6. *(a)* Show the fine structure energy levels for the $n = 2$ and $n = 3$ states of Hydrogen atom. Calculate the fine structure energy corresponding to $n = 1$ and $j = 1/2$ in unit of $E_n = -13{\cdot}6 / n^2$ eV.

(b) How many new transitions are expected between different fine structure levels corresponding to $n = 3$ and $n = 2$ (i.e., H- α line)? Assume the selection rule to be $\Delta\ell = \pm 1$.

(c) Calculate Landé g-factor for the following states : $2^2D_{3/2}, 2^2F_{5/2}$. [(3+2)+3+2]=10

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Ex/SC/PHY/UG/CORE/TH/11/2024

Bachelor of Science Examination, 2024

(3rd Year, 1st Semester)

Physics

Paper : Core/TH/11

(Quantum Mechanics and Applications)

Time : *Two Hours* *Full Marks* : 40

*Answer from **both** the Groups as directed.*
The figures in the margin indicate full marks.

GROUP—A

Answer ***any two*** of the following questions : 10×2=20

1. *(a)* Explain the idea of wave packet in one– dimension.

(b) Consider a particle moving along positive x- direction through a potential barrier of height $V_0 (E > V_0)$ in the range of $x = 0$ to l. *(i)* Write down time independent Schrödinger equations for different regions. *(ii)* Outline how you obtain the stationary states of the particle for different regions.

(c) How do you define the tunnelling coefficient for $E < V_0$? 2+2+4+2=10

2. *(a)* Consider the Hamiltonian H corresponding to a central force problem. Calculate $[L_i, H]$ in which L_i is the component of the usual orbital angular momentum operator.

(b) Write down the Hamiltonian operator for a hydrogen atom. With the help of the radial part of the wave function, how do you explain the quantization of the energy in terms of the principal quantum number? Derivation is required. Given the form of the confluent hypergeometric series as follows, ${}_1F_1(a;c;x) = \sum [(a)_\nu x^\nu]/[(c)_\nu \nu!]$ 5+5=10

3. (a) Starting from the normalised eigenfunctions of the one dimensional Harmonic Oscillator (HO), how do you get the creation and annihilation operators, $a^\dagger$ and a? Hence using suitable recurrence relations of the Hermite polynomials obtain actions of operators *(i)* $a^\dagger$ and *(ii)* a on the eigenstate, Ψ_n .

(b) What are the symmetry properties of eigenfunctions corresponding to *(i)* the ground state and *(ii)* first excited state of the HO?

(c) Write down the Hamiltonian of the HO in terms of number operator N. 6+3+1=10

GROUP—B

Answer ***any two*** of the following questions : 10×2=20

4. (a) The unperturbed ground state wave function for an infinite square well ($V(x) = \infty$, at $|x| \geq a$ and $V(x) = 0$, otherwise) is $\psi_0(x) = \sqrt{1/a}\cos(\pi x/2a)$. If the potential well is slightly perturbed by $V'(x) = V_0 \cos(\pi x/2a)$, then calculate the first order correction to the ground state energy.

(b) Show (in diagram) splittings of the energy levels corresponding to $n = 1$ and $n = 2$ of a hydrogen atom when placed in a strong external magnetic field. You may ignore effects due to the spin-orbit coupling. 5+5=10

5. (a) The lowest order relativistic correction to the Hamiltonian of Hydrogen atom is $H'_r = -\dfrac{\hat{p}^4}{8m^3c^3}$, where p is relativistic momentum. Show that the correction to energy levels E_n is $E_r^1 = -\dfrac{E_n^2}{2mc^2}\left[\dfrac{4n}{l+1/2} - 3\right]$. How many energy levels the state corresponding to $n = 2$ will be split into due to the above correction?

(b) Probability of finding a particle at time $t = 0$ follows a Gaussian function corresponding to wave function $\psi(x) = \left[\dfrac{1}{\sqrt{2\pi}b}\right]^{1/2} \exp\left[-\dfrac{x^2}{4b^2}\right]$. Show that this can be constructed by superposing waves of different momenta with amplitude

$$A(p) = \left[\sqrt{\frac{2}{\pi}}b\right]^{1/2} \exp\left[-\frac{p^2b^2}{\hbar^2}\right].$$ Further show that uncertainty in momentum (Δp) and position (Δx) of the above particle follows the relation $\Delta x \Delta p \sim \hbar$.

[(4+1)+(4+1)]=10