Derive the equation of motion for the Drude electrons :

$$\frac{d\vec{p}}{dt} = \vec{F} - \frac{\vec{p}}{\tau}$$

(d) Show that in the presence of both electrical and magnetic fields $\vec{E}$ and $\vec{B}$, the relation

$$\vec{E} = \frac{1}{\sigma_0}\vec{j} + \frac{e\tau}{m\sigma_0}(\vec{j} \times \vec{B})$$

holds in the steady state, $\frac{d\vec{j}}{dt} = 0$. Symbols have their conventional meanings. (CO3) [3+2+3+2=10]

5. *(a)* State the assumptions of the Sommerfeld model.

(b) Write down the Schrödinger equation for the electron in the Sommerfeld model. Obtain the expressions for eigenfunction and the eigenvalue.

(c) Show that the volume in the k-space required to accommodate a single state is

$$\Delta k = \frac{8\pi^3}{V}$$

(d) Show that at zero temperature, concentration of electron n and Fermi wave vector k_F satisfy the relation :

$$n = \frac{1}{3\pi^2} k_F^3$$

(e) For copper $n = 8{\cdot}5 \times 10^{28}$ electrons/m^3. Estimate the value of Fermi energy (E_F) in eV. Given that, mass of the electron $m = 9{\cdot}11 \times 10^{-31}$ kg, reduced Planck constant $\hbar = 1{\cdot}054 \times 10^{-34}$ Js and 1eV = $1{\cdot}6 \times 10^{-19}$ J. (CO2)

[2+2+2+2+2=10]