Ref. No.: Ex/M.Sc./P/II/112/20/2018

MASTER OF SCIENCE EXAMINATION 2018

(2nd Year, 1st Semester)

PHYSICS

SOLID STATE PHYSICS AND X-RAY

PHY/TG/112

Time: Two hours **Full marks: 40**

Answer any FOUR questions.

1. (a) Consider the classical scattering of unpolarized X-rays by a single free electron and obtain an exprèssion for the scattered intensity at a given point P. [7]

   (b) Explain using a clean diagram as to why base centered tetragonal lattice does not exist. [3]

2. (a) Explain what is the point group symmetry for crystals? [1]

   (b) What are the advantages of stereographic projection over spherical projection? [2]

   (c) Draw the stereographic projections of the following point groups, (i) $2mm$, (ii) $4/mmm$, (iii) 4. [4.5]

   (d) Explain with a neat diagram how a third mutually perpendicular 2-fold axis is automatically generated when two 2-fold axes are placed perpendicular to each other. [2.5]

3. (a) Show that the inter-planar separation of the lattice planes (hkl), satisfies the relation

$$d_{hkl} = \frac{2\pi}{G_{hkl}},$$

   where G_{hkl} is the magnitude of shortest reciprocal Bravais vector normal to the lattice plane (hkl), *i. e.*, $\vec{G}_{hkl} = h\,\vec{b}_1 + k\,\vec{b}_2 + l\,\vec{b}_3$. $\vec{b}_i$ are the primitive vectors of reciprocal lattice. [4]

   (b) Derive the von Laue equation for X-ray diffraction and hence obtain the Bragg's law from it. [4+2=6]

4. (a) State and prove the Bloch's theorem. [1+2=3]

   (b) Show that the equation of motion of the Block electron in one-dimensional periodic potential $V(x+a) = V(x)$ is $\hbar\frac{dk}{dt} = F(t)$, where F is the external force. [4]

   (c) Show that the wave vector of a hole originated due to the missing of an electron from a otherwise filled band is given by $\vec{k}_h = -\vec{k}_e$, where $\vec{k}_e$ is the wave vector of the missing eletron. But velocity of the hole is the same to that of the missing lectron. [2+1=3]

5. (a) Prove the Sommerfeld expansion,

$$\int_0^\infty f(E)\,\frac{\partial F}{\partial E}\,dE = F(E_F) + \frac{\pi^2}{6}\,(k_B T)^2 \left(\frac{\partial^2 F}{\partial E^2}\right)_{E_F},$$

where $f(E)$ is the Fermi-Dirac distribution function and $F(E)$ is a regular function and such that $F(0) = 0$. [4]

(b) By using the density of states, $g(E) = \frac{3}{2}n(E_F(0))^{-\frac{3}{2}}\sqrt{E}$, and Fermi level

$$E_F(T) \approx E_F(0)\left[1 - \frac{\pi^2}{12}\left(\frac{K_B T}{E_F(0)}\right)^2\right],$$

for the three-dimensional free electron systems, derive the expression of Pauli paramagnetic susceptibility at finite temperature, $\chi_P(T)$. Symbols have their usual meanings.

[6]

6. (a) Consider a thermodynamic system of fixed volume under the magnetic field $\vec{H}$ at finite temperature T. Show that in equilibrium, the entropy, S and the magnetic induction, $\vec{B}$ of this system can be obtained by using the expressions,

$$S = -\left(\frac{\partial G}{\partial T}\right)_{\vec{H}}, \quad \vec{B} = -\left(\frac{\partial G}{\partial \vec{H}}\right)_T,$$

respectively, where G is the Gibbs potential. [2]

(b) Show that the Gibbs potential for the superconducting state, G_s is related to that of the normal state, G_n by the equation,

$$G_s(T, \vec{H}) = G_n(T, \vec{H}) + \frac{1}{2}\mu_0\left(H^2 - H_c^2(T)\right).$$

[3]

(c) Explain that the superconducting state to normal metallic state transition is second order at the points $T = 0$ and $T = T_c$, and otherwise first order, since the critical field of transition is

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right].$$

[2]

(d) Find the expression for specific heat difference between normal and superconducting states and obtain the Rutgers formula. [3]