M.SC. 1ST YEAR 1ST SEMESTER 2018

QUANTUM MECHANICS I **Time: 2 hours** **Full Marks: 40**

Group A

Answer question no. 1 and any one from rest. **10 x 2 = 20**

1. i) The state of a system is described by $|\psi>$. What will be the possible result of the measurement of an observable A? Specify the normalized state of the system immediately after the measurement in terms of $|\psi>$.

   ii) Show that the wave function $\psi(\vec{r},t)$ is the component of the state vector $|\psi(t)>$ along the basis vector $|\vec{r}>$.

   iii) A set of functions labeled by a continuous index is defined by $\xi_{\vec{r}_0}(\vec{r}) = \delta(\vec{r}-\vec{r}_0)$. Can we form a basis set to describe a state of a system using them? Explain your answer with mathematical support. Is this function space a Hilbert space?

   iv) Write down the complete set of commuting observables (C.S.C.O) for the electron in hydrogen atom. How do you understand whether your chosen set is complete or not? Are the eigen-functions of this system common to all operators of the above set? b) Are they unique? Explain.

2. i) Express the energy of the electron in hydrogen atom classically in terms of radial velocity, squared of orbital angular momentum $\vec{L}^2$ and Coulomb potential.

   ii) Then, write down the Hamiltonian operator using the eigenvalue equation of $\vec{L}^2$, and show that the equation of motion of the electron reduces to the motion of a particle in one dimension under the potential $V_{eff}(r) = V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}$. Find the solution of the wave function at $r = 0$.

3. i) Write down the transformation relations from Schrodinger picture to Heisenberg picture for an operator $A_S(t)$ and a state vector $|\psi_S(t)>$, which are given in Schrodinger picture.

   ii) Obtain the time evolution equation of an operator in Heisenberg picture.

   iii) Define the state vector of the system in interaction picture. Obtain the time evolution equation of the state vector in interaction picture.

4. i) Prove the Schwarz inequality $|<\phi_1|\phi_2>|^2 \leq <\phi_1|\phi_1><\phi_2|\phi_2>$.

   ii) Prove that the first order correction to the non-degenerate energy eigenvalue is simply the mean value of the perturbation term W in the unperturbed energy eigenstate.

GROUP-B

Answer any Two Questions (2x10=20).
All parts of a question must be written in one place.

5. (a) Using the results of the Stern-Gerlach experiment of spin $\frac{1}{2}$ system, find out the following spin operators
S_x, S_y S_z and $S_\pm$ in $|S_z; \pm> = |\pm>$ basis.
(b) Find also the Eigen vectors $|S_x; ->$ and $|S_y; +>$ in the same $|S_z; \pm> = |\pm>$ basis. [(2+2+2+2)+2]

6. (a) The z-component of the spin of an electron is measured to be such that its z-component is found to be +1/2 (in the unit of $\hbar$). (i) If a subsequent measurement is made of the x-component of the spin what are the possible results? (ii) What is the probability of finding these results?
(b) Consider the spin-precession problem of a spin $\frac{1}{2}$ system with magnetic moment $(\frac{e\hbar}{2m_e c})$ (terms have their usual meaning) subjected to an external magnetic field $\vec{B}$. We take $\vec{B}$ to be a static uniform field in the z-direction. The problem can also be solved in the Heisenberg picture. Using the Hamiltonian
$H = -(\frac{eB}{m_e c})S_z = \omega S_z$ ($e < 0$ for electron).
Write down the Heisenberg equation of motion for the time dependent operators $S_x(t), S_y(t)$ and $S_z(t)$. Solve then to obtain $S_x(t)$, $S_y(t)$ and $S_z(t)$ as a function of time. [(2+2)+(3+3)]

7 (a). Using the angular momentum commutation relations $[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$ and $[J^2, J_k] = 0$, show that the maximum eigen value of J_k is either an integer or a half integer.
(b) Find the Clebsch Gordan Coefficient for $S_1 = S_2 = 1/2$. [5+5]