

B.E. ELECTRICAL ENGINEERING FOURTH YEAR SECOND SEMESTER - 2025

ADVANCED CONTROL THEORY

Time: 3 Hours
Full Marks: 100

Answer both parts on the same answer script

Part-I

In this part

Answer Question no. 1 which is compulsory

AND

Answer any one question from Question nos. 2 and 3

AND

Any one question from Question nos. 4 and 5

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|-------|--|-----|------|
| 1. a) | Define the term “static nonlinearity”. Give two examples of static nonlinearity. | CO1 | [4] |
| b) | Correct or justify the statement “ <i>Saturation is an example of static nonlinearity</i> ”. | CO2 | [2] |
| c) | Distinguish between the terms “ <i>backlash</i> ” and “ <i>hysteresis</i> ”. | CO2 | [4] |
| d) | A nonlinear system is expressed as follows: | CO4 | [10] |
| | $\dot{x}_1 = -x_1 + x_2(1 + x_1)$ $\dot{x}_2 = -x_1(1 + x_1)$ | | |
| i. | Determine the equilibrium point(s) of the above system. | | |
| ii. | Linearize the above system about its equilibrium point at the origin. | | |
| iii. | Comment about the asymptotic stability of the system at $x=0$. | | |

[Turn over

2. a) State and Explain Lyapunov's 2nd theorem. CO2 [5]
 b) The dynamics of an unforced nonlinear system is described by CO4 [10]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - x_1(x_1^2 + x_2^2) \\ -x_1 - x_2(x_1^2 + x_2^2) \end{bmatrix}$$

Using the function $V = \frac{1}{2}(x_1^2 + x_2^2)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

OR

3. a) Given a second order system $G(s) = \frac{100}{s^2 + 100}$. Explain how the method of CO2 [5]
 isoclines may be used to sketch the phase plane plot of this system. Hence sketch the phase plane plot for the above system.
 b) A satellite attitude control system has forward-reverse type of thrusters with a dead zone and a proportional plus derivative controller. With the help of a CO4 [10]
 phase plane plot, investigate the stability of the system assuming standard notations for the parameters and variables.
4. a) An on-off type controller is used for temperature control of an oven. Develop CO3 [7]
 expressions for the time response of the above system considering a first order plant with delay.
 b) Derive approximate expressions for on-time, off-time, duty cycle and CO4 [8]
 maximum level.

OR

5. a) The output of an ideal (without hysteresis) bi-directional relay element is as CO3 [7]
 follows:

$$y(\omega t) = \begin{cases} +1; & \forall 0 < \omega t < \pi \\ -1; & \forall \pi < \omega t \leq 2\pi \end{cases}$$

Find the describing function for the above relay element.

- b) A closed loop unity feedback control system has the relay element described CO4 [8]
 in 5(a) above, along with a linear process with transfer function

$$G(s) = \frac{1}{s(0.5s + 1)^2} \text{ in the forward path. Investigate the stability of the}$$

closed loop nonlinear control system by using the describing function obtained in 5(a) and comment about the presence of limit cycle, if any, on the basis of the above analysis.

Part II**Answer any one question from Question nos. 6 and 7****AND****Any one question from Question nos. 8 and 9****AND****Question no. 10 which is compulsory**

6. a) Distinguish between the terms '*Additive Unstructured Uncertainty*' and '*Multiplicative Unstructured Uncertainty*'. **CO2 [4]**
- b) Analyze the robust stability of the system with the characteristic polynomial **CO4 [12]**
- $$p_6 s^6 + p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0,$$
- where
- $p_6 \in [1,1], p_5 \in [8,10], p_4 \in [2,3], p_3 \in [5,11], p_2 \in [6,8], p_1 \in [4,7]$ and $p_0 \in [1,5]$.

OR

7. a) Distinguish between the terms '*Stability Robustness*' and '*Performance Robustness*'. **CO2 [4]**
- b) A process plant given by $G_1(s) = \frac{10}{(s+1)(0.005s+1)}$ is modeled by **CO4 [12]**
- using the transfer function $G_2(s) = \frac{10}{s+1}$.

Analyze the suitability of using the above model for the given process plant on the basis of (i) the open loop unit step responses, (ii) the frequency responses (Bode plots) of the plant and its model.

[Turn over

8. a) What is an observer? Explain why a reduced order observer is used. CO2 [4]
 b) Design a reduced order observer for observing the second and third state variables of the following continuous time system so that the observer poles are located at $-10 \pm j10$ CO5 [12]

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u; \quad y = [1 \quad 0 \quad 0] \mathbf{x}.$$

OR

9. a) Explain the physical significance of the term “quadratic performance index”. CO2 [2]
 b) With the help of an example, explain what is meant by “minimum energy control problem”. CO2 [2]
 c) Design an optimal controller for the following linear quadratic regulator by finding CO5 [12]
 (i) the Ricatti matrix P, and
 (ii) the optimal control law.

The plant is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and has the performance index

$$J = \int_0^{\infty} \left\{ \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + u^2 \right\} dt.$$

- 10 a) For a system with plant transfer function P(s) and controller transfer function C(s), show that the largest value of complementary sensitivity is CO3 [6]

$$\text{given by } M_t = \left\| \frac{PC}{1+PC} \right\|_{\infty}.$$

- b) Given a transfer function

$$G(s) = \frac{12}{(s+1)(s+2)^2(s+3)}. \quad \text{CO4 [6]}$$

Determine $\|G\|_2$.

- c) For the system with transfer function $G(s) = \frac{s+10}{s+2}$, determine $\|G\|_{\infty}$. CO4 [6]