Ref. No. Ex/M.Sc/P/I/104/17/2018

MASTER OF SCIENCE EXAMINATION, 2018
(1^{st} year, 1^{st} Semester)
PHYSICS
Statistical Mechanics- I
Paper - PHY/TG/104

Time : Two hours Full Marks: 40

Answer any four questions.

1. (a) Define Boltzmann H function. Show that, for an isolated system which is not likely to be found in any one of its accessible microstates, $\frac{\partial H}{\partial t} < 0$ and hence state the condition for an equilibrium state. 5 + 1

   (b) Establish the general expression for Boltzmann transport equation 4

2. (a) Define density matrix. Show that the dynamics of the density matrix is governed by the quantum mechanical analog of the Liouville's theorem. 4

   (b) Write down the density matrix representation of canonical ensemble and hence derive the expression for expectation value of a dynamical variable. discuss the characteristics of density matrix for an equilibrium ensemble. 2 + 2 + 2

3. (a) Derive the configurational partition function of an non-ideal van der Waal gas. Use mean field interaction potential described by the following equation.

$$\begin{aligned} u(r) &= \infty, \text{ for } 0 \leq V_m \leq V_{ex} \\ &= -u, \text{ for } V_{ex} < V_m \leq V \end{aligned}$$

   Where, V_{ex} is the excluded volume and V is the size of the system. 5

   (b) The universal equation of state for a van der Waal gas is given by

$$(P_r + \frac{3}{v_r})(3v_r - 1) = 8T_r.$$

   where, $P_r = \frac{P}{P_C}$, $v_r = \frac{v}{v_C}$ and $T_r = \frac{T}{T_C}$. Determine the behaviour of isothermal compressibility near the critical point. 5

4. (a) Write down the Hamiltonian of the Ising system assuming the nearest neighbour interaction between Ising variables. Derive the expression for configurational energy of the system and determine the phase transition temperature of this system in the mean field approximation. 1 + 3 + 3

   (b) Establish the critical behaviour of the order parameter in terms of critical exponent in the above system. 3

5. (a) The partition function of an ideal gas of N particle is given by

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}{}' g\{n_\epsilon\} e^{-\beta \sum_\epsilon n_\epsilon \epsilon}$$

where $g\{n_\epsilon\}$ is the statistical weight factor appropriate to the distribution set $\{n_\epsilon\}$. Use single particle states as individual states, calculate the grand canonical partition function for Bose Einstein statistics. Hence calculate the mean occupation number $< n_\epsilon >$. where, symbols have their usual meaning. 3 + 3

(b) Calculate the internal energy (U) of the Bose system. Show that the Bose system, in general, satisfies the relationship $P = \frac{2}{3}(\frac{U}{V})$. 4

6. (a) Write down the Landau free energy in the presence of ordering field. Determine the critical exponent associated with the susceptibility of the system. 5.

(b) Explain the origin of Landau levels in the context of Landau diamagnetism. 3

(c) What is De Haas-Van Alphen effect? 2