Master of Science, 1st Year, 1st Semester, Examination 2018 (Day)

Classical Mechanics I (PHY/TG/101)

Time - Two hours Full Marks : 40

Answer any four questions

1. (a) Define Lagrangian in terms of generalised coordinates, velocities etc. Write down the Lagrange's equations.

   (b) Define the conjugate momenta.

   (c) Show how one can use Legendre Transformation to introduce Hamiltonian, that depends only on positions and conjugate momenta; and not on the velocities $\dot{q}_i$. Clearly identify how the relevant conditions become necessary to execute this transformation.

   (d) Define cyclic coordinates.

   (e) If the system is cyclic in all the coordinates, what can be said about the solution $q_i = q_i(t)$. 1+1+4+1+3

2. Consider an inertial Lab frame and a rotating frame with common origin. A vector $\vec{r}$ is represented in the two frames as $\vec{r} = \sum_{i=1}^{3} x_i \cdot \widehat{e}_i = \sum_{i=1}^{3} x'_i \cdot \widehat{e}'_i$. Where x_i and x'_i are the components of vector $\vec{r}$ in the Lab and rotating frames respectively; $\widehat{e}_i$ and $\widehat{e}'_i$ are the instantaneous unit vectors along the axes in the abovementioned frames.

   (a) Find the equations connecting velocities as observed in the Lab and rotating frames.

   (b) Find the equations connecting accelerations in the above frames.

   (c) Show that Newton's Law as observed from the rotating frame, requires introduction of fictitious forces. Identify them. 2+3+5

3. A given mechanical system is described by a Hamiltonian $H(q_i, p_i)$ that depends on a set of canonically conjugate positions and momenta $\{q_i, p_i\}$, $i = 1, \ldots, n$.

   (a) Find the general condition for transforming to a new set of canonically conjugate positions and momenta $\{Q_i, P_i\}$, $i = 1, \ldots, n$. Clearly identify the role of the generating function of such transformations.

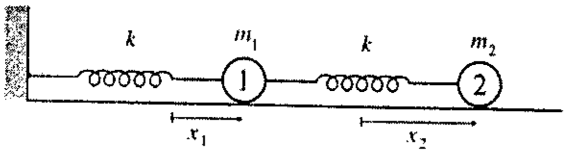
   (b) Argue how many types of such generating functions are possible.

   (c) Define Poisson Bracket $\{F, G\}_{q,p}$ of two differentiable dynamical functions $F(q_j, p_j, t)$ and $G(q_j, p_j, t)$.

   (d) Find relation connecting $\{F, G\}_{\{q,p\}}$ and $\{F, G\}_{\{Q,P\}}$, where $\{Q_i, P_i\}$ are the canonically transformed positions and momenta.

4+2+1+3

4. Two masses, m_1 and m_2 are attached to each other by a spring of spring constant k. The mass m_1 is also attached to a wall with an identical spring. The masses execute one dimensional motion on a frictionless horizontal table.



   (a) Find the Lagrange's equations for the system.

   (b) Find the normal modes (frequencies and coordinates).

   (c) Physically interpret your result in the two limits, (i) $m_1 >> m_2$ and (ii) $m_2 >> m_1$.

2+5+3

5. (a) For a rigid body moving with one point fixed, define Euler angles to describe its motion. Sketch diagram to explain the angles.

(b) Write down the rotation matrix $R_3(\psi)$ for rotation by angle ψ about z-axis, and $R_1(\theta)$ for rotation by θ about x-axis.

(c) Obtain a general rotation matrix R, corresponding to Euler rotations (ϕ, θ, ψ) (rotation about z, x' and z' axes respectively).

(d) In the final expression for R in 5.(c) above , put $\phi = \psi = 45^0$, and $\theta = 0^0$. Interpret this rotation matrix in terms of the actual rotation process. (2+2+4+2)

6. (a) Find the conservation law associated with the isotropy of space (rotational symmetry).

(b) Find a second conservation law that follows from the homogeneity of space (translational symmetry).

(c) Find the conservation law associated with the homogeneity of time. (4+3+3)