

M. Sc (BIOTECHNOLOGY) Part – I EXAMINATION , 2017

Mathematics & Biostatistics

Paper - 1 / 6

Time - Four hours

Full Marks - 100

Use a separate Answer script for each group

GROUP - A

Answer any five questions

10 x 5 = 50

1 (a) Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$ 6

(b) Prove , without expanding that

$$\begin{vmatrix} a & d & 3a - 4d \\ b & e & 3d - 4e \\ c & f & 3c - 4f \end{vmatrix} = 0$$
 4

2 (a) If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(2I + 3E)^3 = 8I + 36E$ 4

(b) Verify $B^T A^T = (AB)^T$ where ,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$
 4

(c) When two matrices are said to be equal ? 2

[Turn over

3 (a) Show that the matrix $A = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \end{bmatrix}$ is orthogonal.

Hence find A^{-1} .

4

(b) Find the eigenvalues and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

6

4 (a) Determine the inverse of the matrix A using Cayley - Hamilton theorem

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

6

(b) Without expanding find the value of the determinant $\begin{vmatrix} 7 & 12 & -3 \\ 9 & 14 & -1 \\ 8 & 13 & -2 \end{vmatrix}$

4

5 (a) Solve the following system of linear equations by Matrix method

$$x + 2y + z = 4$$

$$x - y + z = 5$$

$$2x + 3y - z = 1$$

5

(b) Solve the following system of linear equations using Cramer's rule

$$X + 2y - z = -3$$

$$3X + y + z = 4$$

$$X - y + 2z = 6$$

5

6. Diagonalise the following matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$$

10

7. Solve the following differential equations (any two)

2 x 5

(a) $y \frac{dy}{dx} + xy^2 = x$

(b) $x \frac{dy}{dx} + 2y = 12x$

(c) $x(x^2 + 2y^2)dx + y(2x^2 + y^2)dy = 0$

(d) $\frac{dy}{dx} - xy = x^3 y^2$

8 (a) Solve the following differential equations by the method of undetermined co-efficient

$$d^2y/dx^2 + 4y = x^2 \quad 5$$

(b) Solve the following differential equation (any one) 5

(i) $(D^2 - D - 2)y = \sin 2x$ (D = d/dx)

(ii) $(D^2 + 3D + 2)y = 3e^{2x}$

9 (a) State Gauss' Divergence theorem 2

(b) Evaluate $\int_0^\pi (1 + \cos x)^3 dx$ (N = 6) , using Trapezoidal formula 4

(c) Use Euler's method with $h = 0.2$ to find the solution of

$$\frac{dy}{dx} = \frac{(y-x)}{x+y} \text{ at } x=1. \text{ Here } y(0)=1 \text{ at } x=0 \quad 4$$

10 (a) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases} \quad 4$$

(b) Evaluate $\int_0^1 \frac{1}{(1+x^2)} \log_e (1 + x^2) dx$, taking 12 intervals by Simpson's $\frac{1}{3}$ rd rule

M. Sc. Bio-Technology Part I Examination, 2017

MATHEMATICS & BIOSTATISTICS

PAPER- 1/6

Full Marks: 50

GROUP-B

Answer any three questions

- 1f. a) What are the characteristic features of Normal or Binomial distribution in statistics?
 b) The mean length of 500 different cells under the microscopy is 151 mm and the Standard deviation is 15 mm. Assuming that the length are normally distributed, find how many cells measured (i) between 120 and 155 mm, (ii) more than 185 mm.
- 6 + 5+5
- (2. Voters in a particular state are surveyed. Each respondent is assigned an identification number, and information about each of the following is recorded: sex of the voter and age

Sex	Age
M	35
F	20
F	35
M	41
M	39
M	59
F	20
M	52
F	44
M	46
F	40
F	34
F	24
M	62
M	44

- (a) Complete the construction of figure to obtain a stem-and-leaf display for males and a stem-and-leaf display for females.
 (b) Obtain an ordered array for males and an ordered array for females.
 (c) Find the five-number summary for males and the five-number summary for females.
 (d) Construct a box plot for males and a box plot for females.
 (e) Does the distribution of "Age" appear to be centred at a different value for males and females? If yes, for which sex does the centred of the distribution appear to be greater?

4+2+4+4+2

13. A principal of a private school claims that the students in his class are above average intelligence. A random sample of nine students IQ scores are: 89, 99, 105, 116, 116, 118, 119, 125 and 128. Is there sufficient evidence to support the principal's claim? The average IQ of all the students of this school is 100 with a standard deviation of 15. (Given $t_{8, 0.05} = 1.860$ and $t_{8, 0.01} = 2.896$)
- Explain how the data for this hypothesis test is appropriate for a Student's t test.
 - Complete the four steps of the hypothesis test
 - Decide whether H_0 would have been rejected or accepted with the following significance level: (i) $\alpha = 0.01$, (ii) $\alpha = 0.05$.
 - Considering the result of the hypothesis test, decide which of the Type I or Type II errors is possible and explain.

2 + 8 + 4 + 2

14. The satellite television company gathers data from customers concerning area of the residence and choice of favorite entertainment channel; results are displayed in the contingency table titled "Contingency table for Residence and Favorite channel"

Table 1: Contingency table for Residence and Favorite channel

Residence	Favorite channel			
	Cartoon	Comedy	Sci Fi	Western
Rural	76	50	34	40
Suburban	114	75	51	60
Urban	190	125	85	100

- Complete the four steps of the hypothesis test by completing the table
- Decide whether H_0 would have been rejected or would not have been rejected with each of the following significance levels: (i) $\alpha = 0.01$, (ii) $\alpha = 0.10$. (Given $Z_{0.01} = 1.28$ and $Z_{0.1} = 2.33$)

8 + 4 + 4

15. The score in the subject Biostatistics of 10 Republicans in a class XI is recorded as follows: 34, 55, 53, 45, 30, 29, 33, 61, 41 and 64 .
- What is the mean and the median of the score.
 - Find the variance and standard deviation of the score for the students.
 - Find the skewness ratio for the incomes of the students, and indicate what this suggests about the shape of the distribution.
 - How could the 10 original score be altered so that the mean remains the same, but the standard deviation is larger?
 - How could the 10 original score be altered so that the mean remains the same, but the standard deviation is smaller?

2+4+4+3+3

Neatness = 2 Marks