

M. Sc. BIO-TECHNOLOGY PART I EXAMINATION, 2018**MATHEMATICS & BIostatISTICS****PAPER - MSBT 1/6**

Time : Four hours

Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

GROUP - AAnswer **any five** questions.

10×5=50

1. a) Prove that (without expanding)

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3 \quad 6$$

b) Show that (without expanding)

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \mu \\ \beta\mu & \mu\alpha & \alpha\beta \end{vmatrix} = (\alpha - \beta)(\beta - \mu)(\mu - \alpha) \quad 4$$

2. a) Show that the transpose of an orthogonal matrix is also orthogonal. 3b) If $A = 2$ and $\text{adj}A =$

$$\begin{bmatrix} -2 & 3 & 1 \\ 6 & -8 & -2 \\ -4 & 7 & 1 \end{bmatrix} \quad \text{then find the matrix } A \quad 7$$

[Turn over

- ii) Can we conclude that there is a significant recovery of the population because of the fertilizer serum uses at significance level ; of (a) 0.01 and (b) 0.05 ? 4+4
- iii) Mention the appropriate hypothesis test for this set of data and why ? 2
- iv) Considering the results of the hypothesis test, decide which of the Type I or Type II errors is possible, and describe this error. 2

Neatness - 2

3. Find eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & - \end{bmatrix} \quad 3+7$$

4. a) Solve by Cramers' rule the following equations

$$x + y + z = 4; \quad 2x - y + 3z = 1; \quad 3x + 2y - z = 1 \quad 7$$

- b) If A be a square complex matrix, then show that $(A + A^*)$ is a Hermitian matrix and $(A - A^*)$ is a skew-Hermitian matrix. 3

5. a) Verify Cayley – Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{bmatrix} \quad 7$$

- b) Show that

$$\begin{vmatrix} y+z & z+x & x+y \\ z+x & x+y & y+z \\ x+y & y+z & z+x \end{vmatrix} = 2 \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} \quad 3$$

6. a) Solve the following system of linear equations by matrix method

$$\begin{aligned} x + 2y + z &= 4 \\ x - y + z &= 5 \\ 2x + 3y - z &= 1 \end{aligned} \quad 7$$

15. At an agricultural station it was desired to test the effects of a given fertilizer on wheat production. To accomplish this, 24 plots of land having equal areas were chosen ; half of these were treated with the fertilizer and the other half were untreated (control group). Otherwise the conditions were the same. The mean yield of wheat on the untreated plots was 4.8 liters with standard deviation of 4 liters, while the mean yield on the treated plots was 5.1 liters with standard deviation of 3.6 litres. (Given if $t_{0.005}$ and $t_{0.025}$ is 2.51 and 1.72 respectively when df is 22).

- i) Can we conclude that there is a significant improvement in wheat production because of the fertilizer if a significance level ; of (a) 1% and (b) 5% is used ? 4+4
- ii) State whether this is appropriate for two tail-test. 2
- iii) Define Type I and Type II errors in hypothesis test. Considering the results of above hypothesis test, decide which of the Type I or Type II errors is possible, and describe this error. 3+3

16. Under the null hypothesis H_0 that the serum has no effect, we would expect 70 people in each group to recover and 30 in each group not to recover.

- i) Construct the appropriate contingency table for this experiment. 4

[6]

**Female students : 65, 50, 55, 18, 53, 41, 50, 34, 67, 56,
44, 4, 54, 57, 39, 52, 45, 35, 51, 63, 42**

- i) Complete the construction of figure to obtain a stem-and-leaf display for males and a stem-and-leaf display for females.
- ii) Find the five-number summary for males and the five-number summary for females.
- iii) Construct a box plot for males and a box plot for females.
- iv) Does the distribution of marks contain outliers for males and females ? If yes, Find the outliers and construct a box plot for them.

4+4+4+4

14. Answer **both** questions :

- i) Two students were informed that they received standard scores of 0.8 and -0.4 respectively in the bioinformatics exam. If their marks were 88 and 64 respectively, find the mean and standard deviation of the examination marks.
- ii) The mean length of 500 cells is 151 mm and standard deviation is 15 mm. Assuming that the length of that cells are normally distributed, find how many leaves measured (a) between 120 and 155 mm, (b) more than 185 mm and (c) 128 mm.

4+4+4

[3]

b) Show that $B^T A^T = (AB)^T$ where the matrices are

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} \quad 3$$

7. Diagonalise the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \quad 10$$

8. a) Define Fourier transform of a function $f(x)$.

Find the Fourier cosine transform of

$$f(x) = x \text{ for } 0 < x < 1$$

$$2 - x \text{ for } 1 < x < 2$$

$$0 \text{ for } x > 2 \quad 2+4$$

b) What do you mean by Vector Field ? State Gauss' divergence theorem. 1+3

9. a) Solve the following differential equations (**any two**) 5×2

$$i) \quad xdy - ydx + x^2dx = 0$$

$$ii) \quad (1 - x^2) \frac{dy}{dx} - 2xy = x - x^3$$

[Turn over

[4]

iii) $D^2y - 3Dy + 2y = x^3 \left(D = \frac{d}{dx} \right)$

(Use method of undetermined coefficients)

10. Solve the following differential equations (**any two**) 5×2

a) $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$

b) $D^2y - 6Dy + 9y = x^4 e^{2x}$

c) $(D^2 + 4)y = \sin 2x$

11. a) Evaluate the integral $I = \int_0^1 e^{-x} \sin 4x \, dx$ by Simpson's one third rule, taking $N = 8$ 5

b) Compute $\int_1^4 \frac{\ln(1 + 0.5x + x^2)}{0.5 + x} \, dx$ using Trapezoidal rule taking $N = 6$. 5

[5]

GROUP - B

Answer **any three** questions.

12. The frequency distribution of the variable X_i are given follows:

Values of X_i	Frequency (f_i)
5	10 (f_1)
7	30 (f_2)
9	- (f_3)
11	- (f_4)

- The mean ($\bar{X}$) of the above distribution is 8.4. The total number of observations is 100. Fill up the gaps marked (–) for the value of f_3 and f_4 .
- Find the value of the Mean Absolute Deviation of the above distribution.
- Find the value of the standard deviation (σ) of the above distribution. 8+4+4

13. The results of 21 male and 21 female students' biostatistics examination are recorded below :

Male students : 30, 31, 49, 19, 62, 50, 24, 45, 23, 51, 32, 48, 55, 60, 40, 35, 54, 26, 57, 37, 43,

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