JADAVPUR UNIVERSITY
MASTER OF SCIENCE EXAMINATION, 2018.
(2nd Year, 1st Semester)

Subject: PHYSICS
Paper: QUANTUM MECHANICS - III
PHY / TG / 113
(Ref. No. *Ex/M.Sc/P/II/113/20/2018*)

Time: Two Hours
Full Marks: 40

Answer any TWO questions

1. (a) Show that the average of the square of the distance between two non-interacting quantum particles is statistically lesser for symmetric wave-functions than antisymmetric ones.

   (b) Construct all the possible full (space part along with spin part) wavefunctions involving two spin-half particles.

   (c) For a Helium atom, show analytically that triplet states are lower in energy than the singlet state.

   Marks: 6 + 4 + 10 = 20

2. (a) A hydrogen atom is placed in a uniform magnetic field $\vec{B} = B\hat{z}$. Show that the part of the Hamiltonian that consists of (i) internal spin-orbit coupling and (ii) coupling of the orbital and spin momenta with $\vec{B}$, commutes with J_z (which is the z-component of the total angular momentum).

   (b) Calculate the matrix elements of this part of the Hamiltonian in a suitably chosen basis.

   (c) Find the corrections to energy to first order in perturbation theory and show the strong-field and weak-field limits.

   Marks: 4 + 6 + 10 = 20

3. (a) Consider a spherical potential barrier with radius r_0. Particles are impinging on it with momentum $\hbar k$ and getting scattered. What can you say about a possible upper bound in the value of l of a partial wave that suffers a significant phase shift ?

(b) Starting from the radial equation satisfied by the l-th radial function R_l, for a particle of mass m and energy E, scattered in a three-dimensional central potential $V(r)$ of short-range, show that you can write the solution of this equation for regions far away from the scattering centre as $R_l(kr) = \frac{A_l(\delta_l)}{kr} \sin\left(kr - \frac{l\pi}{2} + \delta_l\right)$, where δ_l is the scattering phase shift.

(c) Imagining that the scatterer is placed at the origin and a plane wave incident from the negative z-direction on the scatterer, we can write the azimuthally symmetric scattered wave function as

$$\psi(r,\theta) = e^{ikz} + f(k,\theta)\frac{e^{ikr}}{r}.$$

In this equation invoke the partial wave expansions $\psi(r,\theta) = \sum_{l=0}^{\infty} R_l(k,r) P_l(\cos\theta)$ and $f(k,\theta) = \sum_{l=0}^{\infty} f_l(k) P_l(\cos\theta)$ to establish that $A_l(\delta_l)\sin\delta_l = i^l k f_l(k)$.

(d) Hence show that you can measure the total scattering cross-section just from the knowledge of the imaginary part of the function $f(k,\theta)$ at $\theta = 0$.

Marks: 4 + 4 + 6 + 6 = 20