SOLITONS AND DOUBLE LAYERS
IN
DIFFERENT TYPES OF PLASMA

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Dedicated To

My Beloved Mother

Late Dr.(MRS.) Anjali Paul

CERTIFICATE FROM THE SUPERVISOR(S)

This is to certify that the thesis entitled- **"Solitons and Double Layers in Different Types of Plasma"**- submitted by **Smt. Indrani Paul** who got her name registered on 7-10-2013 for the award of PhD (Science) degree of Jadavpur University is absolutely based on her own works under the supervision (s) of **Dr Basudev Ghosh**, Professor, Department of Physics, Jadavpur University, Kolkata and **Dr Chandra Das**, Associate Professor, Department of Physics, Uluberia College, presently at Netaji Nagar Day College, Kolkata and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.

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CONTENTS

CHAPTER- 1

GENERAL INTRODUCTION

1.1.INTRODUCTION

Plasma is an ionized gas consisting of equal numbers of positively charged ions and negatively charged electrons, which shows some sort of collective behaviour. The characteristics of plasmas are significantly different from those of ordinary neutral gases so that plasmas are considered as a distinct state of matter commonly called "fourth state of matter." In addition to externally imposed fields, the plasma is acted upon by electric and magnetic fields created within the plasma itself through localized charge concentrations and electric currents that result from the differential motion of the ions and electrons. It is estimated that about 99% of the matter in the observable universe is in the plasma state. Stars, stellar and extragalactic jets, and the interstellar medium are examples of astrophysical plasmas. In our solar system, the Sun, the interplanetary medium, the magnetospheres and/or ionospheres of the Earth and the comets all consist of plasmas. Again the plasma can also be produced in the laboratory. In recent years, the interest in plasma has grown primarily because of its many applications in electrical and aeronautical engineering, astrophysics and space science and, above all, in the possible development of thermonuclear power generation. Plasma physics is one of the more advanced disciplines of classical physics, drawing its methodology from diverse fields such as statistical mechanics, classical electricity and Magnetism, hydrodynamics, atomic physics, and the kinetic theory of gases.

1.2. DIFFERENT TYPES OF PLASMA

The plasma may be composed of i) electrons and positive ions, ii) electrons, positrons and positive ions, iii) electrons, positrons, positive ion and negative ions, iv) electrons, positive ions and negatively charged dust particles, v) electrons, positrons, positive ions and negatively charged dust particles. Moreover, the plasma particles may be non-relativistic and relativistic depending on the physical situations.

In general, the plasma is considered as non-degenerate for both laboratory and space. But, under some special situations, the plasma is considered as degenerate. In non-degenerate plasma the electrons are considered as i) Maxwellian (isothermal) and ii) Non-Maxwellian e.g. non-isothermal, nonthermal, super thermal (kappa distributed), Q-nonextensive. But, in degenerate plasma the ionized particles are assumed to follow F.D. statistics**.**

i). Electron-Ion Plasma

When matter becomes sufficiently hot, it becomes ionized and forms plasma. This process breaks matter into its constituent particles which includes negatively charged electrons and positively charged ions. Fully ionized plasma has only electrons and ions. It is called electron-ion plasma. The electron density is Maxwellian and given by

$$n_e = n_{e0} \exp(e\varphi / K_B T_e)$$

Where ne0 is the equilibrium electron density, φ is electrostatic potential, K_B is the Boltzmann constant, T_e is the temperature of electrons and e is the charge of an electron which is negative.

In non-dimensional form

$n_e = n_{e0} \exp(\bar{\varphi})$, where $\bar{\varphi} = e\varphi / K_B T_e$

ii). Electron-Ion-Positron Plasma

The plasma may be composed of electrons, ions and positrons. Such plasmas have been observed in the polar regions of neutron stars, in pulsar magnetospheres, in active galactic nuclei, as well as in the intense laser fields. Positron density is assumed to be Maxwellian,

$$n_p = n_{p0} \exp(-e\varphi / K_B T_p)$$

where, n_{p0} is equilibrium positron density, T_p is the temperature of positrons and φ is electro-static potential.

In non-dimensional form

$$n_p = \chi \exp(-\sigma_p \bar{\varphi})$$

where, $\bar{\varphi} = e\varphi / T_e, \sigma_p = T_e / T_p, \chi = n_{p0} / n_{e0}$, T_e and T_p are temperature of electrons and positrons (For simplicity the bar over $\bar{\varphi}$ will be omitted subsequently).

iii). Negative Ion Plasma

In plasma, both positive ions and negative ions together with electrons may be present and their collective behaviour is interesting. When free electrons are attached to neutral particles, negative ions are produced in the plasma. Then negative ions are present in the earth ionosphere and in space. It can be produced in the laboratory. Different types of positive ions and negative ions, e.g. (H^+, O^-) ions, (H^+,O_2^-) ions, (H^+,SF_5^-) ions ,(He^+, Cl^-) ions, (Ar^+,O^-) ions are found to exist.

iv). Dusty Plasma

The dusty plasma is composed of electrons, ions and negatively charged tiny dust particles. Dusty plasmas are found in various plasma media e.g. cometary tails, asteroid belts, planetary ring system, magnetosphere, the lower part of the earth's ionospheres and do expect in various laboratory plasmas too. In dusty plasma, the dust particles get negatively charged due to the attachment of the background electrons and ions on the surface via collisions.

v). Quantum (Degenerate) Plasma

Quantum effects in plasma may become important in a variety of environments such as in intense laser-solid density plasma interaction experiments, in dense astrophysical and cosmological environments, in ultra-small electronic devices and metal nanostructures. Quantum plasma is be found in a variety of environments such as metal nanostructures, astrophysical systems, ultra-small electronic devices, bio-photonics, cool vibes, neutron stars, laser-produced plasmas, quantum wells and quantum diodes. Quantum plasma is a relatively new and fast-developing field of research. The physics of quantum plasmas is now a very rapidly growing subject of plasma physics.

vi). Relativistic Plasma

In some astrophysical plasma, it has been found that plasma particles are ejected with high velocities. For the dynamics of such plasma particles, the mass variation due to the relativistic effect should be considered.

The mass variation of the plasma species (electrons and ions) due to the motion of plasma particles is

$$m = m_0 / (1 - v^2 / c^2)^{1/2}$$

Where m_0 is the rest mass; v is the velocity of the particles and $\boldsymbol{c}$ is the velocity of light.

1.3. DIFFERENT TYPES OF ELECTRONS IN PLASMA

To study the propagation of waves in plasma the electrons are considered to have different types of distribution.

i). **Isothermal Electrons**

The distribution of electrons is assumed Maxwell- Boltzmann distribution,

$$n_e = n_{e0}\exp(\bar{\varphi}) \equiv n_{e0}\exp(\varphi)$$

where, n_{e0} is the equilibrium density of electrons.

ii). **Non-Isothermal Electrons**

When the electrons are trapped, electron distribution is called non-isothermal and is assumed as

$$n_e = n_{e0}[1+\varphi-\frac{4}{3}b_1\varphi^{3/2}+\frac{1}{2}\varphi^2-\frac{8}{15}b_2\varphi^{5/2}+\frac{1}{6}\varphi^3+\cdots\cdots]$$

where, $b_1 = \frac{1-\bar{\beta}_1}{\sqrt{\pi}}, b_2 = \frac{1-\bar{\beta}_1^2}{\sqrt{\pi}}, \bar{\beta}_1 = \frac{T_{ef}}{T_{et}}$, T_{ef} is temperature of free electrons and T_{et} is the temperature of trapped electrons.

iii). Two-Temperature Isothermal Electron

The electrons in plasma may exist at different states with two different temperatures obeying the Maxwell-Boltzmann distribution. The electron densities at low and high temperatures are described as

$$n_{ec} = \mu\exp(\varphi/\mu+\nu\bar{\beta})$$
$$n_{eh} = \nu\exp(\bar{\beta}\varphi/\mu+\nu\bar{\beta})$$

where, $\mu = n_{ec0}$ and $\nu = n_{eh0}$ are the equilibrium density of electrons at low and high temperature; $\bar{\beta} = T_{el}/T_{eh}$; T_{el} and T_{eh} are the temperature of electrons at low and high temperatures.

iv). Two-Temperature Non-Isothermal Electron

Due to the nonisothermality of two groups of electrons, the densities of cold and hot electrons can be assumed as:

$$n_{el} = \mu\left[\exp\left[\frac{T_{ef}}{T_{el}}\varphi\right]\mathrm{erfc}\left[\frac{T_{ef}}{T_{el}}\varphi\right]^{1/2} + \bar{\beta}_l^{-1/2}\exp\bar{\beta}_l\left[\frac{T_{ef}}{T_{el}}\varphi\right]\mathrm{erfc}\left[\bar{\beta}_l\frac{T_{ef}}{T_{el}}\varphi\right]^{1/2}\right],$$

$$n_{eh} = \nu\left[\exp\left[\frac{T_{ef}}{T_{eh}}\varphi\right]\mathrm{erfc}\left[\frac{T_{ef}}{T_{eh}}\varphi\right]^{1/2} + \bar{\beta}_h^{-1/2}\exp\bar{\beta}_h\left[\frac{T_{ef}}{T_{el}}\varphi\right]\mathrm{erfc}\left[\bar{\beta}_h\frac{T_{ef}}{T_{eh}}\varphi\right]^{1/2}\right],$$

where $\bar{\beta}_l$ and $\bar{\beta}_h$ are the ratios of the temperatures of free and trapped electrons in the low- and high-temperature groups of electrons.

These parameters are expressed in the following manner:

$$\bar{\beta}_l = \frac{T_{el}}{T_{elt}},\ \bar{\beta}_h = \frac{T_{eh}}{T_{eht}},$$

where T_{elt} and T_{eht} are the temperatures of the trapped electrons in the low- and high-temperature groups of electrons T_{el}, T_{eh} are the same for free electrons.

Expanding the expressions of n_{el} and n_{eh} in ascending powers of ϕ we obtain the expressions for cold and hot electrons as

$$n_{el} = \mu\left[1+\left[\frac{\varphi}{\mu+\nu\bar{\beta}}\right]-\frac{4}{3\pi^{1/2}}\left(1-\bar{\beta}_l\right)\left[\frac{\varphi}{\mu+\nu\bar{\beta}}\right]^{3/2}+\frac{1}{2}\left[\frac{\varphi}{\mu+\nu\bar{\beta}}\right]^{2}\right.$$

$$\left.-\frac{8\left(1-\beta_l^2\right)}{15\pi^{1/2}}\left[\frac{\varphi}{\mu+\nu\bar{\beta}}\right]^{5/2}+\frac{1}{6}\left[\frac{\varphi}{\mu+\nu\bar{\beta}}\right]^{5}+\ldots\right],$$

$$n_{eh} = \nu\left[1+\left[\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}}\right]-\frac{4}{3\pi^{1/2}}\left(1-\bar{\beta}_h\right)\left[\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}}\right]^{3/2}+\frac{1}{2}\left[\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}}\right]^{2}\right.$$

$$-\frac{8\left(1-\bar{\beta}_h\right)^2}{15\pi^{1/2}}\left[\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}}\right]^{5/2}+\frac{1}{6}\left[\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}}\right]^3\ldots\Bigg],$$

where, $\bar{\beta}=T_{el}/T_{eh}$.

Therefore, we find

$$n_{el}+n_{eh}=1+\varphi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\bar{\beta}\right)^{3/2}}{\left(\mu+\nu\bar{\beta}\right)^{3/2}}\varphi^{3/2}+\frac{1}{2}\frac{\left(\mu+\nu\bar{\beta}^2\right)}{\left(\mu+\nu\bar{\beta}\right)^2}\varphi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\bar{\beta}^{5/2}\right)}{\left(\mu+\nu\bar{\beta}\right)^{5/2}}\varphi^{5/2}+\ldots$$

where

$$b_l=\frac{1-\bar{\beta}_l}{\pi^{1/2}},\ b_h=\frac{1-\bar{\beta}_h}{\pi^{1/2}},\ b_l^{(1)}=\frac{1-\bar{\beta}_l^2}{\pi^{1/2}},\ b_h^{(1)}=\frac{1-\bar{\beta}_h^2}{\pi^{1/2}},$$

v). Nonthermal Electron

In the presence of a population of fast energetic electrons together with a population of Maxwellian distributed electrons, the model of the velocity distribution function is called non-thermal electrons which are Non-Maxwellian. This type of velocity distribution has been considered by many authors in the studies of different collective processes in plasmas and dusty plasmas.

The nonthermal electron distribution function can be taken as

$$f_e(\mathrm{v})=\frac{n_0}{(1+3\alpha)\sqrt{2\pi v_e^2}}\left[1+\alpha\left(\frac{V^2}{V_e^2}-2\phi\right)^2\right].\exp\left[-\frac{1}{2}\left(\frac{V^2}{V_e^2}-2\phi\right)\right]$$

where $V_e=\sqrt{\frac{K_BT_e}{m_e}}$, m_e is the mass of electron, K_B is the Boltzmann constant and T_e is the temperature of electron.

The electron density is given by

$$n_e=n_{e0}(1-\beta\varphi+\beta\varphi^2)\exp(\varphi)$$

where, β is the nonthermal parameter, $\beta=\frac{4\alpha}{1+3\alpha}$.

where α is a parameter determining the number of nonthermal electrons present in the plasma model. β measures the deviation from the thermalized state. The parameter β cannot exceed a certain range ($0 \leq \beta \leq 1.3$), beyond which α is negative which is not acceptable. When $\beta \rightarrow 0$, n_e reduces to the Boltzmann distribution of electrons.

vi). Superthermal or Kappa Distributed Electron

Various experimental data indicate that the Non-Maxwellian particles play the main role in astrophysical situations. Space and laboratory plasmas often possess an excess population of superthermal (Kappa distributed) electrons, a fact which is reflected in a power-law distribution at high velocity (above the electron thermal speed). The superthermal electron distribution is given by

$$f_\kappa(v) = \frac{n_{e0}}{(\pi\kappa\theta^2)^{1/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1+\frac{v^2}{\kappa\theta^2}\right)^{-\kappa-1}$$

where kappa-factor κ measures the deviation from the thermal equilibrium, κ is called as spectral index and

$$\theta^2 = \frac{(\kappa-\frac{3}{2})}{\kappa}(2K_B T_e / m_e)$$

K_B is Boltzmann constant, T_e is the temperature of electrons and m_e is the mass of an electron.

The electron density of superthermal electrons in normalized form is given by:

$$n_e = n_{e0}\left(1-\frac{\varphi}{(\kappa-3/2)}\right)^{-(\kappa-\frac{1}{2})}$$

where, φ is normalized as $\varphi = \frac{e\bar{\varphi}}{K_B T_e}$.

vii). Q-Nonextensive Electron

In a complex regime of cosmological or astrophysical plasmas, Tsallis q-nonextensive particle distribution is considered as a most general distribution which can describe this complexity more appropriately. In one-dimensional equilibrium q- distributed function is given by $f_e(v_x) = C_q \left\{1-(q-1)\left[\frac{m_e v_x^2}{2T_e} - \frac{e\varphi}{T_e}\right]\right\}^{\frac{1}{2}}$

The constant of normalization is

$$C_q = \begin{cases} n_{e0} \dfrac{\Gamma(\frac{1}{q-1})}{\Gamma(\frac{1}{q-1}-\frac{1}{2})} \sqrt{\dfrac{m_e(1-q)}{2\pi T_e}}, \; for -1 < q < 1 \\ n_{e0} \dfrac{1+q}{2} \dfrac{\Gamma(\frac{1}{q-1}+\frac{1}{2})}{\Gamma(\frac{1}{q-1})} \sqrt{\dfrac{m_e(1-q)}{2\pi T_e}}, \; for 1 < q < 0 \end{cases}$$

Here, the parameter q stands for the strength of nonextensive electrons and the quantity

Γ stands for the standard gamma function; T_e is the temperature of electron, m_e is the mass of an electron and v_x is the velocity of nonextensive electrons. In the extensive limit case $q \to 1$, the distribution reduces to Maxwellian case.

$$f_e(v_x) = \frac{n_{e0}}{(2\pi T_e / m_e)^{1/2}} \exp\left(\frac{v_x^2 - 2e\varphi / m_e}{2K_B T_e / m_e}\right)$$

Integrating $f_e(v_x)$ over the velocity space and noting that $q > 1$, the distribution function exhibits a thermal cut off on the maximum value allowed for the velocity of the particles, given by

$$v_{\max} = \sqrt{\{\frac{2T_e}{m_e(q-1)} - \frac{2e\varphi}{m_e}\}}$$

Then we get,

$$n_e(\varphi) = n_{e0}\left[1+(q-1)\frac{e\varphi}{T_e}\right]^{\frac{1}{2}+\frac{1}{q-1}}$$

The nonextensive electron density in normalized form is written as

$$n_e = \mu[1+(q-1)\varphi)^{\frac{(q+1)}{2((q-1)}} = \mu(1+q_1\varphi)^{q_2}$$

where,

$$q_1 = q-1, q_2 = (q+1)/2(q-1), \mu = 1, \varphi = e\varphi / K_B T_e$$

1.4. NONLINEAR EFFECTS IN PLASMA

The plasma is a highly nonlinear medium and various types of nonlinear phenomena occur due to interactions of waves in it. In the earlier stage of plasma research, the plasma physicists were interested in the linear theory of plasma but later on, they realized the necessity of nonlinear theory to explain many nonlinear phenomena in plasma. Nonlinearities may be strong or weak, depending on the slowness of the variation of amplitude concerning to some characteristic time scale in plasma. It is difficult to deal with the problems of wave propagation in plasma retaining the full nonlinearity in most cases. Thus weakly nonlinear systems are considered to overcome the limitations of linear theory as a first step but strong nonlinearities are indispensable for a thorough understanding of the nonlinear plasma dynamics. Generally, particles and waves coexist in plasma and these particles interact with each other and with the waves also. The waves are generated self-consistently by the particle motions and may also be excited by the externally imposed fields. The predominating interactions in plasma are wave-particle interactions and wave-wave interactions of which the first may be due to both linear and nonlinear interaction but the second is inherently nonlinear.

The nonlinear theory of waves may be broadly classified into two kinds as coherent and incoherent or turbulent interactions. The coherent nonlinear phenomena retain its initial phase information while in an incoherent (or turbulent) case, the presence of many waves of different frequencies and phases randomizes the phase information and allows one to treat the system as a statistical ensemble. The important focal point in nonlinear plasma phenomena is solitons or solitary waves, cavitons, spikons etc. Also, there are many important nonlinear optical phenomena in plasma physics such as 1) Self-action effects, 2) Inverse Faraday effects, 3) Stimulated light scattering, 4) Multiphonon transitions etc. Various interesting non-linear effects are observed due to the interactions of intense electromagnetic waves with plasmas. Modulation instabilities in plasma are important for the study of nonlinear dispersion relation together with the non-linear shift of wave parameters. Nonlinear effects are found in experiments involving plasma instabilities, heating and confinement of plasmas, laser-induced fusion, conversion of energy and some other consequences. In order to investigate the non-linear phenomena, one has to understand clearly the basic characteristics of the nonlinearities prevailing in plasma.

1.5. MOTIVATION OF THE Ph.D THESIS

In the thesis, I have theoretically investigated **“Solitons and Double Layers in Different Types of Plasma”** using standard mathematical methods such as Reductive perturbation method, Pseudo-potential / Sagdeev potential (SP) method and Krylov-Bogoliubov-Mitropolsky (KBM) method. For numerical analysis, we have used software like MathCAD, Origin etc. We have considered both non-relativistic and relativistic plasma consisting of positive and negative ions, isothermal electrons, non-isothermal two-temperature electrons, nonthermal electrons, positrons, superthermal (kappa distributed) electrons, Q-nonextensive electrons, charged dust particles, beam ions, degenerate electrons, degenerate positrons, ultra-relativistic degenerate electrons and positrons, relativistically degenerate electrons and positrons etc.

The thesis has two parts: **In Part One,** different kinds of solitons, e.g. small amplitude KdV solitons, arbitrary amplitude higher-order solitons, dressed solitons, envelope solitons etc. have been studied in different types of plasma. **In Part Two,** double layers in unbounded and bounded plasmas have been theoretically investigated in different types of plasma.

Part- One (Solitons in Different Types of Plasma)

In Chapter-2, the exact analytical solutions of the nonlinear equations governing the propagation of ion-acoustics solitons in a plasma consisting of warm streaming positive ions and nonthermal electrons has been obtained with necessary and sufficient conditions for the existence of ion-acoustic solitons. The structure of the solitary waves as a function of the non-thermal electron parameter has been analyzed which gives the deviation from the thermalized state. **In Chapter-3,** the generation of W-type ion-acoustic solitons has been theoretically studied in plasma consisting of cold ions and nonthermal electrons. The plasma consisting of cold positive ions can support the formation of compressive as well as W-type solitary waves in second-and-third- order for a certain value of the nonthermal parameter of electrons. **In Chapter-4,** using the pseudo-potential method, arbitrary amplitude ion-acoustic solitons in an electron-ion-positron plasma has been theoretically investigated. It is found that the presence of positrons has a significant impact on the excitation of slow and fast small amplitude and large amplitude solitary waves in the plasma. The profiles of small amplitude and large amplitude solitons for fast-mode and slow-mode of the wave are drawn and discussed. **In Chapter-5,** using the reductive perturbation method, the existence and characteristics of ion-acoustic solitons have been studied in self-gravitating dusty plasma consisting of warm positive ions, weak non-isothermal two-temperature electrons and negatively charged dust particles with charge variation. From the uncoupled nonlinear equation, the solution of the ion-acoustic wave has been obtained and the effects of two-temperature electrons, gravity and dust charge fluctuations on the ion-acoustic solitary waves have been discussed. **In Chapter-6,** using the Krylov-Bogoliubov-Mitropolsky (KBM) method, the dressed solitons has been theoretically investigated in dense dusty plasma having ultra-relativistic degenerate plasma consisting of cold inertial ions, degenerate two-temperature electrons and negatively charged dust particles. The profiles of dressed solitons are drawn and discussed for different values of density and temperature of ultra-relativistic degenerate two-temperature electrons. **In Chapter-7,** the effect of the ion beam on the ion-acoustic solitons in unmagnetized quantum plasma has been studied using the reductive perturbation technique. The Korteweg-deVries (KdV) equation has been derived and the solution of ion-acoustic solitary waves is obtained in quantum plasma in presence of an ion beam. It is seen that the formation and structure of solitary waves are significantly affected by the ion beam in quantum plasma. **In Chapter-8**, modulation instability and envelope soliton of high-frequency surface waves on warm plasma half-space has been studied using multiple scale perturbation method. From the evolution equation, we have derived the condition of instability and have shown that high-frequency surface waves on a temperate plasma half-space are modulationally unstable. The dependence of the growth rate of instability on temperature and wave number is computed and presented graphically.

Part-Two (Double Layers in Different Types of Plasma)

In Chapter-9, using the Sagdeev potential (SP) method, ion-acoustic double layers in a multi-component plasma consisting of positive ions, negative ions and nonthermal electrons filled in a cylindrical waveguide has been theoretically investigated. It is seen that finite geometry of bounded plasma, nonthermal electrons, the density of negative ions, stream velocity of positive and negative ions have significant contributions to the structure of double layers. The double layers may be compressive or rarefactive in plasma depending upon the values of the plasma parameters. **In Chapter-10**, the influence of nonthermal electrons and ion beams on the ion-acoustic double layers has been studied using the SP method. It is found that both compressive and rarefactive double layers are excited in the plasma depending on the values of nonthermal electrons and ion beam parameters. The width of double layers is also graphically discussed. **In Chapter-11,** the effects of positrons and nonthermal electrons on the ion-acoustic double layers in multi-component plasma with drifting positive and negative ions have been theoretically studied using the SP method. It has been observed that both compressive and rarefactive double layers are excited in presence of positrons, nonthermal electrons and negative ions in the plasma.

In Chapter-12, the effect of relativistically degenerate electrons and positrons on the ion-acoustic double layers in warm ion plasma has been theoretically studied using the SP method. It is seen that double layers are compressive and the relativistically degenerate electrons and positrons have significant contributions to the excitation of double layers in relativistically degenerate plasma.

PART-ONE

SOLITONS IN DIFFERENT TYPES OF PLASMA

SOLITONS IN PLASMA

A). Historical Background:

In physical science, a **soliton** is a self-reinforcing solitary wave packet that maintains its shape while it propagates at a constant velocity. Solitons are caused by a balance between nonlinear and dispersive effects in the medium. Solitons are the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems. Solitons have the following properties:

(i). Solitons are of permanent form.
(ii). Solitons are localized within a region.
(iii). Solitons can interact with other solitons and emerge from the collision unchanged, except for a phase shift.

The soliton phenomenon was first described in 1834 by John Scott Russell (1808–1882) who observed a solitary wave in the Union Canal in Scotland. He reproduced the phenomenon in a wave tank and named it the "Wave of Translation“.John Scott Russell describes his *wave of translation*. The discovery is described here in Scott Russell's own words:

“I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped – not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation”.

Lord Rayleigh published a paper in Philosophical Magazine in 1876 to support John Scott Russell's experimental observation with his mathematical theory. In his 1876 paper, Lord Rayleigh mentioned Scott Russell's name and also admitted that the first theoretical treatment was by Joseph Valentin Boussinesq in 1871. Joseph Boussinesq mentioned Russell's name in his 1871 paper. Thus Scott Russell's observations on solitons were accepted as true by some prominent scientists within his own lifetime of 1808–1882.

Korteweg-deVries [1895] derived an equation (KdV equation) which describes the theory of water waves in shallow channels, such as a canal. It is a non-linear equation which exhibits special solutions, known as solitons, which are stable and do not disperse with time. Furthermore there are solutions with more than one soliton which can move towards each other, interact and then emerge at the same speed with no change in shape (with a time "lag" or" speed up"). However, Korteweg and de Vries did not mention John Scott Russell's name at all in their 1895 paper but they did quote Boussinesq's paper of 1871 and Lord Rayleigh's paper of 1876. The paper by Korteweg and deVries in 1895 was not the first theoretical treatment of this subject but it was a very important milestone in the history of the development of soliton theory.

In 1965, Norman Zabusky of Bell Labs and Martin Kruskal of Princeton University first demonstrated soliton behaviour in media subject to the Korteweg–de Vries equation (KdV equation) in a computational investigation using a finite difference approach. They also showed how this behaviour explained the puzzling earlier work of Fermi, Pasta, Ulam, and Tsingou.

In 1967, Gardner, Greene, Kruskal and Miura discovered an inverse scattering transform enabling analytical solution of the KdV equation. The work of Peter Lax on Lax pairs and the Lax equation has since extended this to solution of many related soliton-generating systems. The most general class of non –linear wave structures is “Soliton” having approximately self consistent wave amplitudes and phases. Solitons are important and extensively studied because a whole class of non-linear differential equations encountered in plasma physics, solid state physics, particle physics, hydrodynamics, non-linear optics and Biology are seen to support solutions in soliton form . Solitons even in plasma physics itself, often appear in various phenomena like plasma turbulence, anomalous plasma resistance , self compression of high intensity waves (like the Langmuir, lower hybrid, drift waves etc.), space plasmas and so on. In plasma dynamics, there are two types of energies; one is decay of energy and the other is growth of energy. If only ‘dispersion’ is present in plasma there will be a decay of energy and if only the ‘nonlinearity’ is present in plasma then there will be a growth of energy. But if both the dispersion and nonlinearity are present then under certain conditions there is a possibility of balancing between the nonlinearity and dispersion giving rise to the formation of solitary waves in plasmas. Therefore, solitons or solitary waves are formed in plasma medium when the nonlinearity and dispersion effects are balanced .In plasma if there is no nonlinearity dispersion will destroy the solitary wave as the various components of the wave propagate at different velocities. On the other hand if there is no dispersion in the medium, non-linearity rules out the possibility of formation of soliton because the pulse energy is continuously injected in the process called harmonic generation into high frequency modes .But with both non-linearity and dispersion, solitary waves can again be formed. Solitons are nothing but solitary waves which neither crest nor dissipate. Some of the solitary waves in plasma are: (i) KdV Solitons, ii) Dressed Solitons, (iii) Alfven solitons, (iv) Langmuir solitons, (v) Envelope solitons, (vi) Drift solitons and vii) Trivelpiece-Gould solitons.

B). Works on Solitons in Plasma

(i). Korteweg-deVries (KdV) solitons:

The solitons (solitary waves) in plasma have been studied by various authors over last fifty years in different types of plasma. We have here mentioned some of the most important works on the solitons in plasma. Washimi and Taniuti [1] were probably the first to give a concise approach known as reductive perturbation technique for deriving the Korteweg-deVries (KdV) equation to study the ion-acoustic solitons (IAS) in plasma .In the derivation of KdV equation by using perturbation method the approximation of small amplitude wave concept has been taken in plasma. In order to study the plasma-acoustic waves of arbitrary amplitude, the nonperturbative approach known as Sagdeev potential [2] analysis is to be followed. In the study of non-linear phenomena, solitons, in plasma most of the earlier observations were limited to simple plasma (cold, collisionless and unmagnified) having only electrons and ions. The solitary wave in plasma were initially observed experimentally by Ikezi and his collaborators [3] and later by Tran [4], Lonngren [5], and others. It is noted that the soliton velocity obtained in experiments was greater than the theoretical predicted velocity from the KdV equation. Subsequently, many authors incorporated different parameters, e.g. ion temperature [6-8] , negative ions [9-11], two-temperature electrons [12-14] non-isothermal electrons [15-17] , nonthermal electrons [18-20] , superthermal electrons [21-23], beam ions [24-26], magnetic field [27-29], Landau damping [30-32], density gradient [33-35] , temperature gradient [36-37], positron [38-40] , q-nonextensive electrons

[41-43], gravitation [44-46] , quantum effect [47-49] , relativistic effect [50-56] etc. for theoretical analysis of IAS in plasma. There are also hundreds and hundreds references on the solitons in different types of plasma which are not mentioned here.

(ii). Dressed Solitons:

From the experimental observations [57-59] it is found that theoretically predicted values of the amplitude, width, and velocity of solitary waves do not agree well with the experimental results. To remove the discrepancy between the theoretical and experimental results, many authors have studied including higher- order perturbation corrections into the Korteweg-deVries (KdV) solitons, and the resulting solution has been termed as dressed solitons (DS). Initially, Ichikawa *et al*. [60] first examined such higher-order effects by the reductive perturbation method (RPM) for solitary waves in plasma consisting of cold ions. Later, the temperature of ions in the plasma was considered by Lai [61] and investigated the higher-order effect on the IAS. Subsequently, Kodama and Taniuti [62] reconsidered this problem and shown by the method of renormalization how to eliminate the secular terms appearing in higher-order terms of the expansion. Later, several authors considered the effect of higher-order nonlinearity for investigating the DS of ion-acoustic waves (IAW) in non-degenerate plasma and obtained interesting results [63-68] considering quantum (degenerate) plasma, Chatterjee *et al*. [69] used the RPM for the study of DS of IAW in dense quantum plasma whose constituents are electrons, positrons, and positive ions. Later, Roy and Chatterjee [70] investigated the DSs in unmagnetized two-species electron-ion quantum plasma by using the RPM. They obtained the nonsecular solution by the use of renormalization procedure. Subsequently, Wang *et al*. [71] presented a theory for the DS of quantum ion-acoustic waves (QIAW) in unmagnetized plasmas composed of positive and negative ions and weakly relativistic beams of electron considering higher-order corrections. They investigated the properties of the QIAW using a quantum hydrodynamic (QHD) model from which a KdV equation has been derived using the RPM. Recently, Paul *et al* [72] studied the DS of IAW in dusty plasma having ultra-relativistic degenerate (URD) two-temperature electrons and negatively charged dust particles using the KBM method. They derived the expression of pseudo-potential and from the nonlinear equation and obtained the solutions of DS of IAW up to second- order approximation in ultra-relativistic plasma. The profiles of DS are drawn and discussed for different values of density and temperature of URD electrons. The solution for the DS at critical state is also obtained and discussed graphically.

C). Nonlinear Schrodinger Equation

When a plasma wave goes nonlinear, the dominant new effect is that the ponderomotive force of the plasma waves causes the background plasma to move away, causing a local depressions in the density called a caviton. The plasma waves are trapped in this cavity then form an isolated structure called envelope soliton. Such solutions are described by the nonlinear Schrodinger (NLS) equation. The NLS equation governs various phenomena like self- focusing, self-modulation, envelope solitons.

The NLS equation is of the form

$$i\frac{\partial\phi}{\partial\tau} + P\frac{\partial^2\phi}{\partial\eta^2} = Q\,|\phi|^2\,\phi \qquad (1)$$

where, the coefficients P and Q are functions of angular frequency, ω and the propagation constant k .This equation represents envelope properties of a nonlinear waves.

(i). Modulation Instability

The solution of the above equation is plane wave type. The product of the coefficients P and Q will indicate whether the wave will be stable or unstable. When PQ is negative, the wave is modulationally unstable i.e. a small ripple on the envelope of the wave will tend to grow. On the other hand, for positive PQ the wave will be stable.

(ii). Envelope Soliton

A "bright soliton" is characterized as a localized intensity peak above a continuous wave (CW) background while a dark soliton is featured as a localized intensity dip below a continuous wave (CW) background. The terms bright and dark soliton are borrowed from optics, where they manifest as bright spots and dark shadows in optical fibres. In water bright solitons were first observed in 1830s. The discovery in the 1960s and 1970s of bright solitons on the surface of deep ocean waters initially stunned oceanographers, but many experiments have been done to explore and confirm the phenomenon. Solitons of both the bright and dark persuasions have now been spotted in fibre optics, plasmas, Bose-Einstein condense- states and elsewhere. But, a dark soliton had never been seen in water until now.

The NLS equation (1) has been widely used to successfully study solitons in dispersive nonlinear optical fiber and for analyzing the properties of the matter-wave solitons shortly after the realization of Bose–Einstein condense state. In general, solitons are formed when the nonlinear term exactly balances the wave packet dispersion, and two different types of solitons, i.e. bright and dark solitons when $PQ < 0$ and $PQ > 0$, respectively, have been observed in experiments and widely studied in theories for the NLS equation. Modulation instability and envelope solitons have been studied by many authors [73-78]

REFERENCES

KdV and mKdV Solitons

[1]. H. Washimi and T. Taniuti, Phys. Rev. Letts. , **17,** p.966 (1966)
[2].R.Z.Sagdeev, Rev. Plasma Phys edited by M A Leontovich, New York (Consultant Bureau), 4, p. 23 (1966)
[3]. H. Ikezi, R. J .Tayler and D. R.Baker, Phys. Rev. Letts., **25**, p. 11 (1970)
[4]. M. Q. Tran, Physica Scripta, **20**, p. 317 (1979)
[5]. K. E. Lonngren, Plasma Phys., **25**, p. 943(1983)
[6]. S. G .Tagare, Plasma Phys., **15**, p. 1247 (1973)
[7]. R. K .Roychoudhury and S. Bhattacharya, Canad. J. Phys. , **65,** p. 699 (1987)
[8]. S. N. Paul, S. Chattopadhyaya, S. K. Bhattacharya & B. Bera, Pramana ,**60**, p. 1217 (2003)
[9].S. Watanabe, J. Phys. Soc. Japan, 53, p. 950 (1984)
[10.S. Chattopadhyaya, FIZIKA A (Zagreb), 19 , p. 31 (2010)
[11]. S. Chattopadhyaya and S. N. Paul, The African Review of Physics, **7**.003, p. 289 (2012)
[12]. K. K. Ghosh, B. Paul, Chandra Das and S. N. Paul, J. Phys. A: Math. Theor., 41, p. 335501 (2008)
[13]. Shalini and N. S. Saini, Phys. Plasmas, **21,** p. 102901 (2014)
[14]. F. Verheest and M. A. Hellberg , Phys. Plasmas, **22**, p. 072303 (2015)
[15].H.Schamel , Plasma Phys., **14,** p. 905 (1972)
[16].K.P.Das, S.R.Majumdar and S.N.Paul, Phys.Rev., **E 51(5)**, p.4796 (1995)
[17].H. R. Pakzad, Astrophys.Sp. Science, 326, p. 77(2010)
[18].S. K. Bhattacharaya, S. N. Paul and B. Chakraborty, Indian J. Phys., **77B** p. 327 (2003)
[19].B. Ghosh, S. Banerjee & S. N. Paul, 1ndian J. Pure & App. Phys., **51,** p 488 (2013)
[20].S. Chattopadhyay, Sri Lankan Journal of Phys., **20,** p. 1 (2019)
[21].N. Saini, I. Kourakis and M. A. Hellberg, Phys. Plasmas, **16,** p. 062903 (2009)
[22]. M. R. Rauhani and Z. E. Abbasi, Iranian J. Phys. Research, **12,** p. 84 (2013)
[23].K. Kumar and M. K. Mishra,AIP Advances, **7**, p.115114 (2017)
[24]. P. S. Abrol and S. G. Tagare, Phys. Letts. , **A75**, p. 74 (1979).
[25].S. K. El-Labany, J. Plasma Phys., **54**, p. 285 (2009)
[26].R. Das, Astrophys. Sp. Science, 341, p. 543 (2012)
[27].K. K. Ghosh, S. N. Paul and S. Chattopadhyaya, FIZIKA, **A 13,** p. 33 (2004)
[28].M. Shahmonsouri, and H . Alinezad, Phys. Plasmas, **20**, p. 033704 (2013)
[29].A. P. Misra and Y. Wang, Comm. Nonlinear Science and Numerical Solution, **22**, p. 1360 (2015)
[30] F. Ott and R.N.Sudan, Phys. Fluids, **13,** p. 1432 (1970)
[31].A.Bandyopadhyay and K.P Das, Phys.Plasmas, **9**, p. 465(2002)
[32].S. Ghosh and R. Bharuthram, Astrophys. Space Sc., **314**, p. 121 (2008)
[33]. K. Nishikawa and P. K. Kaw, Phys Letts., **A 50,** p. 445(1975).
[34]. R. Tomar, H.K.Malik and R. P. Dahiya, J.Theor. Appl. Phys., **8** , p. 126 (2014).
[35]. S A Elwakil, M A Zahua, Sk El-Shamy and A E Mowly, Advances in Space Res., 48, p. 1067 (2014).
[36]. B..N..Goswami and M. Sinha, Pramana ,**7** , p. 141 (1976)
[37]. N. N.. Rao and R..K.Varma, Pramana, **10**, p. 247 (1978)
[38]. S. I. Popel, S. V. Vladimirov, and P. K. Shukla, Phys. Plasmas, **2,** p. 716 (1995)
[39]. Indrani Paul, B. Ghosh, G. Pakira, S. N. Paul and S. Chattopadhyay, Indian J. Phys., **86**, p. 395 (2012)

[40]. B. Ghosh, S. Banerjee and S. N. Paul , Indian. J. Pure & Appl. Phys., **51**, p. 488 (2013).
[41]. N. Akhtar, W. F. Al-Taibany, and S. Mahmood, Phys. Letts., **A 377,** p.1282 (2013).
[42]. H. Sahoo , S. Chandra and B. Ghosh, The African Rev. Phys., **10:**0032, p. 235 (2015).
[43]. J. Sarkar, J. Goswami, S. Chandra, B. Ghosh, Laser and Particle Beams, **35 (4),** p. 641(2017)
[44].S. N. Paul, G. Pakira, B. Paul and B. Ghosh, Acta Physica Polonica, **A 122**, p. 116 (2012)
[45].S. N. Paul, A .Chatterjee, G.Pakira and Indrani Paul, J. Appl. Phys. Sci. International (JAPSI), **8(2),** p. 77 (2017)
[46].S. N. Paul, A. Chatterjee and Indrani Paul, Indian J. Phys., **91**, p. 101 (2017)
[47]. M.Akbari-Moghanjoughi, Phys. Plasmas, **17,** p. 082315 (2010)
[48].S.Chandra, S.N.Paul and B. Ghosh , Astrophys. Sp. Science, **343 ,** p.213 (2013).
[49]. Indrani Paul, A.Chatterjee and S.N.Paul , Laser and Particle Beams, **37,** p. 370 (2019)
[50]. G.C.Das, and S.N.Paul, Phys. Fluids, **28,** p. 823 (1985)
[51].Lee, N. C. and Choi, C. R. , Phys. Plasmas,**14,** p. 022307 (2007)
[52].A.A. Mamun and P.K. Shukla , Phys. Plasmas, **17**, p. 104504 (2010)
[53]. A.A. Mamun and P.K. Shukla , Phys. Letts., **A 374**, p. 4238 (2010).
[54]. N. Roy , M.S. Zobaer and A.A. Mamun, Journal of Modern Phys., **3,** p. 850 (2012)
[55].S.A El-Wakil, E.M.Abulwafe, E.K.El-Shewy, H.G.Abdelwahid and H.M.Abd-El-Hamid, Advances in Mathematical Phys., p. 52921(2012)
[56].A. Rahman, I. Kourakis and A. Qamar , IEEE Trans. on Plasma Sci., **43(4)**, p. 974 (2015).

Dressed Solitons

[57].I.kezi, R. J. Tayler and D. R. Baker, Phys. Rev. Letts., **25** p.11 (1970)
[58]. M. Q.Tran, Physica Scripta, **20,** p.317 (1979)
[59]. K E. Lonngren, Plasma Phys., 25 , p. 943(1983)
[60]. Y. Ichikawa, M . Matsuhasi and K. Konno J. Phys. Soc. Japan, **41**, p. 1382 (1976).
[61]. C. S. Lai, Canadian J. Phys., **57**, p 490 (1979).
[62]. Y. Kodama and T. Taniuti, J. Phys. Soc. Japan, **45**, p. 298 (1978).
[63]. R. S. Tiwari and M. K. Mishr,a Phys. Plasmas, **13,** p. 062112 (2006).
[64]. S. K. El-Labany, W. F. El-Taibany, O M El-Abbasy, Chaos Solitons Fractals, **33** p. 813 (2007).
[65]. T. S. Gill, P. Bala and H. Kaur, Phys. Plasmas, **15,** p. 122309 (2008).
[66]. K. Roy , P. Chatterjee and S. Kundu, Advances in Space Research, **50,** p 1288 (2012).
[67]. R. Amour, L. A .Gougam and M. Tribeche, Physica, **A 436,** p. 856 (2015).
[68]. E. K. El-Shewy, N. F. Abdo and M. S. Yousef, Indian J. Phys, **90,** p. 959 (2016).
[69]. P. Chatterjee, K. Roy, S. V. Muniandy and C. S. Wong, Phys. Plasmas, **16**, p. 112106 (2009).
[70]. K. Roy and P. Chatterjee, Indian J. Phys., **85**, p. 1653 (2011).
[71]. Y. Wang,Y. Dong and B. Eliasson , Phys. Letts., **A 377**, p. 2604 (2013).
[72].Indrani Paul, A. Chatterjee and S. N.Paul, Indian J. Phys. (Published on 31 October,2020) https://doi.org/10.1007/s12648-020-01899-w

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[73].B. Ghosh and S. Banerjee, Journal of Astrophys., **2014**, Article ID 785670 (2014).
[74]. B.Ghosh , S.Banerjee, African Rev. Physics, **10,** p.031 (2015).
[75]. B. Ghosh, S. Chandra and S. N. Paul, Phys. Plasmas, **18,** p.012106 (2011)
[76]. B.Ghosh , S.N.Paul and S.Chandra, Pramana, **78**, p.779 (2012)
[77]. S. Chandra and B. Ghosh, Astrophy.Sp. Science, **342**, p.417 (2012)
[78]. B. Ghosh and S. Banerjee, J. Plasma Phys., **81,** p.905810308 (2015)

CHAPTER-2

ANALYTICAL STUDY OF ION-ACOUSTIC SOLITONS IN PLASMA CONSISTING OF WARM STREAMING IONS AND NONTHERMAL ELECTRONS

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2.1. INTRODUCTION

Ion-acoustic solitary waves (solitons) (IAS) have been studied by various authors because it is important for wide applications in laboratory and space plasma. In cold unmagnetized and non-dissipative plasma, Washimi and Taniuti [1] have first theoretically studied the IAS using the reductive perturbation method (RPM). Later, many authors have studied IAS considering the effect of ion temperature [2,3], negative ions [4,5], two-temperature electrons [6,7], magnetic field [8,9] etc. It is seen that these parameters have a considerable impact on the excitation of IAS in the plasma. But, the presence of non-Maxwellian electrons, nonisothermal electrons [10,11], superthermal electrons distributions [12,13] etc., in a plasma gives rise to more interesting results in the nonlinear propagation of waves, particularly the IAS in plasma.

The nonthermal electrons which are also non-Maxwellian give rise to various interesting behaviour on the IAS in plasma. Cairns *et al* [14] have shown that solitary waves are excited in plasma consisting of nonthermal electrons with an excess of energetic particles and ions. They were motivated by the recent observation of solitary structures with density depletion made by the Hall *et al*. [15] observed in Freja satellite. Such electron distribution with an enhanced population of energetic electrons has been observed in the magnetosphere. Nonthermal distribution is a common feature of the auroral zone [16]. This type of distribution is common to many space and laboratory plasmas in which wave damping produces an electron tail. Considering the presence of nonthermal electrons in plasma, Mamun [17, 18], Bandyopadhyay and Das [19-21] and other authors have studied the IASW. In bounded plasma, Bhattacharya *et al*. [22] have theoretically investigated the effects of nonthermal electrons on the IAS using pseudo-potential technique and have shown that both compressive and rarefactive IAS would be excited for any value of the nonthermal electron parameter in plasma. The streaming motion of ions and electrons may become important in some physical situations such as solar wind plasma, beam plasma interactions and cometary plasmas. The cometary plasmas are mixtures of usual solar wind plasma and the cometary matter, which are escaped as neutral gas from the comet's nucleus and became ionized afterwards by various physical mechanisms. In these plasmas, constituents have their drifts. Kourakis and Shukla [23] studied the nonlinear propagation of ion-acoustic wave in collisionless three-component plasma consisting of inertial positive ions moving in a background of two thermalized electron populations. The streaming motions of the electrons or ions are expected to introduce new effects [24, 25]. It has been found that the electron and ion distributions can play a crucial role in the formation of nonlinear wave structures. Further, these distributions considerably influence the conditions for the existence of IAS. Subsequently, Paul *et al*. [26] have obtained exact analytical solutions of the nonlinear equations governing the propagation of IAS in plasma consisting of warm streaming positive ions and nonthermal electrons with the necessary and sufficient conditions for the existence of IAS. Later, Ghosh *et al*. [27] have theoretically investigated the effect of nonthermal electrons and warm negative ions on the IAS in multi-component drifting plasma.

In this Chapter, we intend to obtain the exact analytical solutions of the nonlinear equations governing the propagation of IAS in plasma consisting of warm streaming positive ions and nonthermal electrons. We have obtained the necessary and sufficient conditions for the existence of IAS. From these conditions, the critical values of the nonthermal parameter, ion temperature and velocity of IAS have been numerically evaluated and their dependence on various plasma parameters has been discussed. The expression of Sagdeev potential has been derived and the conditions of solitary wave solution are obtained. The structure of the IAS as a function of the nonthermal electron parameter are analyzed which gives the deviation from the thermalized state. Moreover, it is seen that there are two modes namely Slow-mode and Fast mode of ion-acoustic wave in presence of ion stream in the plasma. So, the profiles of IAS for Slow-mode and Fast-mode are drawn for different values of ion stream velocity, nonthermal parameter of electrons and ion temperature in the plasma.

2.2. BASIC EQUATIONS

We consider collisionless unmagnetized plasma consisting of warm positive ions with streaming motion in presence of nonthermal electrons. Both the ions and electrons are assumed to be non-relativistic. The system of basic equations governing the one-dimensional propagation of ion-acoustic wave in such plasmas are given by

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}\left(n_i u_i\right)=0 \tag{2.1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2.2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial u_i}{\partial x}=0 \tag{2. 3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i \tag{2. 4}$$

where $n_i(n_e)$ is the ion (electron) number density normalized by the equilibrium ion density n_0 ; $\sigma_i=T_i/T_e$ with $T_i(T_e)$ being ion (electron) temperature ; u_i is the ion fluid velocity normalized by the ion-acoustic speed $(K_B T_e/m_i)^{1/2}$ in which K_B is the Boltzmann constant and m_i is ion mass ; p_i is the ion pressure normalized by $n_0 K_B T_i$; ϕ is the electrostatic potential normalized by $K_B T_e/e$, e being the magnitude of electronic charge ; the space variable x is normalized by the Debye length $(K_B T_e/4\pi n_0 e^2)^{1/2}$ and time t by the ion plasma period $(m_i/4\pi n_0 e^2)^{1/2}$.

The model of nonthermal distribution function used first to study the ion-acoustic solitary structures is non-Maxwellian with the presence of fast-moving energetic electrons together with a population of Maxwellian distributed electrons. Such a distribution arises in space plasmas from the force field present there. This type of distribution is called nonthermal distribution. Following Cairn's model, the non-thermal electron distribution function is assumed as

$$f_e\left(v\right)=\frac{n_{eo}}{\left(1+3\alpha\right)\sqrt{2\pi V_e^2}}\left[1+\alpha\left(\frac{v}{V_e^2}-2\phi\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{v}{V_e^2}-2\phi\right)\right] \tag{2.5a}$$

where $\alpha(\geq 0)$ is a parameter that determines the proportion of the fast energetic electrons, v is the velocity of electrons in phase space and is normalized by the average thermal speed $V_e=\sqrt{K_B T_e/m_e}$, m_e is the mass of electron, n_{e0} is the equilibrium electron density, φ is electrostatic potential, K_B is the Boltzmann constant, T_e is the temperature of electrons and e is the charge of an electron which is negative .

From (2.5a) the normalized electron density n_e is obtained as

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)e^{\phi} \tag{2.5b}$$

where

$$\beta = \frac{4\alpha}{1+3\alpha} \tag{2.5c}$$

β is the nonthermal parameter of electrons and α is a parameter determining the number of nonthermal electrons present in our plasma model.

To obtain the solution of IAS , we introduce the single independent variable η defined by the relation $\eta = x - Vt$, where V is the velocity of the solitary wave.

Now, using the boundary conditions

$$n_i \to 1`, u_i \to u_{io}, p_i \to 1, \phi \to 0 \quad \text{at} \quad |\eta| \to \propto \tag{2. 6}$$

and the charge neutrality condition ($n_{io} = 1$).

Eqs. (2.1) – (2.4) in the stationary frame can be integrated to give,

$$n_i = \frac{V - u_{i0}}{V - u_i} \tag{2.7a}$$

$$p_i = \frac{(V - u_{i0})^3}{(V - u_i)^3} \tag{2.7b}$$

$$\phi(u_i) = Vu_i - \frac{1}{2}u_i^2 - \frac{3\sigma_i(V - u_{i0})^2}{2(V - u_i)^2} - a_0 \tag{2.7c}$$

where, $a_0 = -Vu_{i0} + \frac{1}{2}u_{i0}^2 + \frac{3\sigma_i}{2}$

Using Eqs. (2.5a) to (2.7c), we obtain from Eq. (2.4),

$$\frac{d^2\phi}{d\eta^2} = \left(1 - \beta\phi + \beta\phi^2\right)\exp(\phi) - \frac{V - u_{i0}}{V - u_i} = G(u_i) \quad \text{(say)} \tag{2.8}$$

2.3. ANALYTICAL STUDY OF ION-ACOUSTIC SOLITONS

Integrating Eq. (2.8), we get

$$(\frac{d\phi}{du_i})^2(\frac{du_i}{d\eta})^2 = H(u_i) - K \tag{2.9}$$

where, K is an arbitrary constant of integration and

$$H(u_i) = 2\int G(u_i)\frac{d\phi}{du_i}du_i \tag{2.10}$$

Now, from Eq. (2.7c) we obtain

$$\frac{d\phi}{du_i} = (V - u_i)[1 - \frac{3\sigma_i(V - u_{i0})^2}{(V - u_i)^4}] \tag{2.11}$$

Using Eqs**.** (2.8) and (2.11), the Eq. (2.10) yields

$$H(u_i) = 2[\{(1+3\beta) - 3\beta\phi + \beta\phi^2\}\exp(\phi) - (V - u_{i0})u_i + \frac{\sigma_i(V - u_{i0})^3}{(V - u_i)^3}] \tag{2.12}$$

It may be noted that the function $H(u_i)$ is highly nonlinear and contains a number of parameters including soliton velocity, nonthermal parameter of the electrons and ion temperature. These parameters enter into the function in a complicated way and thus restrict the range of possible solitary wave solutions.

2.3.1 Necessary and Sufficient Conditions

For ion-acoustic solitary wave in a nonthermal electron plasma with warm positive ion, the physically admissible solution of Eq. (2.9) is obtained with some special observations:

(i) $\left(\frac{d\phi}{du_i}\right)^2\left(\frac{du_i}{d\eta}\right)^2$ must be positive, (2.13a)

and

(ii) u_i and $\frac{du_i}{d\eta}$ must be bounded. (2.13b)

From Eqs**. (** 2.7c) and (2.8), it is easy to make the following observations:

Observation 1: $G(u_{i0}) = 0$ (2.14)

Observation 2: $\phi(u_{i0}) = 0$ (2.15)

Observation 3: If $(V - u_{i0})^2 < 3\sigma_i$, then (i) $\phi'(u_i) > 0$ for $V < u_i \le u_{i0}$

and (ii) $\phi^{/}(u_i)\langle 0$ for $u_{i0} \le u_i < V$ (2.16)

Observation 4: If $(V-u_{i0})^2 > 3\sigma_i$,

Then, (i) $\phi^{/}(u_i) > 0$ for $V < u_i \le u'$ (2.17a)

and (ii) $\phi^{/}(u_i)\langle 0$ for $u' \le u_i < V$ (2.17b)

where u' is given by $\phi^{/}(u') = 0$,that is,

$$u' = V - (3\sigma_i)^{1/4}.\sqrt{V-u_{i0}} \tag{2.18}$$

Therefore, for physical solution of Eq. (2.9) we need following requirements;

Requirement 1: There exists $u_{\max}$ or $u_{\min}$ such that

$$H(u_{i0}) = H(u_{\max}) = K \quad \text{for} \quad u_{i0} \le u_{\max} < u' \tag{2.19}$$

Or $H(u_{i0}) = H(u_{\min}) = K$ for $u' \le u_{\min} < u_{i0}$
(2.20)

Requirement 2: $H(u_i)\rangle K$ for either $u_{i0} \le u_i < u_{\max}$ or $u_{\min} < u_i \le u_{i0}$ (2.21)

For real and bounded solution of Eq. (2.9) the following conditions must be satisfied:

Necessary condition: $H''(u_{i0}) > 0$ (2.22)

Sufficient condition: $H(u') - H(u_{i0}) < 0$ (2.23)

By virtue of observations (2.3) and (2.4) it follows from the condition (2.22) that

$$G'(u_{i0}) > 0 \quad \text{for} \quad V > u_{i0} \tag{2.24}$$

$$G'(u_{i0}) < 0 \quad \text{for} \quad V < u_{i0} \tag{2.25}$$

Now using Eqs. (2.8) and (2.11) we get the following simple conditions for real and bounded solution of Eq. (2.9):

$$V > u_{i0} + \sqrt{\frac{1}{1-\beta} + 3\sigma_i} \qquad \text{for} \quad V > u_{i0} \tag{2.26a}$$

and $$u_{i0} > V + \sqrt{\frac{1}{1-\beta} + 3\sigma_i} \quad \text{for} \quad V < u_{i0} \tag{2.26b}$$

For a non-streaming plasma with Boltzmann distributed electrons the condition (2.26a) reduces to the well-known result

$$V \rangle \sqrt{1+3\sigma_i} \qquad (2.27)$$

Using Eq.**(**2.12) the condition (2.23) becomes

$$\left[\{(1+3\beta)-3\beta\gamma+\beta\gamma^2\}e^{\gamma}-(V-u_{i0})(u'-\frac{1}{3}\sqrt{3\sigma_i})\right]<\left[\{(1+3\beta)-(V-u_{i0})u_{i0}+\sigma\}\right] \qquad (2.28)$$

For non-streaming plasma with cold ions and Boltzmann distributed electrons conditions (2.27) and (2.28) together require that 1<V<1.6 which agrees with the condition established by Sagdeev [28].

2.3.2. Critical Values of Plasma Parameters for Solitary Wave Solutions

For the existence of IAS in a plasma the conditions (2.26a) and (2.26b) must be satisfied. So, in the plasma consisting of warm ions and nonthermal electrons, the critical values of phase velocity (*V*), nonthermal parameter (β_c), ion temperature (σ_{ic}) are obtained by

$$V=V_F=u_{i0}+(3\sigma_i+\frac{1}{1-\beta})^{1/2} \qquad (2.29)$$

$$V=V_S=u_{i0}-(3\sigma_i+\frac{1}{1-\beta})^{1/2} \qquad (2.30)$$

$$\beta_c=1-\frac{1}{(V-u_{i0})^2-3\sigma_i} \qquad (2.31)$$

$$\sigma_{ic}=\frac{1}{3}[(V-u_{i0})^2-\frac{1}{1-\beta}] \qquad (2.32)$$

It is seen from (2.29) - (2.30) that the streaming motion (u_{i0}) of ions has significant effects on the critical values of the phase velocity and other plasma parameters. It is interesting to note that two distinct modes (fast and slow) of the waves exist in presence of streaming motion of ions.

2.4. SAGDEEV POTENTIAL AND ION-ACOUSTIC SOLITONS

In previous section, we have discussed the conditions for the existence of IAS in terms of ion fluid velocity (u_i) instead of electrostatic potential ϕ or number density (n_i) of ions. However, expressing the term G in Eq. (2.8) as a sole function of ϕ and integrating the resulting equation one can find out the Sagdeev potential and then one can use it in usual way to discuss the properties of IAS structures.

The Eq. (2.8) can be written in the form

$$\frac{\partial^2 \varphi}{\partial \eta^2} = (1-\beta\varphi+\beta\varphi^2)\exp(\varphi) + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}[(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}] \qquad (2.33a)$$

where,

$$A_1 = (V-u_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}}, A_2 = 2\left(\frac{3\sigma_i p_{i0}}{n_{i0}}\right)^{\frac{1}{2}}(V-u_{i0})$$

Eq. (2.33a) can be written in the form

$$\frac{\partial^2 \varphi}{\partial \eta^2} = -\frac{d\psi}{d\varphi} \qquad (2.33b)$$

where $\psi(\phi)$ is known as the Sagdeev potential and is given by

$$\psi(\phi) = (1+3\beta) - e^{\phi} + \beta(\phi-1)e^{\phi} - \beta(\phi^2-2\phi+2)e^{\phi} -$$

$$\frac{1}{6\sqrt{3\sigma_i}}[\left\{\left(V-u_{io}+\sqrt{3\sigma_i}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left\{\left(V-u_{io}-\sqrt{3\sigma_i}\right)^2 - 2\phi\right\}^{\frac{3}{2}}$$

$$-\left(V-u_{io}+\sqrt{3\sigma_i}\right)^3 + \left(V-u_{io}-\sqrt{3\sigma_i}\right)^3] \qquad (2.34)$$

The form of the pseudo-potential $\psi(\varphi)$ would determine whether a soliton like solution of Eq. (2.33a) will exist or not. The conditions for the existence of soliton solution are:

i) $\psi(\varphi) = 0$ at $\varphi = 0$ and $\varphi = \varphi_c$ (2.35)

ii) $\frac{d\psi}{d\varphi} = 0$ at $\varphi = 0$ (2.36)

and

iii) $\dfrac{d^2\psi}{d\varphi^2} < 0$ at $\varphi = 0$ (2.37)

An analytical solution for the IAS can be obtained by expanding the right hand side of Eq.(2.33a) in terms of ϕ keeping the terms up to second order, third order or even next higher orders. After a straightforward calculation one can get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 + R\varphi^3 + \cdots\cdots \tag{2.38}$$

where

$$P = (1-\beta) + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i}}\left(\frac{1}{G_1^{1/2}} - \frac{1}{G_2^{1/2}}\right) \tag{2. 39a}$$

$$Q = -\frac{1}{2}\left[1 + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i}}\left(\frac{1}{G_1^{3/2}} - \frac{1}{G_2^{3/2}}\right)\right] \tag{2.39b}$$

$$R = \frac{1}{2}\left[\frac{(1+3\beta)}{3} + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i}}\left(\frac{1}{G_1^{5/2}} - \frac{1}{G_2^{5/2}}\right)\right] \tag{2.39c}$$

where, $G_1 = A_1 + A_2, G_2 = A_1 - A_2$.

Now, for small amplitude IAS, the terms up to φ^2 in (2.38) are taken into consideration, neglecting next higher order terms of φ i.e. φ^3 , φ^4 , etc. So, we get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 \tag{2.40}$$

Using the standard mathematical technique, the solution of Eq. (2.30) for the small amplitude IAS is obtained as:

$$\varphi_1 = \frac{3P}{2Q}\sec h^2\theta \tag{2.41}$$

where $\theta = [(P/4)^{1/2}\eta]$.

The amplitude (φ_{01}) and the width (δ_1) of the IAS are given by

$$\varphi_{01} = \frac{3P}{2Q} \quad \text{and } \delta_1 = \frac{2}{\sqrt{P}} \tag{2.42}$$

From Eq.(2.39a-c) and Eq.(2.40) we see that ion-acoustic wave propagates with two modes in presence of an ion stream in e-i-p plasma., one is Slow- mode (V_S) and the other is Fast-mode (V_F). So, for the solution for IAS of Slow-mode and Fast-mode, the phase velocities V_S and V_F should be used instead of V in Eqs. (2.38) to (2.39c). So, we obtain

$$\begin{aligned} G_{1S} &= A_{1S} + A_{2S}, G_{1F} = A_{1F} + A_{2F}, \\ G_{2S} &= A_{1S} - A_{2S}, G_{2F} = A_{1F} - A_{2F}. \end{aligned} \tag{2.43}$$

$$\begin{aligned} A_{1S} &= (V_S - u_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}}, A_{1F} = (V_F - u_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}}, \\ A_{2S} &= 2\left(\frac{3\sigma_i p_{i0}}{n_{i0}}\right)^{\frac{1}{2}} (V_S - u_{i0}), A_{2F} = 2\left(\frac{3\sigma_i p_{i0}}{n_{i0}}\right)^{\frac{1}{2}} (V_F - u_{i0}) \end{aligned} \tag{2.44}$$

So, the solution of Eq. (2.30) of small amplitude IAS for Slow-mode and Fast-mode in the plasma will be

$$\varphi_{1S} = \frac{3P_S}{2Q_S} \sec h^2\theta_S \qquad \text{and } \varphi_{1F} = \frac{3P_F}{2Q_F} \sec h^2\theta_F \tag{2.45}$$

where $\theta_S = [(P_S/4)^{1/2}\eta]$ and $\theta_F = [(P_F/4)^{1/2}\eta]$

The amplitude (φ_{01}) and the width (δ_1) of IAS for Slow-mode and Fast-mode in the plasma are:

$$\varphi_{01S} = \frac{3P_S}{2Q_S}, \varphi_{01F} = \frac{3P_F}{2Q_F} \tag{2.46a}$$

$$\text{and } \delta_{1S} = \frac{2}{\sqrt{P_S}}, \delta_{1F} = \frac{2}{\sqrt{P_F}} \tag{2.46b}$$

2.5. RESULTS AND DISCUSSION

2.5.1. Variation of H with u_i

In Eq. (2.12) it is seen that the parameter H is highly nonlinear and it depends on the plasma parameters. To see the effects of u_i on H for different values of the plasma parameters e.g. nonthermal parameter of electrons (β), ion temperature (σ_i) and ion stream velocity (u_{i0}), numerical estimation is made considering model plasma and the graphs shown in Figs. 2.1(a), 2.1(b) and 2.1 (c) are drawn.

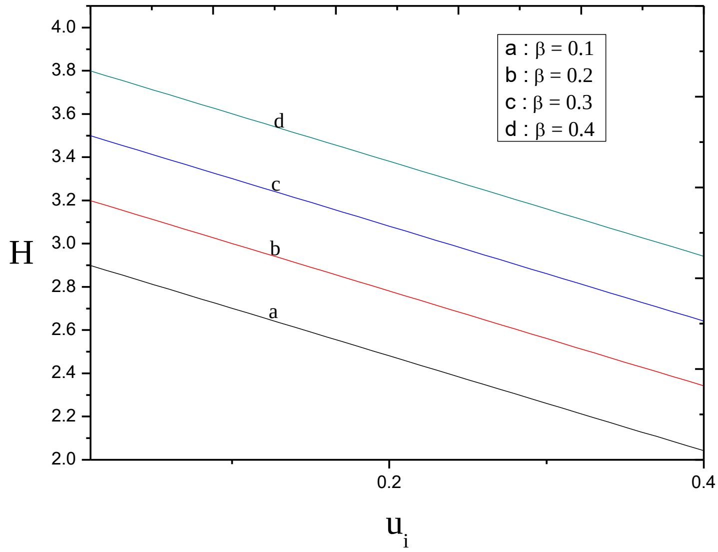

Fig. 2.1(a). Variation of H with u_i at different values of β with fixed values of the plasma parameters σ_i = 0.001, u_{i0} = 0.5, φ =0.01, p_i = 1 and V=1.6. Graphs a, b, c and d represent β = 0.1, 0.2, 0.3 and 0.4 respectively.

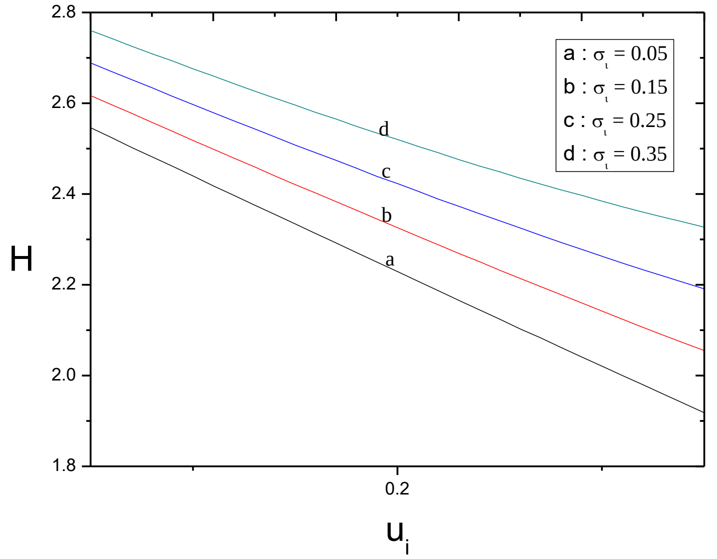

Fig. 2.1(b)-Variation of H with u_i at different values of σ_i with fixed values of the plasma parameters β = 0.1, u_{i0} = 0.5, φ =0.01, p_i =1 and V=1.6. Graphs a, b, c and d represent σ_i = 0.05, 0.15, 0.25 and 0.35 respectively.

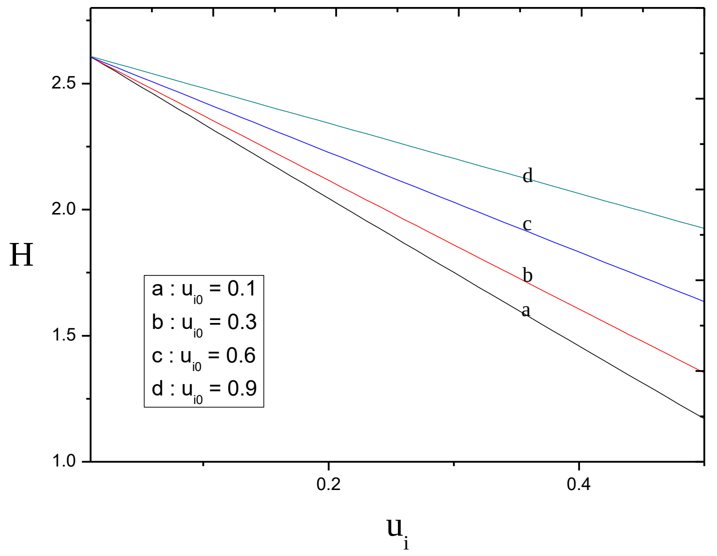

Fig. 2.1(c). Variation of H with u_i at different values of u_{i0} with fixed values of the plasma parameters σ_i = 0.01, β = 0.1, φ = 0.01, p_i = 1 and V=1.6. Graphs a, b, c and d represent u_{i0} = 0.1, 0.3, 0.6 and 0.9 respectively.

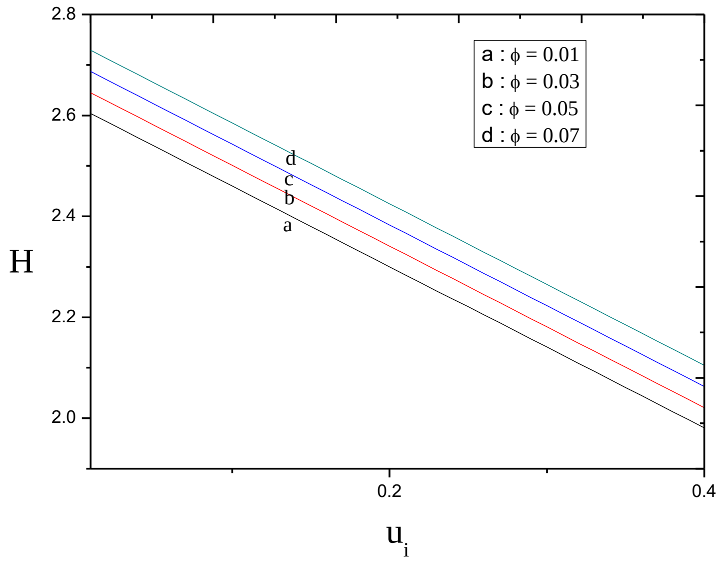

Fig. 2.1(d). Variation of H with u_i at different values of φ with fixed values of plasma parameters σ_i = 0.001, β = 0.1, φ =0.01, p_i =1 and V=1.6. Graphs a, b, c and d represent φ = 0.01, 0.03, 0.05 and 0.07 respectively.

From Fig. 2.1(a) it is seen that H decreases as u_i increases and H is large for large values of β. Fig. 2.1(b) shows that H decreases with the increase of u_i but H large for large values of σ. The variations of H with u_i for different values of u_{i0} are shown in Fig. 2.1(c). It is seen that H sharply decreases with the increase of u_{i0} and H is large for large values of u_{i0}. The effects of ϕ on the variation of H with u_i are shown in Fig. 2.1(d). It is observed that H is large for large values of ϕ. Moreover, H decreases with the increase of u_i.

2.5.2. Critical Values of Plasma Parameters

For the existence of IAS in plasma consisting of warm ions and nonthermal electrons, the critical values of phase velocity (V), nonthermal parameter (β_c) and ion temperature (σ_{ic}) should be considered.

i). Critical Value of Phase Velocity

Let us first the critical values of phase velocities of fast- and slow- modes of the ion-acoustic wave for different values of the nonthermal parameter, ion temperature and ion stream velocity of ions are numerically estimated considering model plasma. The variation of phase velocities with the plasma parameters are graphically shown in Fig 2.2(a) and Fig. 2.2(b).

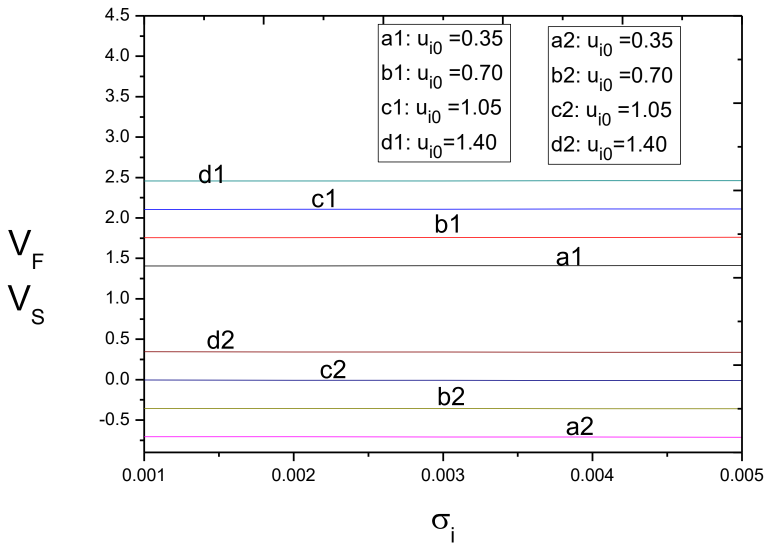

Fig 2.2(a). Variation of phase velocity of Fast- mode (V_F) and Slow -mode (V_S) of the ion-acoustic wave with ion temperature (σ_i) for different values of ion stream velocity (u_{i0}) with fixed values of plasma parameters β =0.1 and p_i =1. With four values u_{i0} = 0.35, 0.70, 1.05 and 1.4, graphs a1, b1, c1 and d1 represent Fast-mode whereas graphs a2, b2, c2 and d2 represent Slow-mode of the wave.

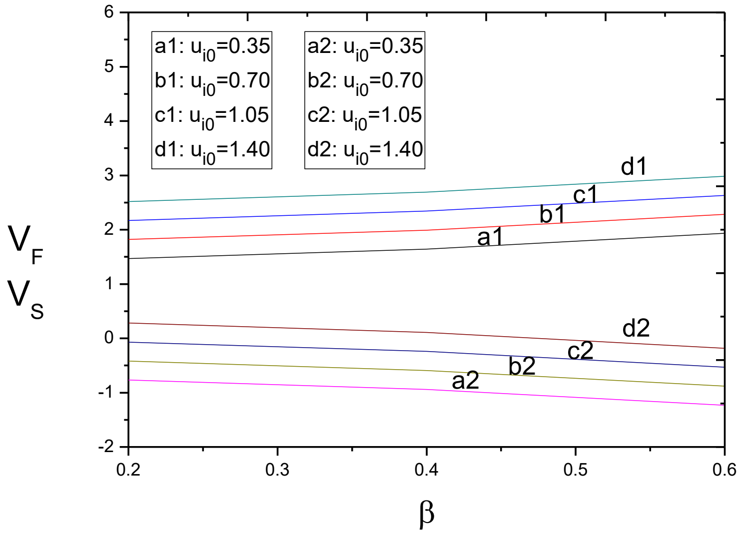

Fig . 2.2(b). Variation of phase velocity of Fast- mode (V_F) and Slow -mode (V_S) of the ion-acoustic wave with nonthermal parameter (β) ion stream velocity (u_{i0}) for different values of ion stream velocity (u_{i0}) with fixed values of plasma parameters σ_i = 0.001 and p_i =1 . With four values u_{i0} = 0.35, 0.70, 1.05 and 1.4, graphs a1, b1, c1 and d1 represent Fast-mode whereas graphs a2, b2, c2 and d2 represent Slow-mode of the wave.

Fig.2.2(a) shows that V_F is always positive and it decreases with the increase of σ_i and increases with increases of v_{i0} in nonthermal plasma. But V_S is always negative and it increases slowly with the increase of σ_i and v_{i0} in the plasma. Similarly, the variations of V_F and V_S with the nonthermal parameter (β) for different values of ion drift velocity (v_{i0}) are numerically estimated and the variation of the phase velocities are shown graphically in Fig. 4.2(b). It is found that V_F is always positive and it increases with the increase of β and v_{i0}. But, V_S is negative it decreases with the increase of β and increases with the increase of v_{i0} . The positive value of V_F indicates that the Fast-mode of the wave is propagating in the plasma. But, the negative value of V_S for v_{i0} = 0.35 to 1.05 means the Slow-mode of the wave is non-propagating in the plasma, whereas positive V_S for u_{i0} = 1.4 indicates that the slow-mode is propagating through the plasma. So, to study the propagation of IAS in nonthermal plasma for Fast mode (V_F) and Slow mode (V_S), the value of ion-stream should be taken as $v_{i0} \approx 1.4$.

ii) Critical Value of Nonthermal Parameter

The critical values nonthermal parameter (β_c) are numerically estimated for different values of plasma parameters and the variations is shown in Fig. 2.3.

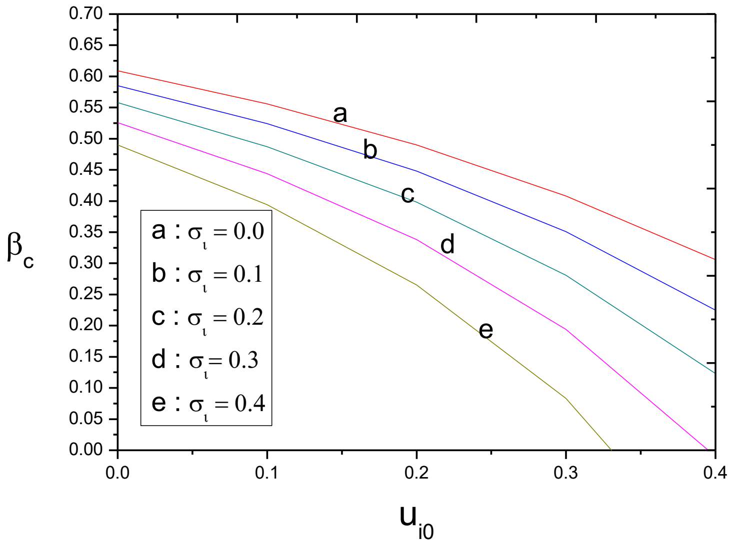

Fig. 2.3. Variation of critical value of nonthermal parameter (β_c) with ion stream velocity (u_{i0}) at different values of ion temperature (σ_i) with fixed values of plasma parameters p_i =1 and V=1.6. Graphs a, b, c, d and e represent five values of σ_i = 0.0, 0.1, 0.2, 0.3 and 0.4.

Fig. 2.3 shows the critical values of the nonthermal parameter of electrons (β_c) for different values of ion temperature (σ_i) and stream velocity of ions (u_{i0}). It is observed that for a given u_{i0}, β_c decreases with the increase of σ_i. So for a given σ_i, β_c decreases with the increase of u_{i0}. Hence we require less number of nonthermal electrons for the existence of solitary waves.

iii) Critical Value of Ion Temperature

The critical value ion temperature (σ_{ic}) is numerically estimated for different values of non-thermal parameters (β) and the variation is shown in Fig. 2.4.

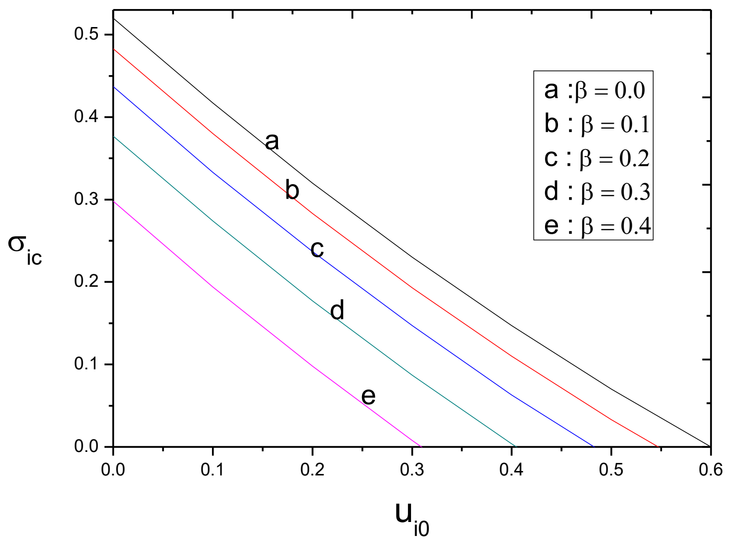

Fig .2.4. Variation of critical value of ion temperature (σ_{iC}) with ion stream velocity (u_{i0}) for different values of nonthermal parameter (β) with constant values of other parameters of the plasma p_i =1 and V=1.6. Graphs a, b, c, d and e represent β = 0.0, 0.1, 02, 0.3 and 0.4 respectively

The critical values of ion temperature (σ_{iC}) for different values of stream velocity of ions (u_{i0}) and nonthermal parameter (β) of electrons are depicted in Fig. 2.4. It is seen that for a given β, σ_{iC} decreases with the increase of u_{i0}. For a given u_{i0}, σ_{iC} decreases with the increase of β.

2.5.3. Profiles of Sagdeev Potential

In order to examine the effects of nonthermal electron (β), stream velocity of ions (u_{i0}) and ion temperature (σ_i) on the IAS structure, we have numerically studied the behaviour of the Sagdeev potential ψ (ϕ) using Eq. (2.34) for different values of the plasma parameters. In Fig. 2.5, the profiles of ψ (ϕ) is shown for an arbitrary value of phase velocity V and different values of nonthermal parameter of electrons.

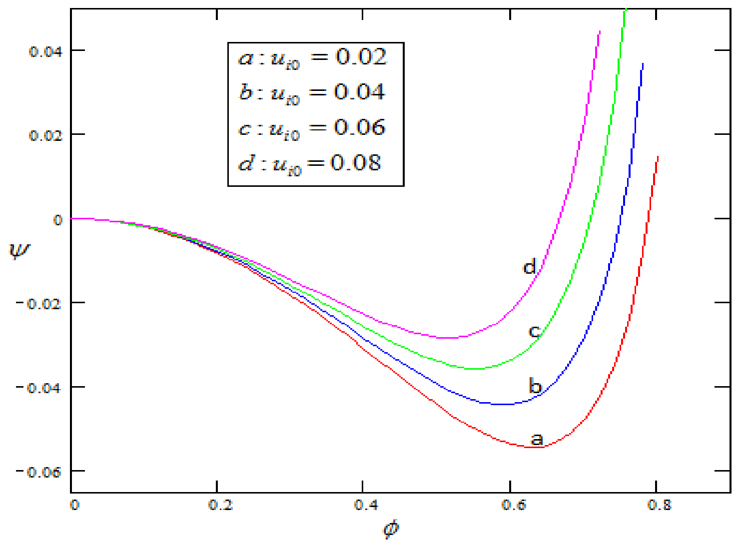

Fig.2.5(a). Profiles of Sagdeev potential $\psi(\phi)$ for different values of ion stream velocity (u_{i0}) with fixed values of plasma parameters σ_i = 0.001, β = 0.01, V=1.35 and p_i = 1. Graphs a, b, c and d represent u_{i0} = 0.02, 0.04, 0.06 and 0.08 respectively.

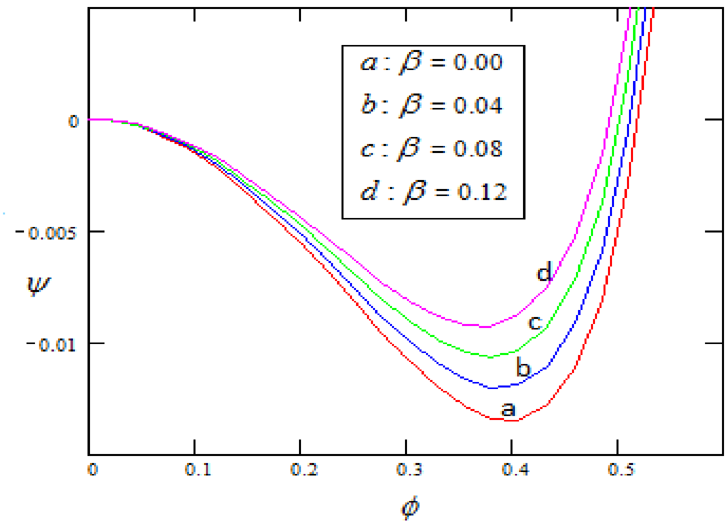

Fig.2.5(b). Profiles of Sagdeev potential $\psi(\phi)$ for different values of nonthermal parameter (β) with fixed values of plasma parameters σ_i = 0.018, V=1.4, p_i = 1 and u_{i0} =0.17. Graphs a, b, c and d represent β = 0.0, 0.04, 0.08 and 0.12 respectively.

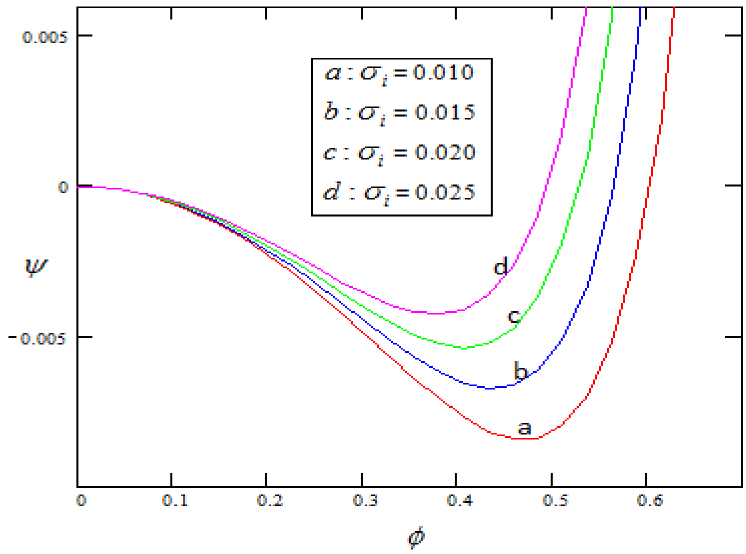

Fig.2.5(c). Profiles of Sagdeev potential $\psi(\phi)$ for different values of ion temperature (σ_i) with fixed values of plasma parameters β = 0.33, V=1.6, p_i = 1 and u_{i0} = 0.24. Graphs a, b, c and d represent σ_i = 0.01, 0.015, 0.02 and 0.025 respectively.

It is observed From Figs. 2.5(a, b, c), that the Sagdeev potential depends on the values of the plasma parameters. The depths of Sagdeev potential in the plasma increase with the decrease of u_{i0} , β and σ_i when the values of other plasma parameters are fixed.

2.5.4. Profiles of IAS for Arbitrary(fixed) Value of Phase Velocity

To see the effects of nonthermal electron (β), ion temperature (σ_i) and ion stream velocity of ions (u_{i0}), on small amplitude IAS in plasma, we have numerically studied (2.41) for an arbitrary value phase velocity V and different values of u_{i0} , β and σ_i.

i). Effect of Ion Stream Velocity:

Fig.2.6(a) is drawn for the profiles of IAS for different values of u_{i0} when other plasma parameters have fixed values.

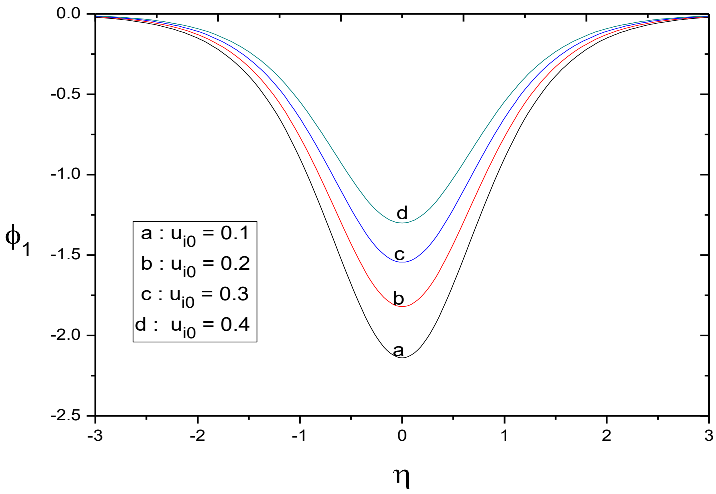

Fig. 2.6(a). Profile of IASW for different value of ion stream (u_{i0}) in plasma with fixed values of plasma parameters V=1.6, β = 0.2 and p_i =1. Graphs a, b, c and d represent u_{i0} = 0.1, 0.2, 0.3 and 0.4 respectively.

In Fig. 2.6(a), the profiles of IAS in plasma is shown for different values of ion stream (u_{i0}) in plasma. It is found that the IAS is rarefactive and the amplitude of IAS increases with the decrease of u_{i0} when all other parameters are fixed.

ii). Effect of Nonthermal Parameter of Electrons:

Fig.2.6(b) is drawn for the profiles of IAS for different values of β when other plasma parameters have fixed values.

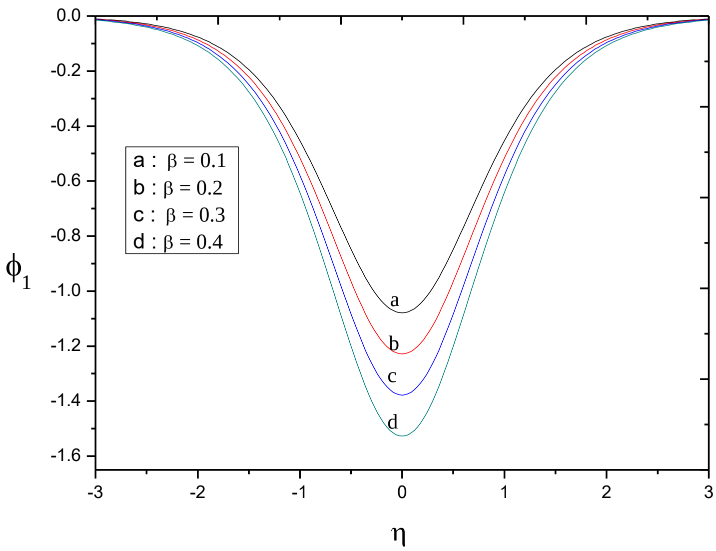

Fig. 2.6(b). Profile of IAS for different value of nonthermal parameter (β) in plasma with fixed values of plasma parameters V=1.6, u_{i0} =0.1, p_i =1 and σ_i = 0.1. Graphs a, b, c and d represent β = 0.1, 02, 0.3 and 0.4 respectively.

In Fig. 2.6(b), the profiles of IAS in plasma is shown for different values nonthermal parameter β of electrons in plasma. It is found that the IAS is rarefactive and the amplitude of IAS increases with the increase of β when all other parameters are fixed. In a similar way, the profiles of IAS can be drawn taking different values of ion temperature and ion stream in the plasma.

iii). Effect of Ion Temperature:

Fig.2.6(c) is drawn for the profiles of IAS for different values of σ_i when other plasma parameters have fixed values.

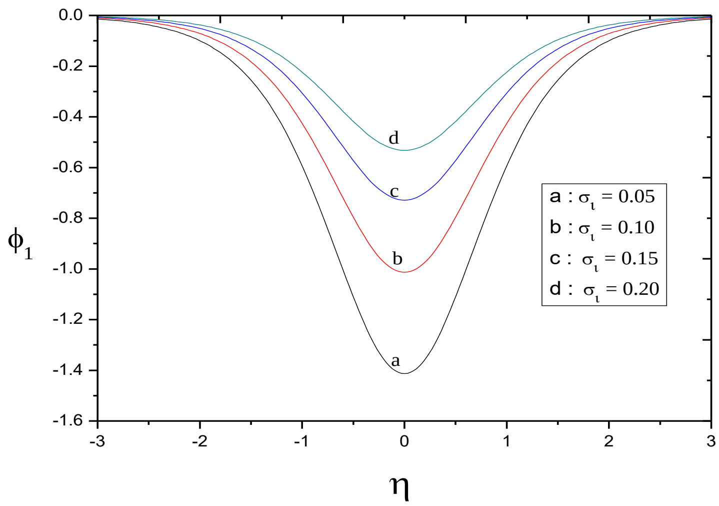

Fig. 2.6(c). Profile of IAS for different values of ion temperature (σ_i) in plasma with fixed values of plasma parameters V=1.6, u_{i0} =0.1, β = 0.2 and p_i =1. Graphs a, b, c and d represent σ_i = 0.05, 01, 0.15 and 0.2 respectively.

In Fig. 2.6(c), the profiles of IAS in plasma is shown for different values of ion temperature (σ_i) in plasma. It is found that the IAS is rarefactive and the amplitude of IASW increases with the decrease of σ_i when all other parameters are fixed.

2.5.5. Profiles of IAS for Slow-Mode and Fast-Mode

So, to see the real nature of IAS in nonthermal plasma in presence of an ion stream, the values of the phase velocities V_S and V_F should be used instead of arbitrary (fixed) value phase velocity for drawing the profiles of the IAS. Using V_S and V_F for our model of nonthermal plasma, the profiles of small amplitude IAS for the Slow-mode and Fast -mode are drawn for different values of the parameters of ion stream velocity, nonthermal parameter of electrons and ion temperature.

i). Effect of Ion Stream Velocity:

The main factor of the excitation of Slow-mode and Fast- mode of IAS in nonthermal plasma is the stream velocity of ions. For the Slow-mode and Fast-mode of the IAS , the profiles of small amplitude IAS are drawn as shown in Fig. 2.7(a) and Fig. 2.7(b).

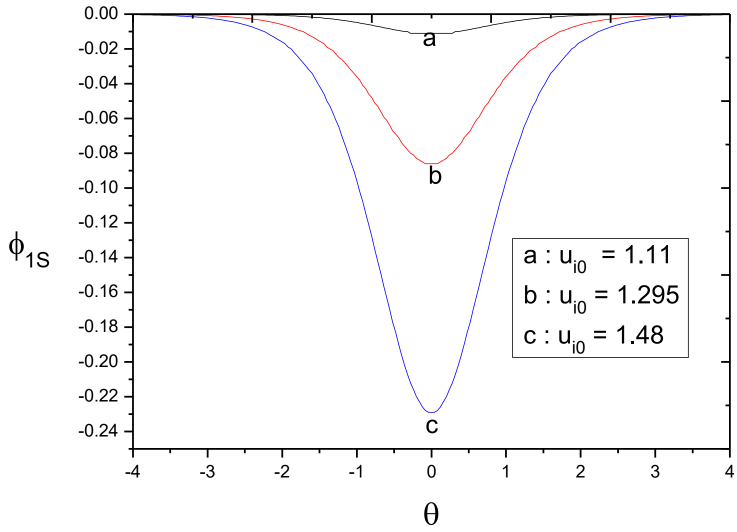

Fig. 2.7(a). Profile of Slow- mode of Small amplitude IAS for different values of ion stream velocity (u_{i0}) in plasma with fixed values of plasma parameters σ_i =0.0001, β = 0.01 and p_i =1. Graphs a, b, c and d represent u_{i0} = 1.11, 1.295 and 1.48 respectively.

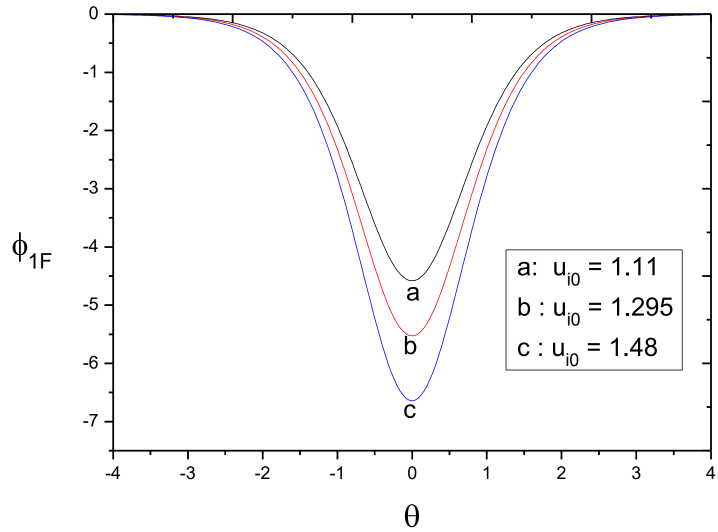

Fig. 2.7(b). Profile of Fast- mode of Small amplitude IAS for different values of ion stream velocity (u_{i0}) in plasma with fixed values of plasma parameters σ_i =0.0001, β = 0.01 and p_i =1. Graphs a, b, c and d represent u_{i0} = 1.11, 1.295 and 1.48 respectively.

Fig. 2.7(a) and 2.7(b) show that Small amplitude IAS for both Slow-mode and Fast-mode are rarefactive. In both modes, the amplitude of IAS increases with the increase of ion stream velocity . The amplitude for Slow-mode is much smaller than that of the Fast-mode of IAS in nonthermal plasma.

ii). Effect of Nonthermal Parameter of Electrons:

In same plasma , the profiles of Small- amplitude IAS for Slow-mode and Fast- mode for different values of nonthermal parameter of electrons are shown in Figs.2.8(a) and 2.8(b).

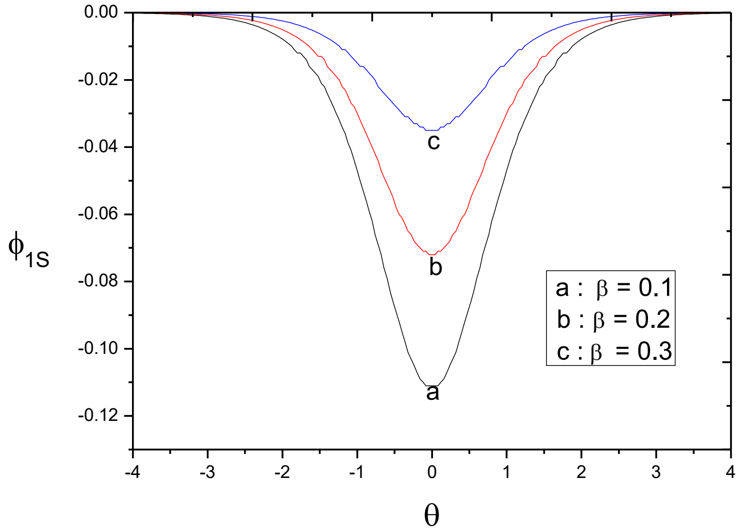

Fig. 2.8(a). Profile of Slow- mode of Small amplitude IAS for different values of nonthermal parameter (β) of electrons in plasma with fixed values of plasma parameters σ_i =0.001, u_{i0} = 1.2 and p_i =1. Graphs a, b and c represent β = 0.1, 0.2 and 0.3 respectively

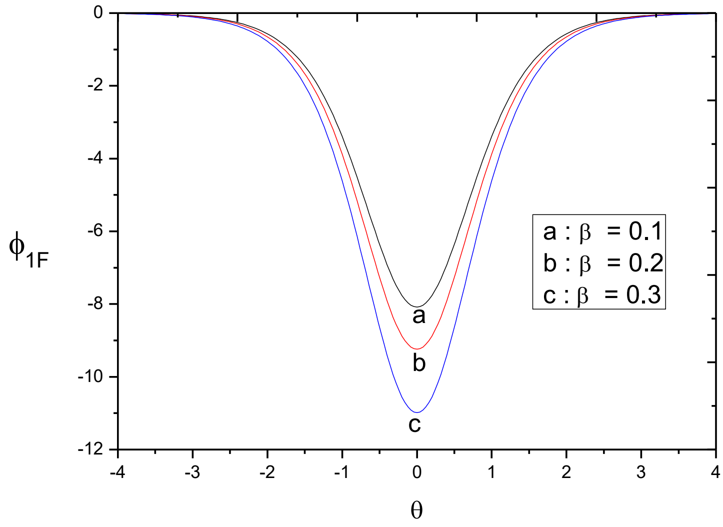

Fig. 2.8(b). Profile of Fast- mode of Small amplitude IAS for different values of nonthermal parameter (β) of electrons in plasma with fixed values of plasma parameters σ_i =0.001, u_{i0} = 1.2 and p_i =1. Graphs a, b and c represent β = 0.1, 0.2 and 0.3 respectively

Fig. 2.8(a) and 2.8(b) show that Small amplitude IAS for both Slow-mode and Fast-mode are rarefactive. In both modes, the amplitude of IAS increases with the decrease of nonthermal parameter of electrons. The amplitude of IAS for Slow-mode is much smaller than that of the Fast-mode of IAS in nonthermal plasma.

iii). Effect of Ion Temperature

Similarly, the profiles of small amplitude IAS in same plasma for different values of ion temperature for Slow-mode and Fast-mode are shown in Figs. 2.9(a) and 2.9(b).

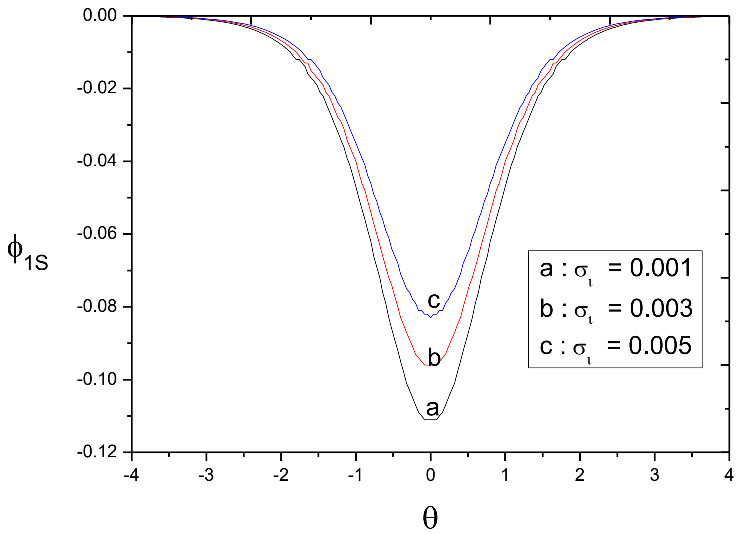

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Fig. 2.9 (a). Profile of Slow -mode of Small amplitude IAS for different values of ion temperature (σ_i) in plasma with fixed values of plasma parameters $\beta = 0.1$, u_{i0} = 1.4 and p_i =1. Graphs a, b and c represent σ_i = 0.001, 0.003 and 0.005 respectively.

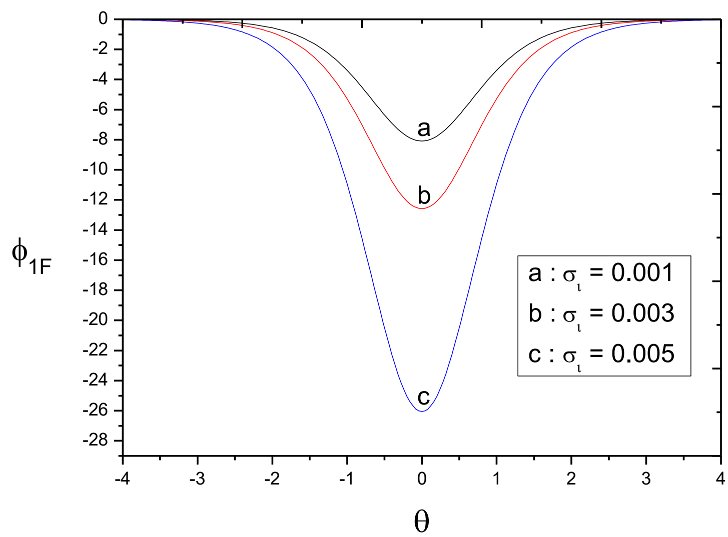

Fig. 2.9 (b). Profiles of Fast- mode of Small amplitude IAS for different values of ion temperature (σ_i) in plasma with fixed values of plasma parameters $\beta = 0.1$, u_{i0} = 1.4 and p_i =1. Graphs a, b and c represent σ_i = 0.001, 0.003 and 0.005 respectively.

It is observed from Fig.2.9(a) and Fig. 2.9(b) that Small amplitude IAS for different values of ion temperature for Slow-mode and Fast-mode of the IAS are rarefactive in nonthermal plasma. For Slow-mode, the amplitude of IAS increases due to decrease of ion temperature. But, For Fast-mode, the amplitude of IAS increases with the increase of ion temperature in the plasma.

2.5.5. Amplitude of IAS for Slow-Mode and Fast- Mode

It is observed from the profiles Fig.2.7(a) to Fig.2.9(b) that the IAS are rarefactive in nature. The amplitudes of Slow-mode and Fast-mode are seen to be almost same, but close observation shows that these are different in magnitude. For this reason, we have numerically estimated the amplitude of the IAS for Slow-mode (φ_{01S}) and Fast-mode (φ_{01F}) using Eq.(2.46a) for different values of the plasma parameters in nonthermal plasma. In Table-1, Table-2 and Table-3 we have shown the amplitudes of IAS for different values of ion stream velocity (u_{i0}), nonthermal parameter of electrons (β) and ion temperature respectively (σ_i).

Table-1

Effect of Ion Stream Velocity (u_{i0})

u_{i0}	φ_{01S} for different u_{i0}	φ_{01F} for different u_{i0}
0.185	**-0.297**	**-0.619**
0.370	**-0.177**	**-0.828**
0.555	**-0.088**	**-1.070**
0.740	**-0.030**	**-1.346**
0.925	**-0.002**	**-1.663**
1.110	**-0.005**	**-2.025**
1.295	**-0.038**	**-2.445**
1.480	**-0.102**	**-2.937**

Table-2

Effect of Nonthermal Parameter (β):

β	φ_{01S} for different β	φ_{01F} for different β
0.1	**-0.015**	**-5.971**
0.2	**-0.001**	**-6.674**
0.3	**-0.002**	**-7.669**
0.4	**-0.003**	**-9.213**
0.5	**-0.040**	**-12.005**
0.6	**-0.139**	**-18.889**
0.7	**-0.386**	**-71.738**
0.8	**-1.062**	**23.277**

Table-3
Effect of Ion Temperature σ_i

σ_i	φ_{01S} for different σ_i	φ_{01F} for different σ_i
0.001	**-0.015**	**-5.971**
0.002	**-0.00931**	**-6.571**
0.003	**-0.05305**	**-7.207**
0.004	**-0.00248**	**-7.933**
0.005	**-0.00075**	**-8.800**
0.006	**-0.00003**	**-9.878**
0.007	**-0.00023**	**-11.274**
0.008	**-0.00115**	**-13.173**

2.6. SUMMARY AND CONCLUSION

We have made an analytical study for the existence of IAS in collisionless, unmagnetized plasma consisting of warm streaming ions and nonthermal electrons. Some necessary and sufficient conditions for the solutions of IAS of the nonlinear equations governing the dynamics of the ion-acoustic waves are also obtained. We have obtained expressions for the critical values of the nonthermal parameter of electrons, ion temperature and velocity of the wave for the existence of IAS. The dependence of these quantities on different plasma parameters has been depicted graphically and discussed. An important observation is that for the presence of streaming ions two distinct wave modes (slow-mode and fast-mode) are possible. To examine the effects of stream velocity of ions, nonthermal electrons and ion temperature on the structure of IAS , we have studied graphically the behaviour of the Sagdeev potential. It is shown that the stream velocity of ions, nonthermal electrons and ion temperature have considerable effects on changing the nature of Slow-mode and Fast-mode of the IAS. Finally, we would like to point out that the analysis presented here for the model plasma with warm streaming ions and nonthermal electrons and it may be of relevance to some space plasmas such as solar wind plasma and cometary plasmas.

REFERENCES

[1]. H. Washimi and T. Taniuti, Phys. Rev. Letts., **17,** p.966 (1966).
[2]. S. G. Tagare, Plasma Phys., **15,** p.1247 (1973).
[3]. R. K . Roychoudhury and S. Bhattacharya, Canadian J. Phys., **65**, p.699 (1987).
[4]. G. C. Das, IEEE Trans. Plasma Sci., **3**, p.168 (1975).
[5]. S. Chattopadhyaya and S. N. Paul, The African Review of Physics, **7.003,** p.289 (2012).
[6]. B. N. Goswami and B. Buti, Phys. Letts. **A 57,** p.149 (1976).
[7]. K. K. Ghosh, B. Paul, C. Das and S. N. Paul, J. Phys. A: Math. Theor., **41**, p.335501 (2008).
[8]. L. C. Lee and J. R . Kan, Phys. Fluids, **24**, p.430 (1981).
[9]. K. K . Ghosh, S. N. Paul and S. Chattopadhyaya, FIZIKA, **A 13**, p.33 (2004).
[10]. H. Schamel, Plasma Phys., **14,** p.905 (1972).
[11]. H. Schamel. J. Plasma Phys., **9,** p.377 (1973).
[12]. N. Saini, I. Kourakis and M. A. Hellberg, Phys. Plasmas, **16,** p.062903 (2009).
[13]. M. R. Rauhani and Z. E. Abbasi, Iranian J. Phys. Research, **12,** p.84 (2013).
[14]. R. A. Cairns, A. A. Mamun, R. Bingham, R. Bostrom, R. O. Dendy, C. M. C. Nairn and P. K. Shukla, Geophys. Res. Letts., **22,** p.2709 (1995).
[15]. D. S. Hall, C. P. Chaloner, D. A. Bryant, D. R. Lepine and V. P. Tritakis, J. Geophys. Res. , **96,** p.7869 (1991).
[16]. R. A. Cairns, A. A. Mamun, R. Bingham and P. K. Shukla, Physica Scripta, **T63,** p.80 (1996).
[17]. A. A. Mamun, Phys. Rev., **E 55,** p.1852 (1997).
[18]. A. A. Mamun, The Euro. Physical J., **D 11**, p.143 (2000).
[19]. A. Bandyopadhyay and K. P. Das, J. Plasma Phys., **62,** p.255 (1999).
[20]. A. Bandyopadhyay and K.P. Das, Physica Scripta, **61**, p.92 (2000).
[21]. A. Bandyopadhyay and K. P. Das, J. Plasma Phys., **68,** p.285 (2002).
[22]. S. K. Bhattacharya, S. N. Paul and B. Chakraborty, Indian J. Phys. , **77B,** p.327 (2003).
[23]. I.Kourakis and P.K. Shukla, J.Phys.A: Math. Gen., **36,** p.11901 (2003).
[24]. G.C.Das and S.N. Paul, Phys. Fluids, **28,** p.823 (1985)
[25]. S. N. Paul, C.Das, I.Paul, B. Bandyopadhyay, S.Chattopadhya and S .S. De, Indian J.Phys., **86 (6)**, p. 545 (2012).
[26]. I. Paul, B. Ghosh, G. Pakira, S. N. Paul and S. Chattopadhyay, Indian J. Phys., **86,** p.395 (2012).
[27]. B. Ghosh, S. Banerjee and S. N. Paul, Indian. J. Pure & Appl. Phys., **51,** p.488 (2013).
[28]. R.Z.Sagdeev, Rev. Plasma Phys edited by M A Leontovich, New York (Consultant Bureau), 4, p. 23 (1966)

CHAPTER-3

GENERATION OF W-TYPE ION-ACOUSTIC SOLITONS IN PLASMA CONSISTING OF COLD IONS AND NONTHERMAL ELECTRONS

[This Chapter is based on the publication: Indian Journal of Physics, Vol. 90 (10), pp. 1195–1205 (2016)]

3.1. INTRODUCTION

In the previous Chapter-2, the analytical solution of the nonlinear equations for the propagation of ion-acoustic solitary waves or solitons (IAS) in plasma consisting of warm streaming positive ions and nonthermal electrons has been studied. Moreover, the expression of Sagdeev potential has been derived and the effects of nonthermal electrons have been discussed graphically. The solution of IAS is also obtained and the structure of solitary wave in the plasma having nonthermal electrons is graphically discussed.

However, from the experimental observations [1- 4], it is found that theoretically predicted values of the amplitude, width, and velocity of the first-order solitary waves in plasma do not obey the experimental results. To remove the discrepancy between theoretical and experimental results researchers realized the necessity of considering higher-order nonlinear and dispersive terms in the propagation of solitary waves in a plasma, Ichikawa *et al* [5] were the first to examine such higher-order effects by the reductive perturbation method (RPM) for IAS in plasma consisting of cold ions. Considering the temperature of ions in plasma, Lai [6] have investigated the higher-order effect on the IAS. Kodama and Taniuti [7] have reconsidered this problem and have shown by the method of renormalization on how to eliminate the secular terms appearing in higher-order terms of the expansion. Considering the effect of higher order nonlinearity the IAS correct up to second order in plasma have been studied by various authors. [8-13]. Taking the electrons as nonisothermal, Kalita and Bujarbarua [8] have investigated the propagation of higher-order IAS in plasma. Tagare and Reddy [9] have considered the effect of higher-order nonlinearity on the IAS in plasma consisting of negative ions and nonisothermal electrons. Subsequently, Moslem [10] has studied higher-order contributions to the IAS in warm multicomponent plasma with an electron beam. Paul *et al*. [11] have considered the *electron*-inertia and drift motion for the propagation of higher order IAS in negative ion plasma. To get better theoretical results for matching the experimentally observed value, Das and Majumdar [12] and Majumdar *et al*. [13]

have considered the effect of higher-order nonlinear and dispersive terms up to the third order approximation and have studied the IAS in a plasma.

However, no authors have studied the IAS critically in presence of the nonthermal electrons in plasma especially considering higher order nonlinearity and dispersive effects. So, in this Chapter, we have investigated theoretically the IAS correction up to third-order having nonthermal electrons in cold plasma using the Bogoliubov-Krylov-Mitropolosky (BKM) method [14]. It is seen that compressive as well as rarefactive W-type IAS would be generated in third-order IAS in the plasma consisting of cold positive ions and nonthermal electrons.

3.2. BASIC EQUATIONS

We consider unmagnetized collisionless plasma consisting of cold ions and nonthermal electrons. The governing equations for such a plasma system can be written in the following dimensionless form:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{3.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \varphi}{\partial x} = 0 \quad . \tag{3.2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i \tag{3.3}$$

where n_i and n_e are, respectively, the number densities of ions and electrons; u_i is the ion-fluid velocity; φ is the electrostatic potential. The quantities n_i, n_e, u_i and φ are made dimensionless by $n_0, n_0, \left(k_B T_e / m_i\right)^{1/2}$ and $k_B T_e / e$, respectively, where n_0 is the unperturbed electron or ion density, m_i is the mass of the ion, and k_B is Boltzmann constant.

Following the model of Cairns *et al* [15] for nonthermal electrons in plasma, the normalized electron density n_e is assumed as

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)\exp(\phi) \tag{3.4}$$

with, $$\beta = \frac{4\alpha}{1+3\alpha} \tag{3.5}$$

where α is a parameter determining the number of nonthermal electrons present in our plasma model. β measures the deviation from thermalized state.

3.3. SAGDEEV POTENTIAL AND CONDITIONS FOR SOLITON SOLUTION

For a solitary-wave solution we assume that the dependent variables depend on a single independent variable $\xi = x - Vt,$ where V is the velocity of the solitary wave. Using the boundary conditions $n \to 1,\ \phi \to 0$ as $\xi \to \pm\infty$ in the Eqs. (3.1)- (3.3), we get:

$$u_i = V\left(1 - \frac{1}{n_i}\right) \tag{3.6}$$

$$n_i = \frac{V}{\sqrt{V^2 - 2\varphi}} \tag{3.7}$$

$$\frac{d^2\varphi}{d\xi^2} = (1 - \beta\varphi + \beta\varphi^2)\exp(\varphi) - \frac{1}{\sqrt{1 - \frac{2\varphi}{V^2}}} \tag{3.8}$$

Eq. (3.8) can be written as

$$\frac{d^2\varphi}{d\xi^2} = -\frac{d\psi}{d\varphi} \tag{3. 9}$$

Ψ is the Sagdeev potential and it is given by

$$\psi(\phi) = (1 + 3\beta) - e^{\phi} + \beta(\phi - 1)e^{\phi} - \beta\left(\phi^2 - 2\phi + 2\right)e^{\phi} + V^2\left(1 - \sqrt{1 - \frac{2\varphi}{V^2}}\right) \tag{3.10}$$

For solitary wave solution of (3.9), the Sagdeev potential $\psi(\phi)$ in equation (3.10) must satisfy the conditions stated in Chapter-2. The critical values of the velocity ($V = V_c$) and the non-thermal parameter ($\beta = \beta_c$) for the existence of solitons are obtained as

$$V_c = \sqrt{\frac{1}{1 - \beta}} \tag{3. 11a}$$

and

$$\beta_c = (1 - \frac{1}{V^2}) \tag{3.11b}$$

3.4. SOLUTIONS OF ION-ACOUSTIC SOLITONS

Expanding the right-hand side of (3. 9) in ascending powers of we get

$$\frac{d^2\varphi}{d\xi^2} = \Delta_1\varphi + \Delta_2\varphi^2 + \Delta_3\varphi^3 + \Delta_4\varphi^4 + ... \quad . \tag{3. 12}$$

where,

$$\Delta_1 = (1-\beta) + \frac{1}{V^2}, \tag{3. 13a}$$

$$\Delta_2 = \frac{1}{2}(1-\frac{3}{V^4}), \tag{3. 13b}$$

$$\Delta_3 = (\frac{1}{6} + \frac{\beta}{2} - \frac{5}{2V^6}), \tag{3. 13c}$$

And

$$\Delta_4 = (\frac{1}{24} - \frac{\beta}{3} - \frac{35}{8V^8}) \tag{3. 13d}$$

An integral of (3. 12) satisfying the conditions $d\varphi/dX \to 0, \varphi \to 0$ as $\xi \to \pm\infty$ is

$$\frac{\varepsilon}{2}\left(\frac{d\varphi}{dX}\right)^2 = \frac{1}{2}\Delta_1\varphi^2 + \frac{1}{3}\Delta_2\varphi^3 + \frac{1}{4}\Delta_3\varphi^4 + \frac{1}{5}\Delta_4\varphi^5 + ... \tag{3. 14}$$

where we have stretched the ξ coordinate according to the relation

$$X = \in^{1/2} \xi \tag{3. 15}$$

.

The parameter ε is dimensionless and small ε < 1. It measures the dispersion and nonlinear effects.

We now make the following perturbation expansions for φ and V.

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + ..., \tag{3. 16}$$

$$V = V_0 + \varepsilon V^{(1)} + \in^2 V^{(2)} + ... \tag{3.17}$$

Now, for linear dispersion relation, $(\Delta_1)_0 = \delta_1^{(0)} = 0$.

Therefore,

$$1-\beta-\frac{1}{V_0^2} = 0, \text{ i.e. } V_0 = \sqrt{\frac{1}{(1-\beta)}} \tag{3.18}$$

With the expansion of V given by (3.17) the expansions for Δ_1, Δ_2, Δ_3 and Δ_4 become

$$\Delta_1 = \delta_1^{(0)} + \varepsilon\delta_1^{(1)} + \varepsilon^2\delta_1^{(2)} + \varepsilon^3\delta_1^{(3)} + \cdots\cdots \tag{3.19a}$$

$$\Delta_2 = \delta_2^{(0)} + \varepsilon\delta_2^{(1)} + \varepsilon^2\delta_2^{(2)} + \cdots\cdots \tag{3.19b}$$

$$\Delta_3 = \delta_3^{(0)} + \varepsilon\delta_3^{(1)} + \cdots\cdots \tag{3.19c}$$

$$\Delta_4 = \delta_4^{(0)} + .\varepsilon\delta_4^{(1)} + \varepsilon^2\delta_4^{(2)} + \cdots\cdots \tag{3.19d}$$

Where

$$\left.\begin{aligned}
&\delta_1^{(0)} = 0, \delta_1^{(1)} = 2V_0^{-3}V^{(1)}\\
&\delta_1^{(2)} = 2V_0^{-3}V^{(2)} - 3V_0^{-4}V^{(1)^2}\\
&\delta_1^{(3)} = 2V_0^{-3}V^{(3)} - 6V_0^{-4}V^{(1)}V^{(2)} + 4V_0^{-5}V^{(1)3}\\
&\delta_2^{(0)} = \frac{1}{2}(1-3V_0^{-4})\\
&\delta_2^{(1)} = 6V_0^{-5}V^{(1)}\\
&\delta_2^{(2)} = 6\ V_0^{-5}V^{(2)} - 15V_0^{-6}\ V^{(1)^2}\\
&\delta_3^{(0)} = (\frac{1}{6} + \frac{\beta}{2} - \frac{5}{2}V_0^{-6})\\
&\delta_3^{(1)} = 15\ V_0^{-7}V^{(1)}\\
&\delta_4^{(0)} = (\frac{1}{24} - \frac{\beta}{3} - \frac{35}{8}V_0^{-8})
\end{aligned}\right\} \tag{3.20}$$

Substituting the expansions (3.16) in (3.14) we get a sequence of equations for the $\varphi^{(i)}$'s and the equation for $\varphi^{(i)}$ at each order becomes a first-order inhomogeneous differential equation for $i > 1$.

For linear dispersion relation, $(\Delta_1)_0 = \delta_1^{(0)} = 0$.

Therefore, $1-\beta-\frac{1}{V_0^2} = 0$, i.e. $V_0 = \sqrt{\frac{1}{(1-\beta)}}$

3.4.1. First-Order Equation and its Solution

In the first order i.e. in the lowest order which is at the order of ε^3 we obtain the following equation for $\varphi^{(1)}$.

$$\frac{1}{2}(\frac{d\varphi^{(1)}}{dX})^2 = \frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2} + \frac{1}{3}\delta_2^{(0)}\varphi^{(1)^3} \tag{3.21}$$

The solution of which is the first- order KdV soliton

$$\phi^{(1)} = l_0 \sec h^2\eta \tag{3.22}$$

where

$$l_0 = -\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}} = 3\bar{V}_0 V^{(1)} \tag{3 22a}$$

$$\bar{V}_0 = (3-V_0^4)^{-1} \tag{3.22b}$$

$$\eta = \frac{\sqrt{\delta_1^{(1)}}}{2}X = \left(\frac{V_0^{-3}V^{(1)}}{2}\right)^{1/2} X \tag{3.22c}$$

$\phi^{(1)}$ vanishes at infinity and it attains a maximum value at η = 0 giving the amplitude of first order solitary wave $l_0, l_0 = \alpha_{01}(say)$.

The width 'D' of the soliton is defined as half the value of width of pulse at a height of 0.42 of its amplitude.
The first- order width *D₁ of* the solitary wave is

$$D_1 = \left(\frac{2}{V_0^{-3}V^{(1)}}\right)^{1/2} \eta_1 \tag{3.22d}$$

where η_1 is the positive root of the equation, $\tanh^2 \eta_1 = 0.58$.

3.4.2. Second-Order Equation and its Solution

At the second- order we get the following equation for $\varphi^{(2)}$ using (3. 16) and (3.22a)

$$\frac{d\phi^{(2)}}{d\eta} - \frac{\phi^{(1)}_{\eta\eta}}{\phi^{(1)}_{\eta}}\phi^{(2)} = \frac{\delta_1^{(2)}}{V_0V^{(1)}}\frac{{\phi^{(1)}}^2}{\phi^{(1)}_{\eta}} + \frac{2\delta_2^{(1)}}{3V_0V^{(1)}}\frac{{\phi^{(1)}}^3}{\phi^{(1)}_{\eta}} + \frac{\delta_3^{(0)}}{2V_0V^{(1)}}\frac{{\phi^{(1)}}^4}{\phi^{(1)}_{\eta}} \tag{3.23}$$

Removing the secular term the solution of (3.23) is obtained as

. .

$$\phi^{(2)} = \left[l_1 + l_2 \tan h^2\eta\right]\sec h^2\eta \tag{3.24}$$

where

$$l_1 = \frac{2l_0^2}{\delta_1^{(1)}}\left[\frac{1}{3}\delta_2^{(1)} + \frac{1}{2}l_0\delta_3^{(0)}\right], l_2 = -\frac{l_0^3}{2}\frac{\delta_3^{(0)}}{\delta_1^{(1)}} \quad , l_0 = -\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}} \tag{3.24a}$$

and $V^{(2)} = \dfrac{3(V^{(1)})^2}{2V_0}$

The amplitude of second- order solitary wave is (at η = 0) $l_1 = \alpha_{02}$ (say),

The second- order width *D₂* of the solitary wave is

$$D_2 = \left(\sqrt{\frac{2}{V_0^{-3}V^{(1)}}}\right)\eta_2 \tag{3.24b}$$

where η_2 is the positive root of the equation

$$(l_2/l_1)\tanh^4\eta_2 - [1 + 2(l_2/l_1)]\tanh^2\eta_2 + [(l_2/l_1) + 0.58) = 0 \tag{3.24c}$$

l_1 and l_2 are given above.

3.4.3. Third-Order Equation and its Solution

We obtain the following equation for $\varphi^{(3)}$ at the third- order, in which we use relation (3.16) and introduce the new variable η given by (3.22c),

$$\frac{d\phi^{(3)}}{d\eta}-\frac{\phi^{(1)}_{\eta\eta}}{\phi^{(1)}_{\eta}}\phi^{(3)}=\frac{2}{V_0V^{(1)}\phi^{(1)}_{\eta}}\left[\frac{1}{2}\delta_1^{(1)}\phi^{(2)^2}+\frac{1}{2}\delta_1^{(3)}\phi^{(1)^2}+\delta_2^{(0)}\phi^{(1)}\phi^{(2)^2}+\delta_2^{(1)}\phi^{(1)^2}\phi^{(2)}+\frac{1}{3}\delta_2^{(2)}\phi^{(1)^3}\right.$$
$$\left.+\delta_1^{(0)}\phi^{(1)^3}\phi^{(2)}+\frac{1}{4}\delta_3^{(1)}\phi^{(1)^4}+\frac{1}{5}\delta_4^{(0)}\phi^{(1)^5}-\frac{1}{4}V_0V^{(1)}\left(\frac{d\phi^{(2)}}{d\eta}\right)^2\right] \tag{3.25}$$

Removing the secular term the solution of (3.25) for $\varphi^{(3)}$ is obtained as

$$\phi^{(3)}=\left(l_3+l_4\tan h^2\eta+l_5\tan h^4\eta\right)\sec h^2\eta \tag{3.26a}$$

where,

$$\left.\begin{aligned}
l_3&=\frac{(l_1+l_2)^2}{2l_6^2}\left[\frac{\delta_1^{(1)}}{2l_0}+\delta_2^{(0)}\right]+\frac{l_0(l_1+l_2)}{2l_6^2}\left[\delta_2^{(1)}+\frac{l_0}{3}\delta_2^{(2)}\right]+\frac{l_0^2}{2l_6^2}\left[(l_1+l_2)\delta_3^{(0)}+\frac{l_0}{4}\delta_3^{(1)}+\frac{l_0^2}{5}\delta_4^{(0)}\right]\\
l_4&=l_2(l_1+l_2)\left[\frac{4}{l_0}+\frac{\delta_2^{(0)}}{l_6^2}\right]+\frac{l_2}{2l_6^2}\left[l_0\delta_2^{(1)}+\frac{l_2}{2l_0}\delta_1^{(1)}\right]+\frac{l_0^2}{2l_6^2}\left[(l_1+2l_2)\delta_3^{(0)}+\frac{l_0}{4}\delta_3^{(1)}+\frac{2}{5}l_0^2\delta_4^{(0)}\right]\\
l_5&=-\frac{l_0^2}{6l_6^2}\left[l_2\delta_3^{(0)}+\frac{l_0^2}{5}\delta_4^{(0)}\right]-l_2^2\left[\frac{4}{3l_0}+\frac{\delta_2^{(0)}}{6l_6^2}\right]\\
l_6^2&=\frac{1}{4}\delta_1^{(1)}
\end{aligned}\right\} \tag{3.26b}$$

After simplification we get

$$\left.\begin{aligned}
l_3&=\frac{6V_0^3V^{(3)}}{5}a_2,\ l_4=\frac{6V_0^3V^{(3)}}{5}b_2,\ l_5=\frac{6V_0^3V^{(3)}}{5}c_2\\
a_2&=-\frac{27\times15V_0^2}{32}-\frac{27\times13V_0^2}{10}+\frac{81\times7V_0^2}{8}-9\\
b_2&=-\frac{81\times7\times9V_0^2}{32}-\frac{27\times13V_0^2}{32}+\frac{81\times19V_0^2}{8}\\
c_2&=-\frac{27\times7V_0^2}{32}+\frac{9\times13V_0^2}{10}\\
V^{(3)}&=\frac{5}{2}\frac{(V^{(1)})^3}{V_0^2}
\end{aligned}\right\} \tag{3.26c}$$

The amplitude of third-order solitary wave is l_3, $l_3 = \varphi_{03}(say)$, $\alpha_{03} = \frac{6}{5} a_2 V_0^3 V^{(3)} = 3a_2 V_0 (V^{(1)})^3$

So using the values of $\varphi^{(1)}, \varphi^{(2)}$, and $\varphi^{(3)}$ given by (3.22), (3.24) and (3.26a) respectively, the solitary wave solutions Φ_1, Φ_2, Φ_3 with corrections become

$$\Phi_1 = \varphi^{(1)} = l_0 \sec h^2\eta \tag{3.27a}$$

$$\Phi_2 = \varphi^{(1)} + \varphi^{(2)} = \alpha_1\left[1 + \alpha_1' \tanh^2 \eta\right]\sec h^2\eta \tag{3.27b}$$

$$\Phi_3 = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} = \alpha_3\left[\ 1 + \alpha_2 \tan h^2\eta + \alpha_4 \tan h^4\eta\right]\sec h^2\eta \tag{3.27c}$$

where,

$$\alpha_1 = l_0 + l_1, \alpha_1' = \frac{l_2}{\alpha_1},\ \alpha_2 = \frac{l_4 - l_2}{\alpha_3}, \alpha_4 = \frac{l_5}{\alpha_3},\ \ \alpha_3 = l_0 + l_1 + l_3, \tag{3.27d}$$

$l_0, l_1, l_2, l_3, l_4, l_5$ are given above.

The amplitude of solitary waves Φ_{01}, Φ_{02}, Φ_{03} are given by

$$\Phi_{01} = l_0 = \alpha_{01},\ \ \Phi_{02} = l_0 + l_1 = \alpha_{01} + \alpha_{02},\ \Phi_{03} = \alpha = l_0 + l_1 + l_3 = \alpha_{01} + \alpha_{02} + \alpha_{03} \tag{3.28}$$

The width D_3 of third- order solitary wave is obtained as

$$D_3 = \left(\sqrt{\frac{2}{V_0^{-3} V^{(1)}}}\right)\eta_3 \tag{3.29}$$

where η_3 can be obtained from the positive root of the following equation of η,

$$\alpha_4 \tanh^6 \eta + (\alpha_2 - \alpha_4)\tanh^4 \eta + (1 - \alpha_2)\tanh^2 \eta - 0.58 = 0 \tag{3.30}$$

α_2 and α_4 in (3.30) are given in Eq. (3.27d).

The phase velocity V_1, V_2 and V_3 of the solitary waves correct up to third- order becomes,

$$V_1 = V_0 + V^{(1)}, \tag{3.31a}$$

$$V_2 = V_0 + V^{(1)} + V^{(2)} = V_0\left[1 + \frac{V^{(1)}}{V_0} + \frac{3}{2}\left(\frac{V^{(1)}}{V_0}\right)^2\right] \tag{3.31b}$$

$$V_3 = V_0 + V^{(1)} + V^{(3)} = V_0\left[1 + \frac{V^{(1)}}{V_0} + \frac{3}{2}\left(\frac{V^{(1)}}{V_0}\right)^2 + \frac{5}{2}\left(\frac{V^{(1)}}{V_0}\right)^3\right] \tag{3.31c}$$

Assuming $\gamma = V_o V^{(1)}$ which can be considered as a parameter and in terms of this parameter the Mach numbers become

$$M_1 = \frac{V_1}{V_0} = [1+\gamma],\ M_2 = \frac{V_2}{V_0} = \left[1+\gamma+\frac{3\gamma^2}{2}\right],\ M_3 = \frac{V_3}{V_0} = \left[1+\gamma+\frac{3\gamma^2}{2}+\frac{5\gamma^3}{2}\right] \tag{3.32}$$

3.5. SPECIAL CASES OF ION-ACOUSTIC SOLITONS

3.5.1. Ion-Acoustic Solitons at Critical Case

In critical case, the nonlinear coefficient vanishes, i.e. $\delta_2^{(0)} = 0$. Under this condition, the solitons will not be excited in plasma. But near the critical value of phase velocity V_0 (i.e. when $\delta_2^{(0)} \approx 0$), solitons will be excited with *sechη* profile instead of usual *sech²η* profile. To study the IAS near-critical state $\delta_2^{(0)} \approx 0$ the following perturbation expansion for ϕ should be used,

$$\varphi = \varepsilon^{1/2}\varphi^{(1)} + \varepsilon\varphi^{(2)} + \varepsilon^{3/2}\varphi^{(3)} + \cdots\cdots \tag{3.33}$$

Substituting (3.16), (3.17) and (3.33) in (3.21) and equating the coefficients of ε^2 from both sides we get

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2} + \delta_2^{(0)}\varphi^{(1)^2}\varphi^{(2)} + \frac{1}{4}\delta_3^{(0)}\varphi^{(1)^4} \tag{3.34}$$

Near critical situation when $\delta_2^{(0)} \approx 0$, Eq.(3.34) is reduced to

$$\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \delta_1^{(1)}\varphi^{(1)^2} + \frac{1}{2}\delta_3^{(0)}\varphi^{(1)^4} \tag{3.35}$$

Eq.(3.35) has the solution for IAS in the form

$$\varphi_c^{(1)} = \lambda \sec h\eta' \tag{3.36}$$

where,

$$\lambda^2 = -\frac{\delta_3^{(0)}}{2\delta_1^{(1)}},\ \eta' = \frac{1}{2}\left(\sqrt{\delta_1^{(1)}}\right)X \tag{3.37}$$

In the critical case ($\delta_2^{(0)} = 0$) , we obtain from (3.20), $V_0 = \sqrt[4]{3}$. Using (3.18) the critical value of β is 0.432 . At $\beta \approx 0.432$, the solution of first- order solitons in plasma with cold ions and nonthermal electrons is given by (3.36).

3.5.2. Arbitrary Amplitude of Ion-Acoustic Solitons

To find the arbitrary amplitude solitons up to second- order we take the terms up to φ^3 of Eq.(3.14a) and obtain

$$\frac{d^2\varphi}{d\xi^2} = S_1\varphi + S_2\varphi^2 + S_3\varphi^3 \tag{3.38}$$

where $S_1 = \Delta_1, S_2 = -\Delta_2, S_3 = \Delta_3$.

The first- and second--order solutions for arbitrary amplitude solitons are

$$\varphi_{1A} = \frac{3S_1}{2S_2}\sec h^2\left(\sqrt{\frac{S_1}{4}}\eta\right) \tag{3.39}$$

and

$$\varphi_{2A} = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}\left[2\cosh^2\left(\sqrt{\frac{S_1}{4}}\eta\right) - 1\right]} \tag{3.40}$$

The amplitudes of the second-order solitons are given by

$$\varphi_{01A} = \frac{3S_1}{2S_2} \quad \text{and} \quad \varphi_{02A} = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}} \tag{3.41}$$

The corresponding widths of the second-order solitons are

$$D_{1A} = \frac{2}{\sqrt{S_1}} \quad \text{and} \quad D_{2A} = D_1\cosh^{-1}\left(\frac{1.381S_2}{\sqrt{4S_2^2 - 18S_1S_3}} + 1.6905\right)^{1/2} \tag{3.42}$$

3.6. RESULTS AND DISCUSSION

To observe the existence of ion-acoustic solitons in plasma, we have plotted Sagdeev potential ψ (ϕ) against electrostatic potential φ for different values of nonthermal parameter (β) of electrons. During our numerical calculations, we have chosen the values of nonthermal parameter in such a way that the conditions (2.35) - (2.37) given in Chapter-2 are satisfied. Moreover , the values of nonthermal parameter are suitable for laboratory as well as space plasma. According to Cairns et. al. [15] the value of nonthermal parameter β is $0 \le \beta \ge 1.3$. But, it is observed from the conditions for the existence of solitons that the nonthermal parameter β should be less than unity, otherwise the phase velocity will be unphysical. So, for all values of β of Cairns model [15], the solitons will not be excited in the plasma. Following the conditions (2.35)-(2.37) of Chapter-2, we have plotted ψ (ϕ) against φ shown Fig.- 3.1 taking the values of β from 0.0 to 0.2.

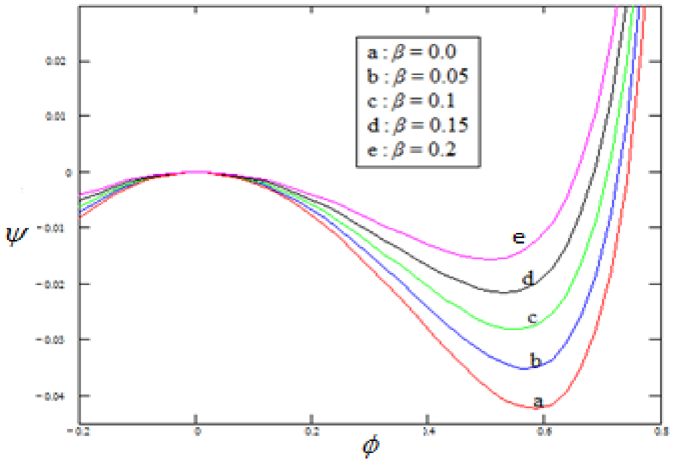

Fig.3.1. The profile of Sagdeev potential for different values of nonthermal parameter of electrons (β) with fixed values of phase velocity V =1.3. Graphs a, b, d and e represent β = 0.0, 0.05, 0.1, 0.15 and 0.2 respectively,

The potential well of Sagdeev potential in Fig.3.1 indicates the existence of ion-acoustic solitons in plasma consisting of nonthermal electrons. It is observed that the depth of the potential well for β = 0.2 is lower than that when no nonthermal electron is present in the plasma. The depth is decreased with the increase of value of the nonthermal parameter (β) in plasma i.e. the amplitude of solitary in plasma increases with the decrease of nonthermal parameter of electrons.

To see the nature of IAS in first- , second- and third- order, the profiles of electrostatic potential $\varphi^{(1)}, \varphi^{(2)}, \varphi^{(3)}$ in plasma having nonthermal electrons are shown in Figs.3.2(a)-3.2(d).

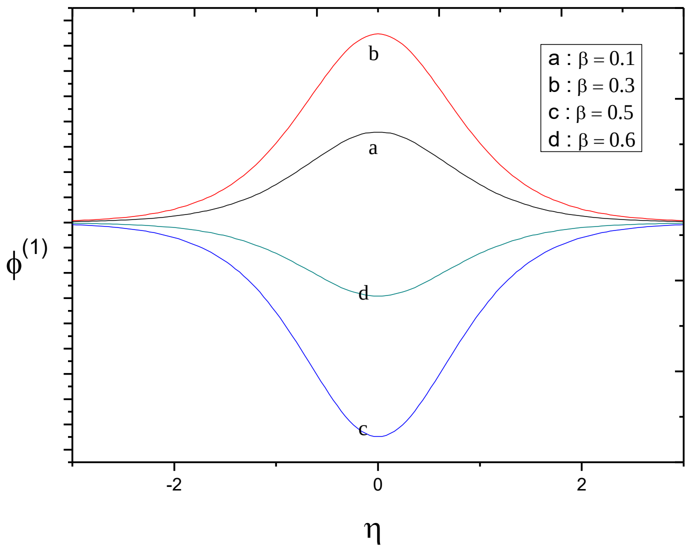

Fig. 3.2(a). The profiles of $\varphi^{(1)}$ of first-order (KdV) solitons for different values of β. Graphs a, b, c and d represent β = 0.1, 0.3, 0.5 and 0.6 respectively.

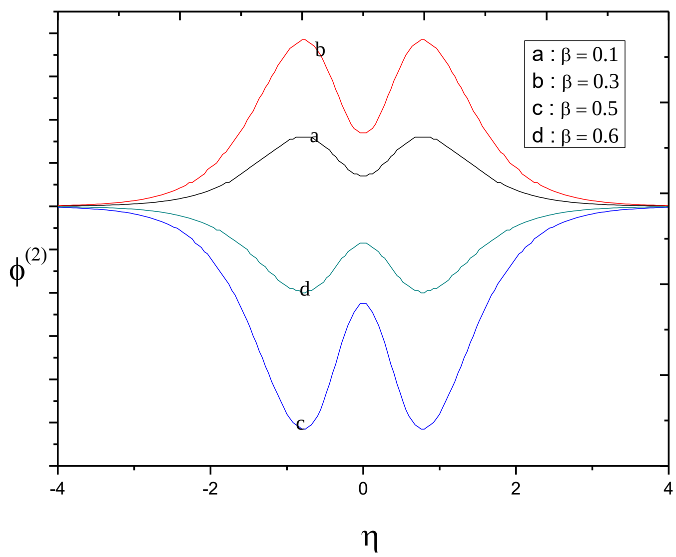

Fig. 3.2(b). The profiles of second -order correction $\varphi^{(2)}$ to the first-order solitons for different values of β. Graphs a, b, c and d represent β = 0.1, 0.3,0.5 and 0.6 respectively.

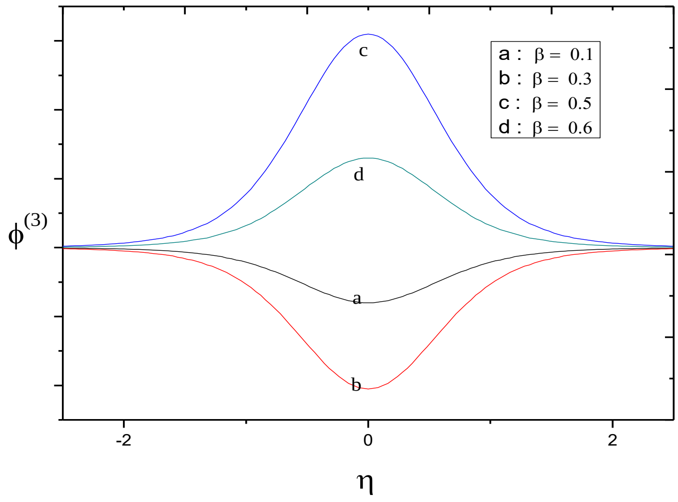

Fig.3.2(c). The profiles of third- order correction $\varphi^{(3)}$ to the first-order solitons for different values of β. Graphs a, b, c and d represent β = 0.1, 0.3,0.5 and 0.6 respectively.

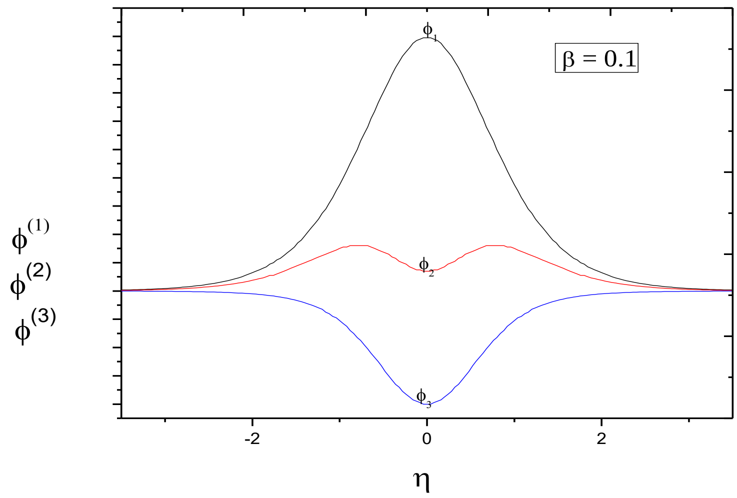

Fig.3.2(d). The profiles of KdV soliton $(\varphi^{(1)})$, second-order correction $(\varphi^{(2)})$ and third- order correction $(\varphi^{(3)})$ for β = 0.1.

It is observed from Fig.3.2(a) that the electrostatic potential $\varphi^{(1)}$ of KdV soliton may be compressive and rarefactive depending on the values nonthermal parameter β of electrons. For β =0.1 and 0.3 the IAS is compressive but for β =0.5 and 0.6 the IAS is rarefactive. The second- order correction $\varphi^{(2)}$ in Fig.3.2(b) is W-type and it is compressive for β =0.1 and 0.3 and rarefactive for the values of β =0.5 and 0.6. The third- order correction term $\varphi^{(3)}$ for different values of β is shown in Fig. 3.2(c). It is not W-type and is compressive for

β =0.5 and 0.6 and rarefactive for β =0.1 and 0.3. The profiles of potential $\phi^{(1)}$,$\phi^{(2)}$ and $\phi^{(3)}$ for β = 0.1 are shown in Fig. 3.2(d) .It is seen that first- order potential $\phi^{(1)}$ of KdV soliton is compressive but the second- order correction $\phi^{(2)}$ is W-type and it is compressive where as the third-order correction $\phi^{(3)}$ is rarefactive in nature.

Now, the corrected second- and third-order electrostatic potentials of solitary waves Φ_2 (= $\phi^{(1)}$ + $\phi^{(2)}$) and Φ_3 (= $\phi^{(1)}$ + $\phi^{(2)}$ + $\phi^{(3)}$) including first order Φ_1 (= $\phi^{(1)}$) for fixed value of β are shown in Figs. 3.3(a) – 3.3(c).

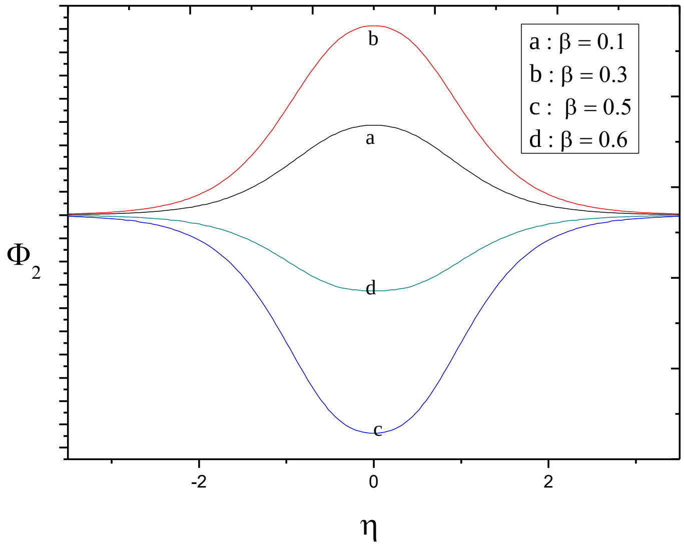

Fig. 3.3(a). The profiles of second-order soliton Φ_2 for different values of β; $\Phi_2 = \varphi^{(1)} + \varphi^{(2)}$. Graphs a, b, c and d represent β = 0.1, 0.3,0.5 and 0.6 respectively.

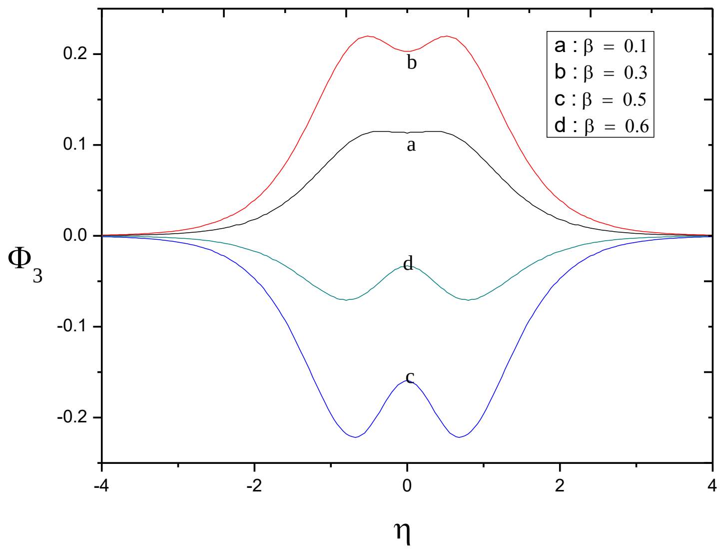

Fig. 3.3(b). The profiles of third-order soliton Φ_3 for different values of β; $\Phi_3 = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)}$. Graphs a, b, c and d represent β = 0.1, 0.3,0.5 and 0.6 respectively.

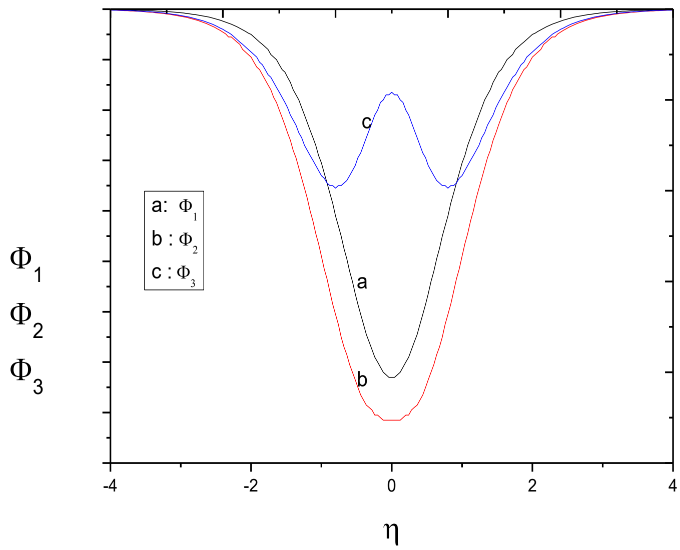

Fig. 3.3(c). The profiles of first-order (Φ_1), second-order (Φ_2) and third-order (Φ_3) solitons for β=0.6. Graphs a, b and c represent Φ_1, Φ_2 and Φ_3 respectively.

From the Fig. 3.3(a) , Fig. 3.3(b) and Fig. 3.3(c) it is observed that the solitary waves Φ_1, Φ_2 , Φ_3 may be compressive and rarefactive depending on the values of nonthermal parameter β. The solitary wave Φ_2 corrected up to second- order correction is compressive for β = 0.1 and 0.3 and rarefactive for β = 0.5 and 0.6. The solitary wave Φ_3 corrected up to third- order correction is W-type and it is compressive for β = 0.1 and 0.3 and rarefactive for β = 0.5 and 0.6. From the Fig. 3.3(c) it is seen that for β = 0.6 the potential Φ_1 of KdV soliton (Φ_1) and Φ_2 of second- order corrected soliton are rarefactive but Φ_3 of third-order corrected soliton is W-type and compressive in nature. It is important to be mentioned that W-type solitary waves are generally observed for second-order solitary waves in negative ion plasma [16] in absence of nonthermal electrons.

3.6.1. Amplitude of Solitary Waves

The amplitudes of first-order (Φ_{10}), second-order (Φ_{20}) and third-order (Φ_{30}) solitons for different values of β are shown in Fig. 3.4.

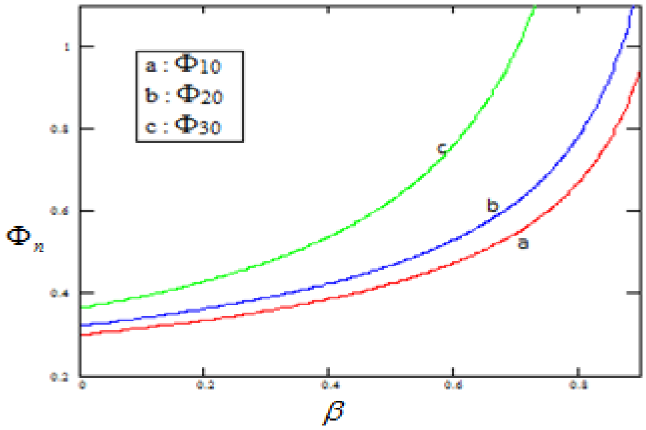

Fig.3.4.Variation of amplitude of IAS with nonthermal parameter of electrons β . Graphs a, b and c represent the amplitude of solitons Φ_n, i.e. Φ_{10}, Φ_{20} and Φ_{30} respectively.

It is seen from Fig. 3.4 that the amplitudes of ion-acoustic solitons Φ_{10}, Φ_{20}, Φ_{30} increase with the increase of nonthermal parameter β . Graphs a, b and c represent the amplitudes Φ_{10} , Φ_{20} and Φ_{30} respectively. The amplitude $\Phi_{30} >$ $\Phi_{20} > \Phi_{10}$.

3.6.2. Critical Cases:

(i). Ion-acoustic solitons near-critical value of the phase velocity

In the critical case, $\delta_2^{(0)} = 0$, i.e. $V_0 = \sqrt[4]{3}$ or β = 0.432 . In critical case, no soliton will be excited in the plasma . But near- critical state i.e. at $\beta \approx 0.432$, the solitons with sechη-profile will be excited the in plasma. The profiles of solitons near critical value of V_0 is shown in Fig.3.5.

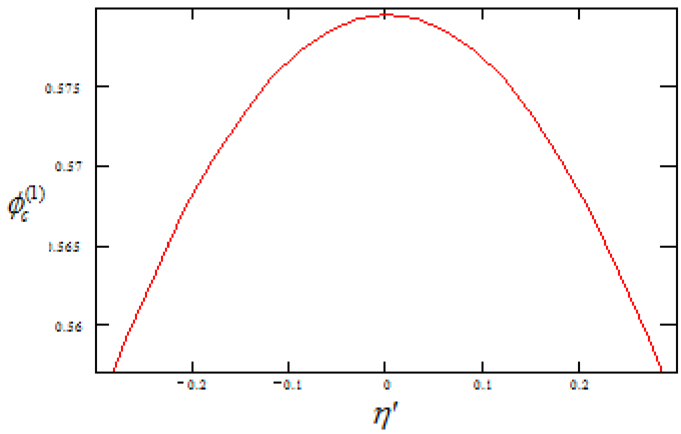

Fig 3.5. The profile of the IAS near critical value of nonthermal parameter β i.e. β = 0.423.

(ii). Arbitrary amplitude solitary waves

The profile of arbitrary amplitude first- and second-order solitary waves in plasma for β = 0.1, 0.3 and 0.9 are shown in Fig. 3.6(a) and 3.6(b).

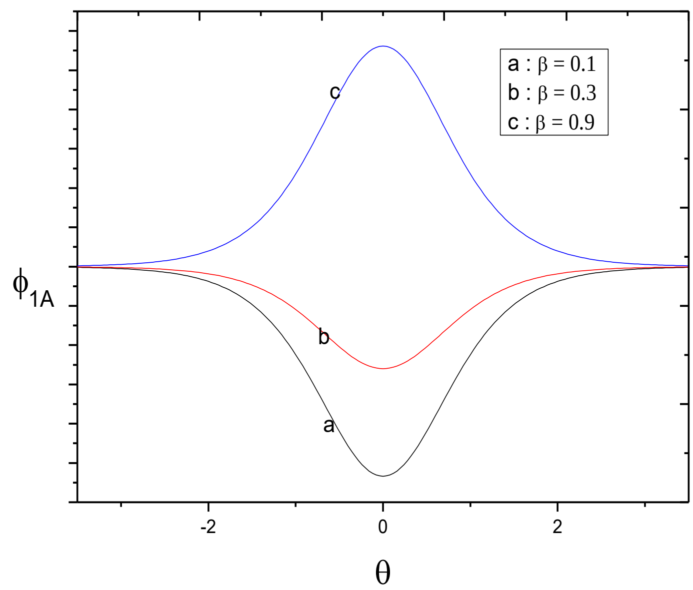

Fig.3.6(a). The profiles of arbitrary amplitude first-order φ_{1A} solitary wave for different values of β. Graphs a, b and c represent β = 0.1, 0.3 and 0.9 respectively.

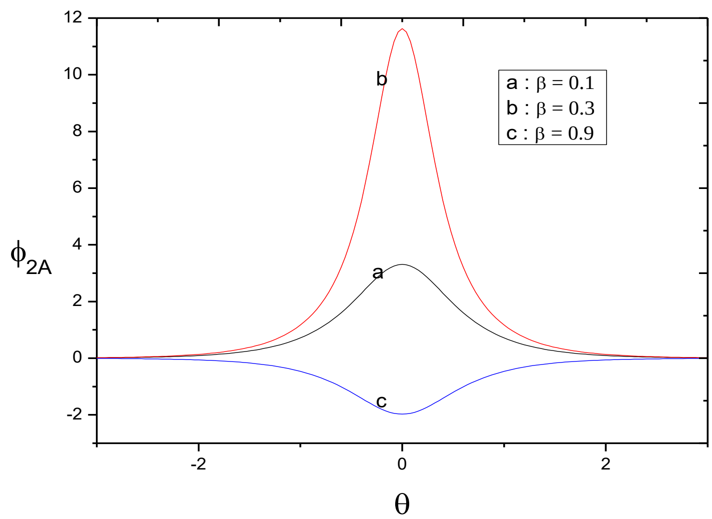

Fig.3.6(b). The profiles of arbitrary amplitude higher - order (ϕ_{2A}) solitary wave for different values of β. Graphs a, b and c represent β = 0.1, 0.3 and 0.9 respectively. V =1.4.

It is seen from Fig.3.6(a) and Fig.3.6(b) that arbitrary amplitude first-order and second-order IAS may be compressive and rarefactive depending on the values of nonthermal parameter of electrons β. But there are no W-type IAS when arbitrary amplitude IAW is considered in presence of nonthermal electrons in the plasma.

3.7. SUMMARY AND CONCLUSION

An integrated form of governing equations in plasma consisting of cold positive ions and nonthermal electrons has been derived and expressed in terms of pseudo-potential for the study of higher order ion-acoustic solitary waves. Nonlinear equation derived from the Sagdeev potential are solved with the help of BKM method [14] for first-, second- and third-order ion acoustic solitary waves and their profiles are depicted for different values of nonthermal parameter of electrons. It is seen that nonthermal electrons has important role on the shape of solitary waves in each order, particularly in third- order where W-shaped solitary wave is excited.

Our present method for the study of higher order ion acoustic solitary wave is advantageous than other mathematical method, because the equation at each order becomes a first-order inhomogeneous differential equation which is easier to solve. But, for the study of solitary waves in plasma using the standard reductive perturbation method [16] we have to solve inhomogeneous second-order differential equation at each order.

Most important and significant achievement in this work is the W-type ion-acoustic solitons in cold plasma in presence of nonthermal electrons when third-order nonlinear and dispersive effects are considered. But, if we consider the effect ion temperature in plasma including nonthermal electrons , we will get more significant results on the third- order ion acoustic solitons.

REFERENCES

[1] .H. Ikezi, R .J .Tayler and D. R.Baker, Phys. Rev. Letts., **25,** p.11 (1970).

[2] .H . Ikezi, Phys. Fluids, **16,** p.1668 (1973).

[3]. M. Q. Tran, Physica Scripta, **20,** p. 317 (1979).

[4]. K. E. Lonngren, Plasma Phys., **25,** p. 943(1983).

[5]. Y. H. Ichikawa, T. Matsuhasi and K. Konno, J. Phys. Soc. Japan., **41,** p. 1382 (1976).

[6]. C. S. Lai, Canadian J. Phys., **57,** p.490 (1979).

[7]. Y. Kodama and T . Taniuti, J. Phys. Soc. Japan, **45,** p.298 (1978).

[8]. M. K . Kalita and S . Bujarbarua, Canadian J. Phys., **60,** p.392 (1982).

[9]. S. G. Tagare and R. V. Reddy, J. Plasma Phys., **35,** p.279 (1986).

[10]. W. M. Moslem, J. Plasma Phys., **63,** p.139 (2000).

[11]. S. N. Paul, S .Chattopadhyay, S. K.Bhattacharya and B. Bera, Pramana, **60,** p.1217 (2003).

[12]. K. P Das and S .R .Majumdar, Canadian J. Phys. , **69,** p. 822 (1991).

[13]. S. R Majumdar, A. Roychowdhury, S. K Bhattacharyya and S. N. Paul, J. Plasma Phys., **47,** p.451 (2005).

[14]. N. N. Bogoliubov and Y. A. Mitropolsky Asymptotic Methods in the Theory of Non-Linear Oscillations New York Gordon and Breach (1961).

[15]. R. A. Cairns, A. A. Mamun, R. Bingham, R. Bostrom, R. O. Dendy, C. M. C. Nairn and P. K. Shukla, Geophys. Res. Letts., **22,** p.2709 (1995).

[16]. H. Washimi and T. Taniuti, Phys. Rev. Letts., **17,** p.966 (1966).

CHAPTER-4

ARBITRARY AMPLITUDE ION-ACOUSTIC SOLITONS IN AN ELECTRON-ION-POSITRON PLASMA

This Chapter is the revised form of the publication: Indian Journal of Physics, Vol. 86 (6), pp. 545-553 (2012)].

4.1. INTRODUCTION

In Chapters-2 and 3, we have discussed about the ion-acoustic solitons (IAS) in plasma having nonthermal electrons. Like nonthermal electrons, the presence of positrons in a plasma gives rise to interesting results in IAS in plasma. In recent years, wave propagation in astrophysical plasma are being investigated in presence of positrons together with electrons and ions [1-4]. It is believed that electron-positron plasma has its origin in the early stage of universe [5-7] and this has important contribution on the physical processes in galactic nuclei [8], pulsar magnetosphere [9], polar caps of neutron star [10]. It has been observed that nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently from the plasma consisting of electrons and ions [11]. To understand the nature of some characteristics of the waves in e-i-p plasma, some works of IAS in a plasma consisting of cold and hot electrons and positrons have been carried out by Pillay and Bharuthram [12]. They have found that both compressive and rarefactive IAS is being excited in the plasma. Later, Popel *et al* [13] have shown that the presence of positron in multispecies plasma can result in the reduction of the IAS amplitudes. Subsequently, Shukla *et al.* [14], Alinejad [15], Ghosh and Bharuthram [16], Massod *et al.* [17] have studied nonlinear propagation of IAS in e-i-p plasma with trapped electrons and have obtained some interesting results. Ghosh and Bharuthram [18] have considered the Landau damping for theoretically investigation of the IAS in e-i-p plasma neglecting the effect of particle trapping using the reductive perturbation method. They have found that Landau damping causes the soliton amplitude to decay with time. They have also discussed the linear wave phase velocity and the soliton amplitude for the increase of positron density in absence of Landau damping whereas these increase with positron temperature in such plasma. Later, Pakzad [19, 20] and some other authors have used pseudo potential method [21] for the study of IAS in e-i-p plasma.

However, the influence of drift motion of ions on the IAS in e-i-p plasma has not been explored by the above authors. So, in this Chapter, we have theoretically studied the arbitrary amplitude IAS in a non-collisional plasma consisting of warm positive ions, isothermal electrons and positrons using Sagdeev potential method. It has been observed that both compressive and rarefactive IAS are excited in presence of positrons in the plasma having the stream of ions. Moreover, using the values of phase velocities of Slow- and Fast-modes of ion-acoustic wave, we have discussed graphically the small amplitude and large amplitude IAS for the Slow-mode and Fast-mode of wave in the e-i-p plasma.

4.2. BASIC EQUATIONS

Unmagnetized and collisionless three-component plasma consisting of isothermal electrons and positrons, singly charged warm positive ions has been considered. Their respective number densities are n_e, n_p and n_i. At equilibrium, $n_{i0} + n_{p0} = n_{e0}$, where n_{e0}, n_{p0} and n_{i0} are the equilibrium densities. Normalizing the densities by n_{e0}, the equilibrium condition becomes $\chi = n_{p0} / n_{e0} = 1 - n_{i0}$. The temperature of ion (T_i) is assumed to be much smaller than the temperature of electrons (T_e) and positrons (T_p) i.e. $T_i << T_e, T_p$ and so the effect of Landau damping can be neglected [22].

The normalized densities of electrons and positrons are given as Boltzmann distribution used by Popel *et al*. [13]:

$$n_e = \exp(\varphi) \tag{4.1}$$

$$n_p = \chi \exp(-\sigma_p \varphi) \tag{4.2}$$

where, φ is the electrostatic potential, $\sigma_p = T_e / T_p$.

For the ions, the dynamics of the plasma are governed by the following equations of continuity and momentum transfer,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{4. 3}$$

and

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\sigma_i}{n_i} \frac{\partial p_i}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{4.4}$$

The pressure equation for the ions is taken as

$$\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial x} + 3 p_i \frac{\partial v_i}{\partial x} = 0 \tag{4.5}$$

For the stated background, Poisson's equation yields

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_p - n_i \tag{4.6}$$

In Eqs. (4.3) - (4.6), v_i is the ion-fluid velocity, p_i is the ion-fluid pressure and $\sigma_i = T_i / T_e$. The velocity v_i is normalized by the characteristic velocity ($K_B T_e / m)^{1/2}$, the length by the Debye-length ($K_B T_e / 4\pi e^2 n_{e0})^{1/2}$ and the time by the inverse of plasma frequency $(4\pi n_{e0} e^2 / m)^{-1/2}$. The potential is normalized to $K_B T_e / e$, T_e and K_B being the electron temperature and Boltzmann's constant, respectively. Other parameters have their usual meanings. It is to be stated that the variables in the plasma are normalized in such a way so that the above equations will appear in totally dimensionless form.

To obtain a solitary wave solution in e-i-p plasma, we introduce the single independent variable η defined by the relation $\eta = x - Vt$, where V is the velocity of the solitary wave.

Now, using the boundary conditions

$$n_i \to 1`, v_i \to v_{io}, p_i \to p_{i0}, \phi \to 0 \ \text{ at } \ |\eta| \to \propto \tag{4.7}$$

Eqs. (4.1) – (4.5) in the stationary frame can be integrated to give,

$$n_i = \frac{(1-\chi)(V - v_{i0})}{(V - v_i)} \tag{4.7a}$$

$$p_i = \frac{p_{i0}(V - v_{i0})^3}{(V - v_i)^3} \tag{4.7b}$$

4.3. SAGDEEV POTENTIAL

For the derivation of Sagdeev potential in e-i-p plasma, it is assumed that the basic equations are supplemented by the boundary conditions mentioned in (4.7).

Using Eq.(4.7) in Eqs.(4.3) - (4.5), the density of positive ion in the plasma is obtained from the following equation,

$$\frac{3\sigma_i p_{i0}}{n_{i0}^3} n_i^4 - [(V - v_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}} - 2\varphi] n_i^2 + n_{i0}^2 (V - v_{i0})^2 = 0 \tag{4.8}$$

Eq.(4.8) yields

$$n_i = \pm \frac{n_{i0}^{3/2}}{\sqrt{6\sigma_i p_{i0}}} \left[(V^2 + a_0 - 2\varphi) \pm \sqrt{\{(V^2 + a_0 - 2\varphi)^2 - \frac{12\sigma_i p_{i0}}{n_{i0}}(V - u_{i0})^2\}} \right]^{1/2} \tag{4.8a}$$

where,

$$a_0 = u_{i0}^2 - 2Vu_{i0} + \frac{3\sigma_i p_{i0}}{n_{i0}}$$

To satisfy the boundary conditions (4.7), the positive value of negative branch of ion density is taken into account i.e. ion density is

$$n_i = \frac{n_{i0}^{3/2}}{\sqrt{6\sigma_i p_{i0}}} \left[(V^2 + a_0 - 2\varphi) - \sqrt{\{(V^2 + a_0 - 2\varphi)^2 - \frac{12\sigma_i p_{i0}}{n_{i0}}(V - u_{i0})^2\}} \right]^{1/2} \tag{4.9a}$$

Or

$$n_i = \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}} [(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}] \tag{4.9b}$$

where

$$A_1 = (V - v_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}}$$

(4.10)

$$A_2^2 = \frac{12\sigma_i p_{i0}}{n_{i0}}(V - u_{i0})^2 \tag{4.11}$$

Now, using Eqs.(4.1), (4.2) and (4.9) in Eq.(4.6), we obtain

$$\frac{\partial^2 \varphi}{\partial \eta^2} = \exp(\varphi) + \chi \exp(-\sigma_p \varphi) + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}} [(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}]$$

(4.12)

Eq.(4.12) is now written as

$$\frac{d^2\varphi}{d\eta^2} = -\frac{d\psi}{d\varphi} \tag{4.12a}$$

where $\psi(\varphi)$ is the Sagdeev potential [21]

Now, using the boundary conditions (4.7) we get from Eqs.(4.12) and (4.12a) after integration

$$\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \tag{4.12b}$$

where

$$\psi(\varphi) = \psi_e(\varphi) + \psi_p(\varphi) + \psi_i(\varphi)$$

(4.13)

$$\psi_e(\varphi) = 1 - \exp(\varphi) \tag{4.14}$$

$$\psi_p(\varphi) = \frac{\chi}{\sigma_p}[1 - \exp(-\sigma_p\varphi)] \tag{4.15}$$

$$\psi_i(\varphi) = \frac{n_{i0}^{3/2}}{6\sqrt{3\sigma_i p_{i0}}}[(A_1 - A_2 - 2\varphi)^{3/2} - (A_1 + A_2 - 2\varphi)^{3/2} - (A_1 - A_2)^{3/2} + (A_1 + A_2)^{3/2}] \tag{4.16}$$

It is seen that $\psi(\varphi)$ of Eq.(4.13) reduces to the Sagdeev potential given by Eq. (4.10) of Popel *et al*.[13] when $\sigma_i = 0$ and $v_{i0} = 0$, if the notations for the plasma parameters of [15] are used. It should be remembered that putting $\sigma_i = 0$ and $v_{i0} = 0$ directly in Eqs. (4.13) or (4.15) the Eq.(4.10) of Popel *et al*.[13] would not be recovered. In an e-i-p plasma consisting of cold and non-drifting ions, n_i should be calculated from our Eq.(4.8) putting $\sigma_i = 0$ and $v_{i0} = 0$ and this value of n_i should be used for calculating $\psi(\varphi)$.

4.4. CONDITION FOR THE EXISTENCE OF SOLITON SOLUTION

The form of the pseudo-potential $\psi(\varphi)$ would determine whether a soliton-like solution of Eq. (4.12b) will exist or not. Following the conditions for the existence of soliton solution mentioned in Chapter-2, it is found that the IAS will exist in an e-i-p plasma only when the inequality is satisfied. The critical values of the phase velocity (V_p) of IAS are obtained from

$$n_{i0}^{3/2}[\frac{1}{\sqrt{A_1 - A_2}} - \frac{1}{\sqrt{A_1 + A_2}}] - (1+\chi\sigma_p)\sqrt{12\sigma_i p_{i0}} = 0 \qquad (4.17)$$

From (4.17) we get after using $n_{i0} = 1-\chi$,

$$V = v_{i0} \pm [\frac{(1-\chi)}{(1+\chi\sigma_p)} + \frac{3\sigma_i p_{i0}}{(1-\chi)}]^{1/2} \qquad (4.18)$$

When the positrons are not present in the plasma (χ = 0) and only electrons and non-drifting ions are present (n_{i0} = n_{e0}), Eq.(4.18) gives the phase velocity of IAW, $V = \sqrt{1+3\sigma_i}$, which is the well known value of the phase velocity of IAS in plasma having warm ions.

Eq. (4.18) shows that the phase velocity of the IAS has two values, one is fast mode (V_F) and other is slow mode (V_S),

$$V_F = v_{i0} + [\frac{(1-\chi)}{(1+\chi\sigma_p)} + \frac{3\sigma_i p_{i0}}{(1-\chi)}]^{1/2} \qquad (4.19a)$$

$$V_S = v_{i0} - [\frac{(1-\chi)}{(1+\chi\sigma_p)} + \frac{3\sigma_i p_{i0}}{(1-\chi)}]^{1/2} \qquad (4.19b)$$

From Eq. (4.18), the critical values of the positron density (χ_C), positron temperature (σ_{pC}), ion temperature (σ_{iC}) and ion density (n_{i0C}) may be obtained for which IAS will exist in plasma in presence of positrons. The critical values of these parameters are:

$$\chi_C = \frac{1}{2(1+\sigma_p M_i^2)}[\gamma \pm \{\gamma^2 - 4(1+\sigma_p M_i^2)(1+3\sigma_i p_{i0} - M_i^2)\}^{1/2}] \qquad (4.20a)$$

$$\sigma_{pC} = \frac{1}{\chi}[\frac{(1-\chi)^2 - M_i^2(1-\chi) + 3\sigma_i p_{i0}}{M_i^2(1-\chi) - 3\sigma_i p_{i0}}]$$

(4.20b)

$$\sigma_{iC} = \frac{(1-\chi)}{3p_{i0}}[M_i^2 - \frac{(1-\chi)}{(1+\chi\sigma_p)}] \tag{4.20c}$$

$$n_{i0C} = \frac{1}{2}[M_i^2(1+\chi\sigma_p) \pm \{M_i^4(1+\chi\sigma_p)^2 - 12\sigma_i p_{i0}(1+\chi\sigma_p)\}^{1/2}] \tag{4.20d}$$

where,

$\gamma = 2-(1-\sigma_p)M_i^2 - 3\sigma_p\sigma_i p_{i0}$ and $M_i = V - v_{i0}$.

4.5. SOLITARY WAVE SOLUTION

An analytical solution of Eq. (4.12) for the IAS can be obtained by expanding the right hand side in terms of φ keeping the terms up to second- order, third- order or even next higher orders of φ. After a straight forward calculation one can get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 + R\varphi^3 + \cdots\cdots \tag{4.21}$$

where,

$$P = 1 + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}(\frac{1}{G_1^{1/2}} - \frac{1}{G_2^{1/2}}) + \chi\sigma_p$$

$$Q = -\frac{1}{2}[1 + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}(\frac{1}{G_1^{3/2}} - \frac{1}{G_2^{3/2}}) - \chi\sigma_p^2] \tag{4.22}$$

$$R = \frac{1}{2}[\frac{1}{3} + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}(\frac{1}{G_1^{5/2}} - \frac{1}{G_2^{5/2}}) + \frac{1}{3}\chi\sigma_p^3]$$

$$G_1 = A_1 + A_2,\ G_2 = A_1 - A_2,\ A_1 = (V - v_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}},\ A_2^2 = \frac{12\sigma_i p_{i0}}{n_{i0}}(V - v_{i0})^2.$$

4.5.1. Small Amplitude Ion-Acoustic Solitons

Now, for small amplitude IAS, the terms up to φ^2 in Eq. (4.21) are taken into consideration, neglecting next higher -order terms of φ i.e. φ^3, φ^4 etc. So, we get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 \tag{4.23}$$

Using the standard mathematical technique, the solution of Eq. (4.23) for the small amplitude IAS is obtained as:

$$\varphi_1 = \frac{3P}{2Q}\sec h^2\theta \tag{4.24}$$

where, $\theta = [(P/4)^{1/2}\eta]$.

The amplitude and width of the IAS are:

$$\varphi_{01} = \frac{3P}{2Q} \quad , \quad \delta_1 = \frac{2}{\sqrt{P}} \tag{4.25}$$

For Slow-mode and Fast-mode of the IAS, the velocity V in Eq. (4.18) is replaced by V_S and V_F of Eq. (4.19a) and Eq. (4.19b).

So, we obtain from Eq. (4.10) and Eq. (4.11)

$$A_{1S,1F} = (V_{S,F} - v_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}} \quad \text{And} \quad A^2_{2F,2S} = \frac{12\sigma_i p_{i0}}{n_{i0}}(V_{F,S} - v_{i0})^2 \tag{4.26a}$$

$$G_{1S,1F} = A_{1S,1F} + A_{2S,2F} \quad \text{and} \quad G_{2S,2F} = A_{1S,1F} - A_{2S,2F} \tag{4.26b}$$

Therefore, for Slow-mode and Fast-mode mode of small amplitude IAS the Eq. (4.24) becomes

$$\frac{d^2\varphi_{S,F}}{d\eta^2} = P_{S,F}\varphi_{S,F} - Q_{S,F}\varphi^2_{S,F} \tag{4.27}$$

The solution of Eq. (4.27) for the small amplitude IAS for Slow-mode and Fast- mode are

$$\varphi_{1S} = \frac{3P_S}{2Q_S}\sec h^2\theta_S, \quad \varphi_{1F} = \frac{3P_F}{2Q_F}\sec h^2\theta_F \tag{4.28}$$

where, $\theta_{S,F} = [(P_{S,F}/4)^{1/2}\eta]$.

The amplitudes and widths for the Small amplitude IAS are given by

$$\varphi_{01S} = \frac{3P_S}{2Q_S} \quad , \qquad \varphi_{01F} = \frac{3P_F}{2Q_F} \tag{4.29}$$

And

$$\delta_{1S} = \frac{2}{\sqrt{P_S}}, \; \delta_{1F} = \frac{2}{\sqrt{P_F}} \tag{4.30}$$

4.5.2. Large Amplitude Ion-Acoustic Solitons

For large values of φ, the terms up to φ^3 are taken in to account and Eq.(4.21) is written in the following form

$$(\frac{d\varphi}{d\eta})^2 = \alpha_1\varphi^2 - \alpha_2\varphi^3 + \alpha_3\varphi^4 \tag{4.31}$$

where $\alpha_1 = P$, $\alpha_2 = (2Q/3)$ and $\alpha_3 = -(R/2)$

Integrating Eq. (4.31), we get the solution for large amplitude IAS in the e-i-p plasma,

$$\varphi_2 = \frac{2\alpha_1}{(\alpha_1^2 - 4\alpha_1\alpha_3)^{1/2}[2\cosh^2\theta - 1] + \alpha_2} \tag{4.32}$$

The amplitude and width of the large amplitude IAS in e-i-p plasma are:

$$\varphi_{02} = \frac{2\alpha_1}{(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2} + \alpha_2} \quad \text{and} \quad \delta_2 = \frac{2}{\sqrt{\alpha_1}}\cosh^{-1}[\{\frac{0.6905\alpha_2}{(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}} + 1.6905\}] \tag{4.33}$$

For Slow-mode and Fast-mode of large amplitude IAS, the velocity V is replaced by V_S and V_F. So, we obtain

$$\varphi_{2S} = \frac{2\alpha_{1S}}{(\alpha_{1S}^2 - 4\alpha_{1S}\alpha_{3S})^{1/2}[2\cosh^2\theta_S - 1] + \alpha_{2S}} \tag{4.34a}$$

and

$$\varphi_{2F} = \frac{2\alpha_{1F}}{(\alpha_{1F}^2 - 4\alpha_{1F}\alpha_{3F})^{1/2}[2\cosh^2\theta_F - 1] + \alpha_{2F}} \tag{4.34b}$$

The amplitudes of small and large amplitude IAS for Slow-mode and Fast-mode in e-i-p plasma are:

$$\varphi_{02S} = \frac{2\alpha_{1S}}{(\alpha_{2S}^2 - 4\alpha_{1S}\alpha_{3S})^{1/2} + \alpha_{2S}} \tag{4.35a}$$

and

$$\varphi_{02F} = \frac{2\alpha_{1F}}{(\alpha_{2F}^2 - 4\alpha_{1F}\alpha_{3F})^{1/2} + \alpha_{2F}} \tag{4.35b}$$

The widths of IAS for Slow-mode and Fast-mode in e-i-p plasma are given by

$$\delta_{2S} = \frac{2}{\sqrt{\alpha_{1S}}}\cosh^{-1}[\{\frac{0.6905\alpha_{2S}}{(\alpha_{2S}^2 - 4\alpha_{1S}\alpha_{3S})^{1/2}} + 1.6905\}] \tag{4.36a}$$

And

$$\delta_{2F} = \frac{2}{\sqrt{\alpha_{1F}}}\cosh^{-1}[\{\frac{0.6905\alpha_{2F}}{(\alpha_{2F}^2 - 4\alpha_{1F}\alpha_{3F})^{1/2}} + 1.6905\}] \tag{4.36b}$$

In Eq. (4.30a) to Eq. (4.32b) the parameter $\alpha_{1S}, \alpha_{1F}, \alpha_{2S}, \alpha_{2F}, \alpha_{3S}, \alpha_{3F}$ are given by

$\alpha_{1S} = P_S, \alpha_{1F} = P_F$, $\alpha_{2S} = (2Q_S/3), \alpha_{2F} = (2Q_F/3)$, and $\alpha_{3S} = -(R_S/2), \alpha_{3F} = -(R_F/2)$

4.6. RESULTS AND DISCUSSION

The character of the Sagdeev potential (Effective potential) given by the Eq. (4.13) will determine the possibility of the existence of IAS in an e-i-p plasma. In fact, the soliton will exist in the plasma if the condition $\left|\frac{d^2\psi}{d\varphi^2}\right|_{\varphi=0} < 0$ is satisfied. It is to be remembered that the IAS will be excited in e-i-p plasma only for the critical values of positron density (χ), positron temperature (σ_p), ion temperature (σ_i) and ion-density (n_{i0}) given by the Eq. (4.20a) to Eq. (4.20d).

4.6.1. Profiles of Sagdeev Potential for Arbitrary (fixed) Phase Velocity

In model e-i-p plasma, the Sagdeev potential Ψ has been numerically estimated and plotted as shown in Fig. 4.1(a) to Fig. 4.1(d) taking different values of plasma parameters.

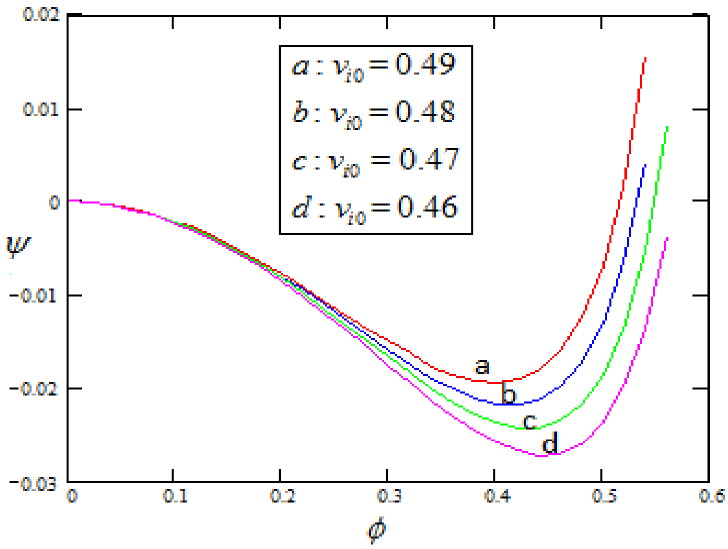

Fig. 4.1(a). Profiles of Sagdeev potential (ψ) for different values of ion stream (v_{i0}) in e-i-p plasma having χ = 0.3, σ_i = 0.001, σ_p = 0.01, V = 1.6 and p =1. Graphs a, b, c and d represent v_{i0} = 0.49, 0.48, 0.47 and 0.46 respectively.

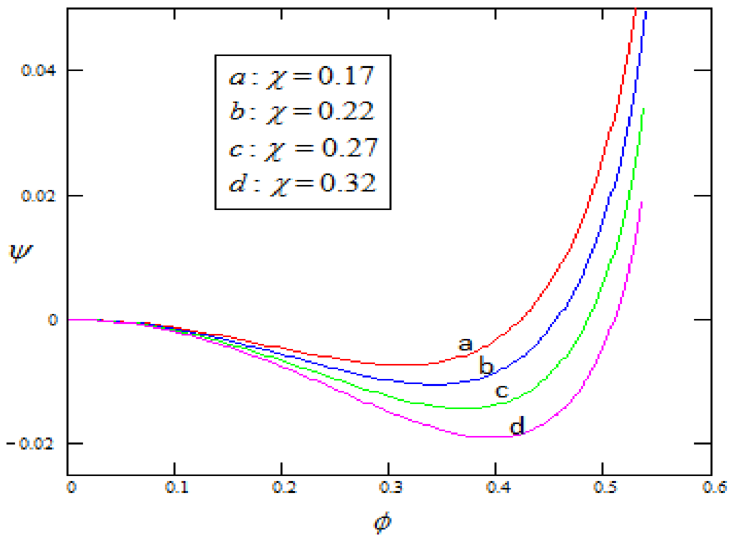

Fig. 4.1(b). Profiles of Sagdeev potential (ψ) for different values of positron density (χ) in e-i-p plasma having σ_p = 0.001, σ_i =0.001, p =1, V = 1.6 and u_{i0} =0.5. Graphs a, b, c and d represent χ = 0.17, 0.22 , 0.27 and 0.32 respectively.

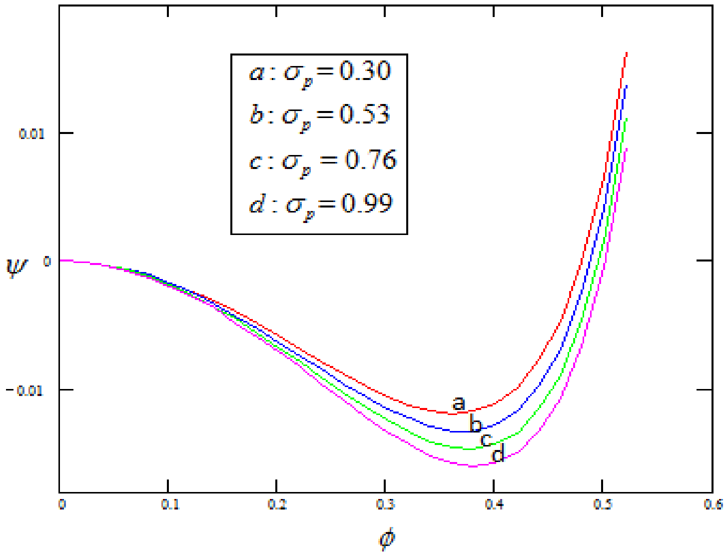

Fig. 4.1(c). Profiles of Sagdeev potential (ψ) for different values of positron temperature (σ_p) in e-i-p plasma having χ = 0.1, σ_i =0.005, p =1, V = 1.6 and v_{i0} =0.44. Graphs a, b, c and d represent σ_p = 0.3, 0.53 , 0.76 and 0.99 respectively.

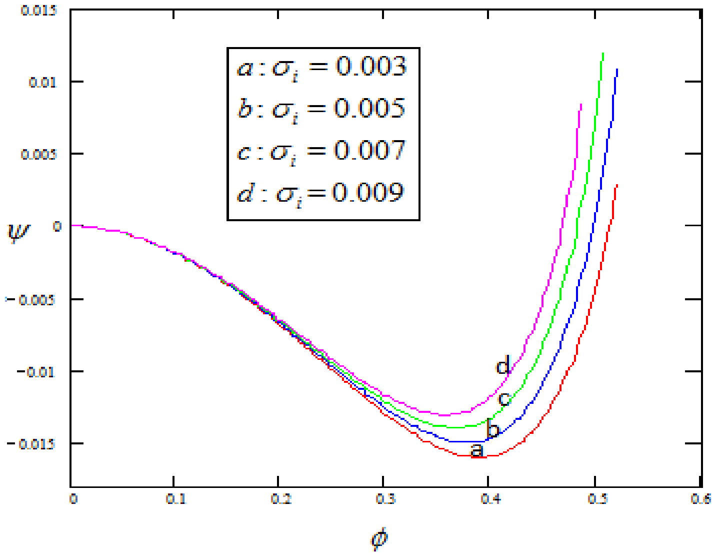

Fig. 4.1(d). Profiles of Sagdeev potential (ψ) for different values of ion temperature (σ_i) in e-i-p plasma having χ = 0.1, σ_p =0.8, p =1, V = 1.6 and v_{i0} =0.44. Graphs a, b, c and d represent σ_i = 0.003, 0.005, 0.007 and 0.009 respectively.

It is seen from Fig. 4.1(a) to Fig.1 (d) that the Sagdeev potential (ψ) is negative in e-i-p plasma for ion stream velocity (v_{i0}), positron density (χ), positron temperature (σ_p) and ion temperature (σ_i). This indicates that the plasma particles are trapped and there is every possibility of the excitation of IAS in e-i-p plasma. It is observed from Fig 4.1(a) to Fig 4.1(c) that the depth of ψ increases with increase of v_{i0}, χ and σ_p. on the other hand, Fig 4.1(d) shows that the depth of ψ increases with decrease of σ_i .

4.6.2. Profiles of Critical Values of Phase Velocity

The critical value of the phase velocity for the excitation of IAS in e-i-p plasma is obtained from the expression (4.17) which gives two values of the phase velocity, one is fast- mode (V_F) and the other is slow- mode (V_S) given by (4.18) and (4.19). The variations of V_F and V_S with the positron density (χ) and ion drift velocity (u_{i0}) are shown in Fig. 4.2(a) and Fig.4.2 (b).

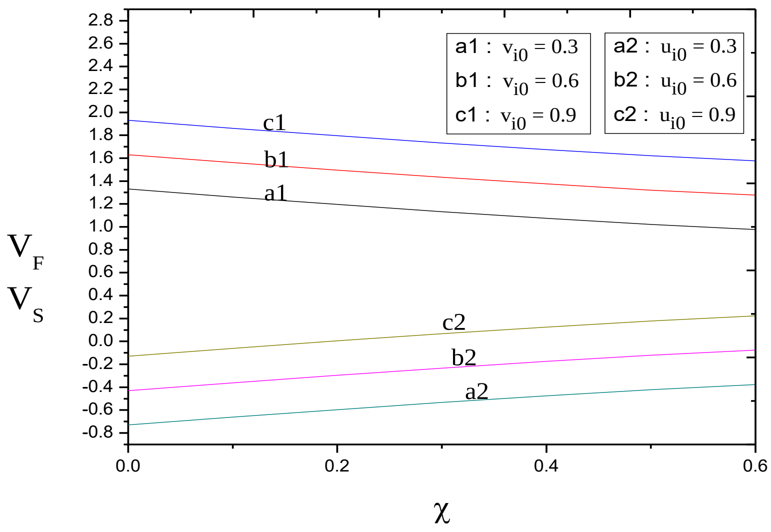

Fig. 4.2(a) Variation of phase velocity of wave with positron density (χ) for different values of ion stream velocity (u_{i0}) in e-i-p plasma having σ_i = 0.02, σ_p = 0.5 and p =1. With v_{i0} = 0.3, 0.6 and 0.9, graphs a1, b1 and c1 represent Fast-mode (V_F) of phase velocity whereas a2, b2 and c2 represents Slow mode (V_S) of phase velocity.

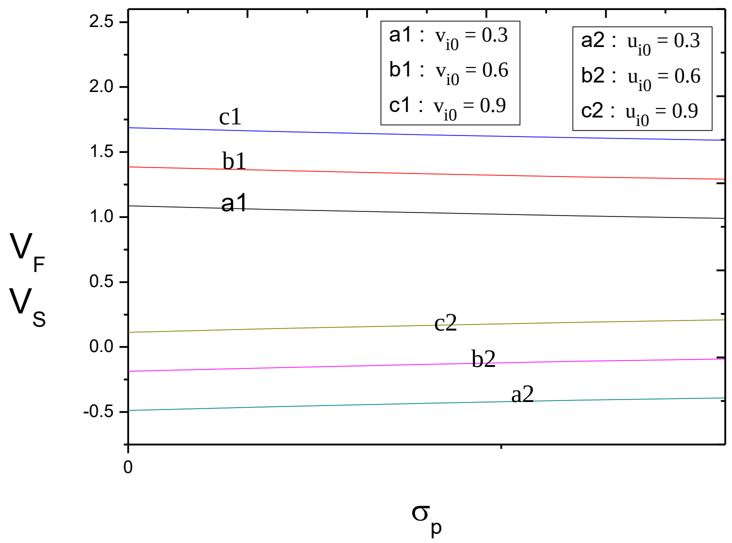

Fig. 4.2(b) Variation of phase velocity of wave with positron temperature (σ_p) for different values of ion stream velocity (v_{i0}) in e-i-p plasma having σ_i =0.02, χ = 0.3 and p =1. With v_{i0} = 0.3, 0.6 and 0.9, graphs a1, b1 and c1 represent Fast-mode (V_F) of phase velocity whereas a2, b2 and c2 represent Slow mode (V_S) of phase velocity.

Fig.4.2 (a) shows that V_F is always positive and it decreases with the increase of χ and increases with increases of v_{i0} in an e-i-p plasma. But V_S is always negative and it increases with the increase of χ and v_{i0} in an e-i-p plasma. Similarly, the variations of V_F and V_S with the positron temperature (σ_p) and ion drift velocity (v_{i0}) are numerically estimated and are shown graphically in Fig. 4.2(b). It is found that V_F is always positive and it decreases with the increase of σ_p. But, V_S is negative it increases with the increase of σ_p and v_{i0} . The positive value of V_F indicates that the Fast-mode of the wave is propagating in e-i-p plasma. But, the negative value of V_S for v_{i0} = 0.3 to 0.6 means the Slow-mode of the wave is non-propagating in the e-i-p plasma, whereas positive V_S for v_{i0} = 0.9 indicates that the slow-mode is propagating through the plasma. So, to study the propagation of IAS in e-i-p plasma for Fast mode (V_F) and Slow mode (V_S), the value of ion-stream must be taken as v_{i0} > 0.9.

4.6.3. Profiles of Critical Values of Plasma Parameters

It is to be noted that IAS will be excited in e-i-p plasma only for certain restricted regions of plasma parameters. The critical values of various plasma parameters for the excitation of IAS in e-i-p plasma may be obtained by using the expressions (4.20a) , (4.20b), (4.20c), (4.20d) and following which the condition for the existence of the solution of IAS are obtained. Fig.4.3 (a) shows the variation of critical positron temperature (σ_p) with positron density for different ion temperature (σ_i) in plasma when other parameters have fixed values. On the other hand, Fig. 4.4(b) shows the variation of critical density of positrons (χ_c) with ion stream velocity (v_{i0}) for different value of ion temperature (σ_i) in plasma when other parameters have fixed values.

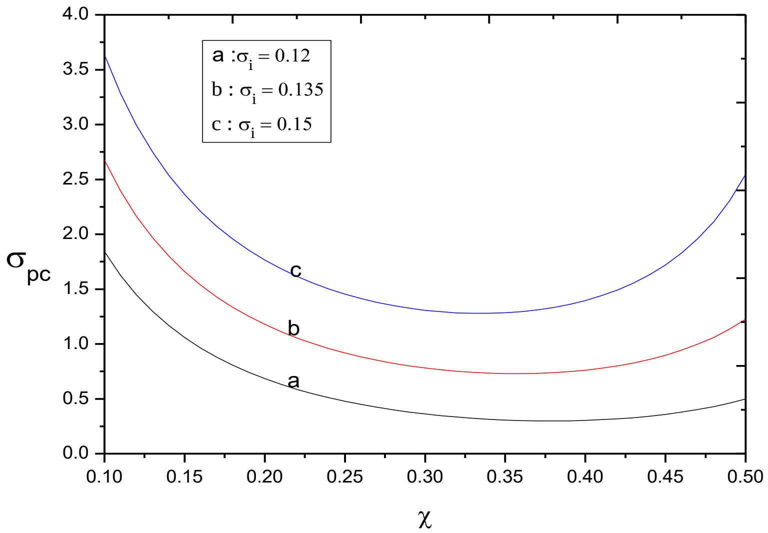

Fig.4.3(a).Variation of critical positron temperature (σ_{pc}) with positron density (χ) for different ion temperature (σ_i) in e-i-p plasma having V=1.2, v_{i0} =0.1 and p=1. Graphs a, b and c represent σ_i =0.12, 0.135 and 0.15 respectively.

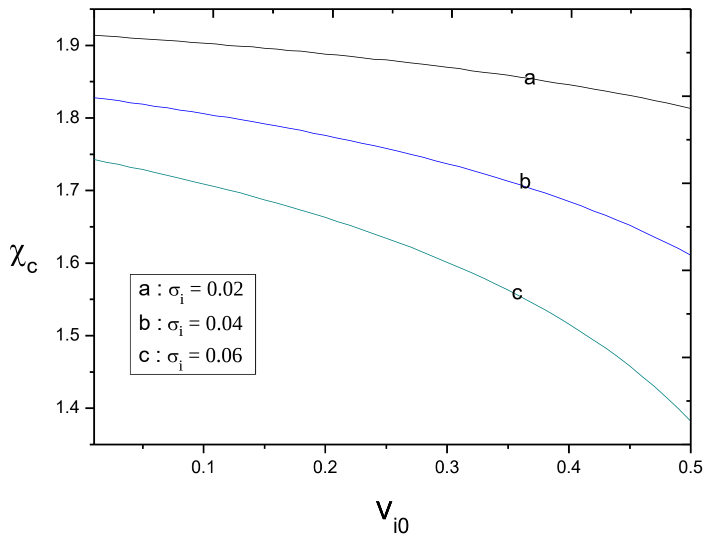

Fig 4.3(b). Variation of critical density of positrons (χ_c) with ion stream velocity (v_{i0}) for different value of ion temperature (σ_i) in plasma having σ_p =0.1, V =1.5 and p=1. Graphs a, b and c represent σ_i =0.02, 0.04 and 0.06 respectively.

Fig. 4.3(a) shows that critical positron temperature (σ_{pc}) decreases with positron density (χ) and reaches to a minimum value at certain value of χ, then it starts to increase. Moreover, σ_{pc} increases with the increase of ion temperature (σ_i). On the other hand, it is seen from Fig. 4.3(b) that the critical positron density (χ_c) decreases with the increase of ion stream velocity (v_{i0}) and ion temperature (σ_i) in the e-i-p plasma.

4.6.4. Profiles of Ion-Acoustic Solitons for an Arbitrary(Fixed) Phase Velocity

The effects of the plasma parameters, e.g. positron density, positron temperature and ion stream velocity on the IAS can be understood from the following profiles. The profiles of IAS are drawn taking an arbitrary value of phase velocity (V=1.6) of ion-acoustic wave.

i). Effect of Positron Density

The profiles of small- amplitude IAS for different values positron density (χ) in an e-i-p plasma are shown in Fig. 4.4(a). The profiles of large- amplitude IAS for different value of positron density in e-i-p plasma having are shown Fig. 4.4(b).

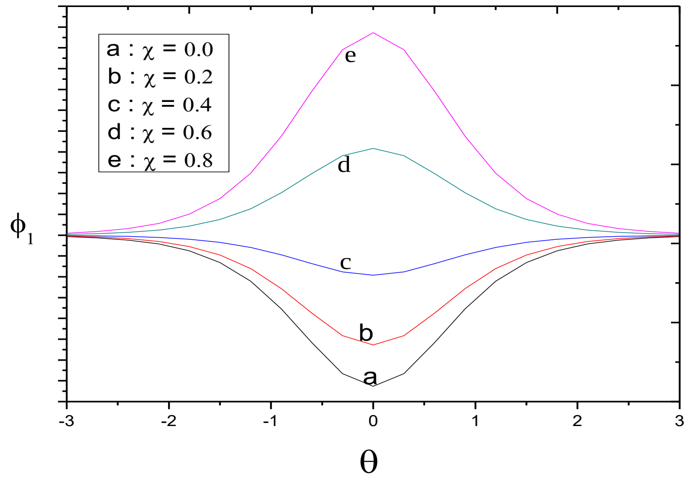

Fig. 4.4(a) Profiles of Small amplitude IAS for different value of positron density (χ) in e-i-p plasma having σ_p = 0.4, σ_i =0.05, V=1.6, v_{i0} =0.1 and p_{i0} =1. Graphs a, b, c ,d, e and f represent χ =0.0, 0.2, 0.4, 0.6 and 0.8 respectively.

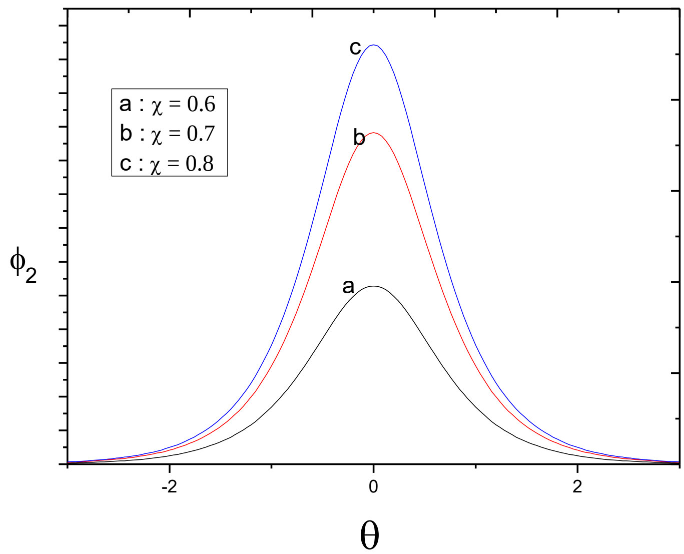

Fig.4.4 (b). Profiles of Large amplitude IAS for different value of positron density (χ) in electron ion positron plasma having plasma parameters σ_p = 0.6, σ_i =0.03, V=1.6, v_{i0} =0.1, p_{i0} =1. Graphs a, b and c represent χ = 0.6, 0.7 and 0.8 respectively.

It is observed from Fig. 4.4(a) that small amplitude IAS may be compressive and rarefactive depending on the values of positron density χ . For χ = 0.0 to 0.4 the IAS is rareefactive, but for χ = 0.6 and 0.8 the IAS is compressive. The amplitudes of rarefactive IAS increases with the decrease of χ , but for the compressive IASW the amplitude increases with the increase of χ in e-i-p plasma. On the other hand, Fig. 4.4(b) shows that large amplitude IAS is compressive for the χ = 0.6 to 0.8. The amplitude of IAS increases with the increase of χ in the e- i-p plasma.

ii). Effect of Positron Temperature.

In an e-i-p plasma, the profiles of small- amplitude IAS for different values of positron temperature (σ_p) are shown in Fig. 4.5(a). The profiles of large- amplitude IAS for different value of positron temperature in e-i-p plasma are shown Fig. 4.5(b).

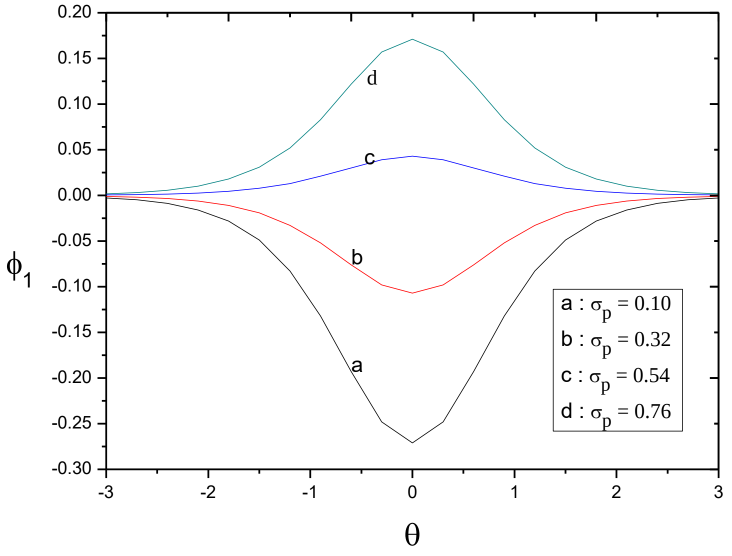

Fig.4.5 (a). Profiles of Small amplitude IAS for different value of positron temperature (σ_p) in e-i-p plasma having plasma parameters χ = 0.5, σ_i = 0.05, V=1.6, v_{i0} =0.2 and p_{i0} =1. Graphs a, b, c and d represent σ_p = 0.1, 0.32, 0.54 and 0.76 respectively.

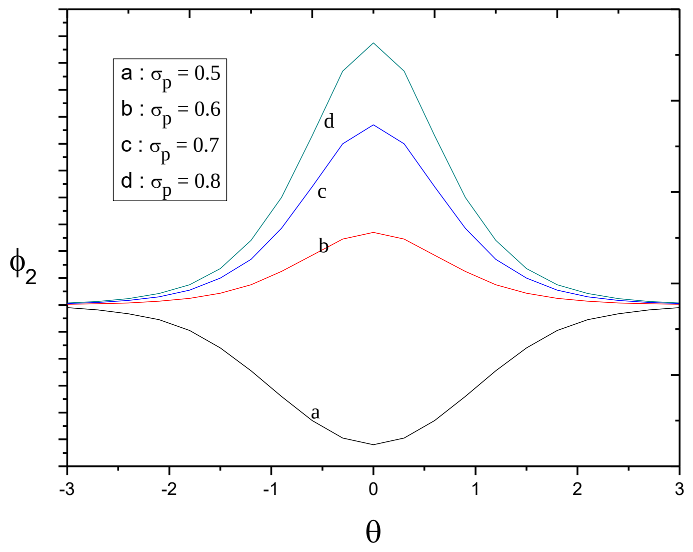

Fig. 4.5(b). Profiles of Large amplitude IAS for different value of positron temperature (σ_p) in e-i-p plasma having χ = 0.5, σ_i =0.06, V=1.6, v_{i0} =0.2 and p_{i0} =1. Graphs a, b, c and d represent σ_p = 0.5, 0.6, 0.7 and 0.8 respectively.

It is seen from Fig.4.5 (a) that small amplitude IAS may be compressive and rarefactive depending on the values of positron temperature σ_p in e-i-p plasma having fixed values of plasma parameters. For σ_p = 0.10 to 0.32 the IAS is rarefactive whereas for σ_p = 0.54 and 0.76 the IASW is compressive. The amplitudes of rarefactive IAS increases with the decrease of positron temperature, but for compressive IAS the amplitude increases with the increase of positron temperature in e-i-p plasma. .

Fig.4.5 (b) shows that large amplitude IAS may also be compressive and rarefactive depending on the values of σ_p in e-i-p plasma having fixed values of plasma parameters. For the positron temperature σ_p = 0.5, the IAS is rarefactive; but for the positron temperature σ_p = 0.6, 0.7 and 0.8, the IAS is compressive. The amplitudes of compressive IASW increase with the increase of σ_p in the e-i-p plasma.

iii). Effect of Ion Stream Velocity

Similarly, the profiles of small amplitude and large amplitude IAS for different values of ion stream velocity (v_{i0}) in e-i-p plasma are drawn as shown in Fig. 4.6(a) and Fig. 4.6(b).

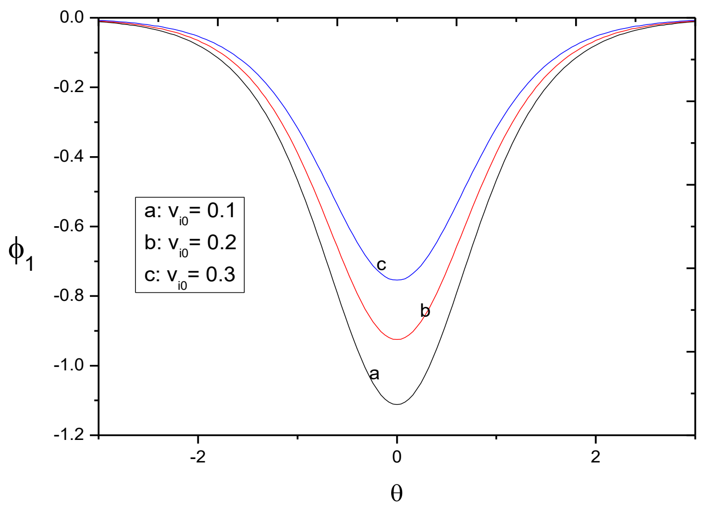

Fig. 4.6(a). Profiles of Small amplitude IAS for different value of ion stream velocity (v_{i0}) in e-i-p plasma having χ = 0.5, σ_i =0.001, σ_p =0.5, V=1.6 and p_{i0} =1. Graphs a, b and c represent v_{i0} = 0.1, 0.2 and 0.3 respectively.

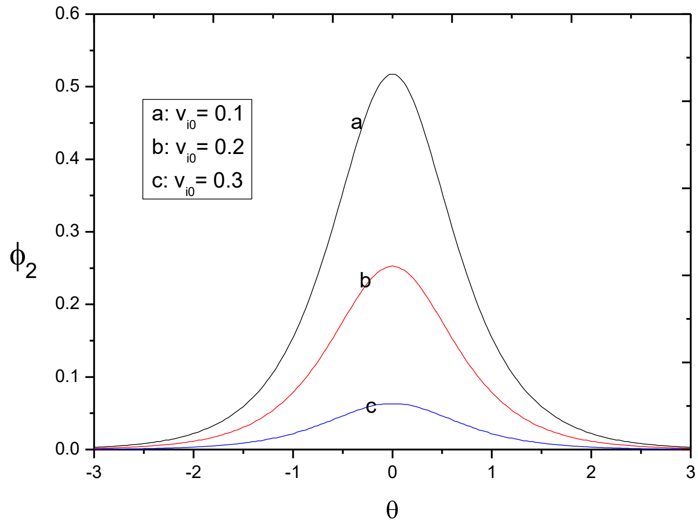

Fig.4.6 (b). Profiles of Large amplitude IAS for different value of ion stream velocity (v_{i0}) in e-i-p plasma having χ = 0.6, σ_i =0.1, σ_p = 0.8, V=1.6 and p_{i0}=1. Graphs a, b and c represent v_{i0} = 0.1, 0.2 and 0.3 respectively.

Fig. 4.6(a) shows that for v_{i0} = 0.1 to 0.3 in e-i-p plasma the small amplitude of IAS are rarefactive, when other plasma parameters have fixed values. But, from Fig. 4.6(b) it is seen that large amplitude IAS are compressive for ion stream velocity v_{i0} = 0.1 to 0.3 when other parameters have fixed values in the e-i-p plasma.

4.6.5. Profiles of Ion-Acoustic Solitons for Phase Velocity of Fast-Mode and Slow-Mode

The profiles of small amplitude and large amplitude IAS in e-i-p plasma are shown in Fig. 4.4(a, b) to Fig. 4.6 (a, b) taking an arbitrary value of the V =1.6 of ion-acoustic wave. But, from Eq.(4.17a) and Eq.(4.17b) we see that ion-acoustic wave propagates with two modes in presence of an ion stream in e-i-p plasma, one is fast- mode (V_F) and the other is slow mode (V_S). The variations of V_F and V_S with positron density (χ) and positron temperature (σ_p) for different values of ion-stream velocity (v_{i0}) on the Fast-mode and Slow-mode of the wave are shown above in Fig. 4.2(a) and Fig. 4.2(b). So, to see the real nature of IAS in e-i-p plasma in presence of an ion stream, the values of the phase velocities of Fast-mode (V_F) and Slow-mode (V_S) should be used for drawing the profiles of the IAS. Using V_F and V_S in e-i-p plasma, the profiles of small amplitude and large amplitude IAS for the fast-mode and slow-mode are drawn using different values of the e-i-p plasma parameters, e.g. ion-stream velocity (v_{i0}), positron density (χ) and positron temperature (σ_p).

i). Effect of ion stream velocity

The main factor of the excitation of Slow-mode and Fast-mode of IAS in e-i-p plasma is the stream velocity of ions. To see the effects of ion stream velocity on the slow-mode and fast-mode of the IAS in e-i-p plasma, the profiles of IAS are drawn as shown in Fig. 4.7(a), Fig. 4.7(b), Fig. 4.7(c) and 4.7(d).

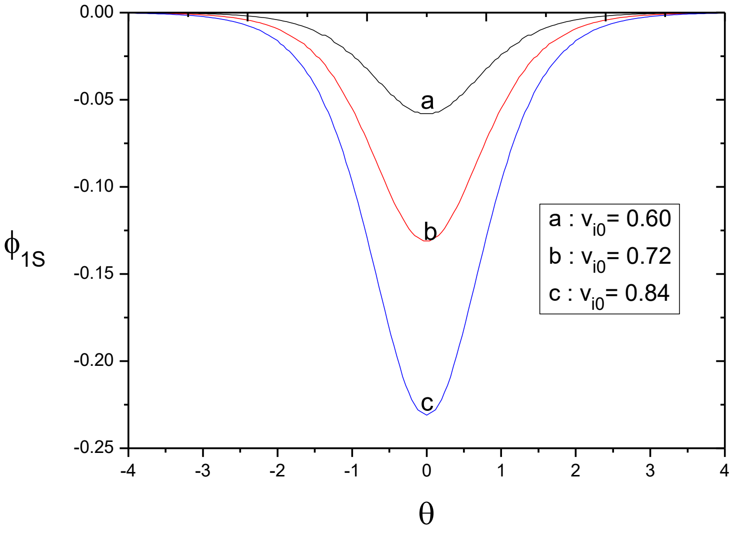

Fig.4.7 (a). Profiles of Small amplitude IAS of Slow-mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i =0.001, σ_p =0.8, χ =0.8 and $p=1$. Graphs a, b and c represent v_{i0} = 0.60 ,0.72 and 0.84 respectively.

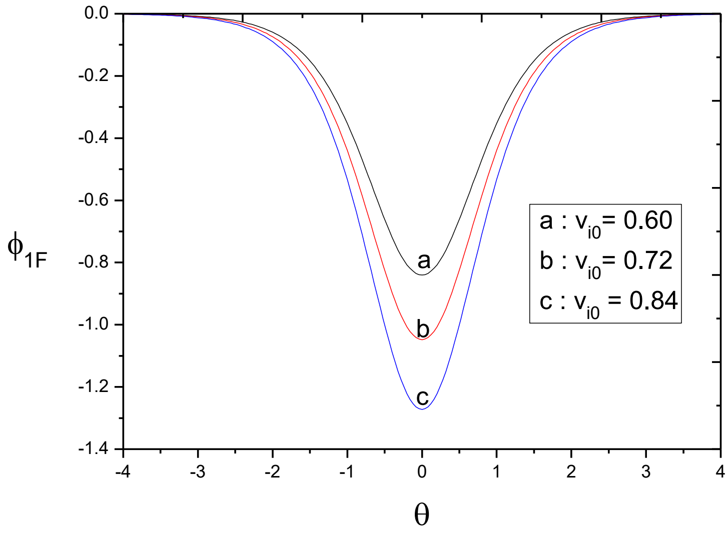

Fig. 4.7(b). Profiles of Small amplitude IAS of Fast-mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i = 0.001, σ_p = 0.8, χ = 0.8 and $p=1$. Graphs a, b and c represent v_{i0} = 0.60, 0.72 and 0.84 respectively.

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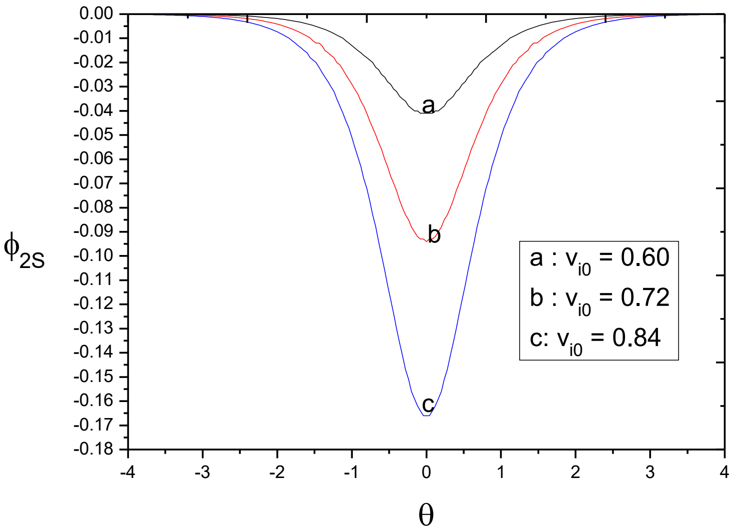

Fig. 4.7(c). Profiles of Large amplitude IAS of Slow-mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i = 0.001, σ_p = 0.8, χ =0.8 and $p = 1$. Graphs a, b and c represent v_{i0} = 0.60, 0.72 and 0.84 respectively.

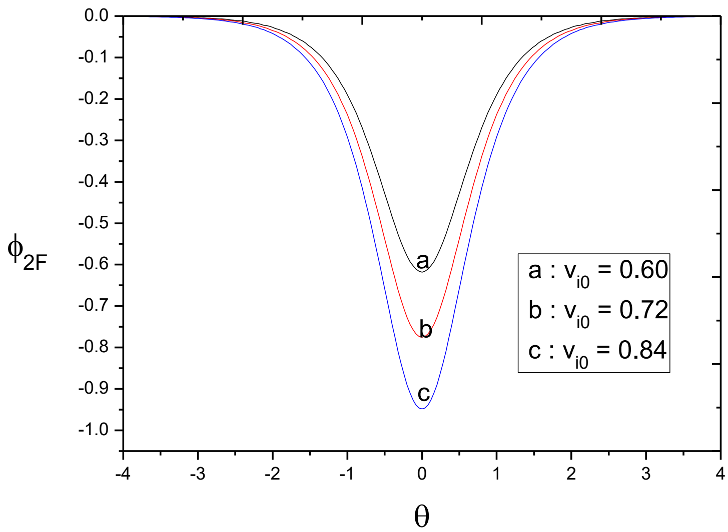

Fig. 4.7(d). Profiles of Large amplitude IAS of Fast-mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i = 0.001, σ_p = 0.8, χ =0.8 and $p = 1$. Graphs a, b and c represent v_{i0} = 0.60, 0.72 and 0.84 respectively.

It is observed from Fig.4.7(a) and Fig.4.7(b) that small amplitude IAS for Fast-mode and Slow-mode are rarefactive. For both Slow-mode and Fast-mode, the amplitudes of IAS increase with the decrease of v_{i0}. Again, Fig.4.7(c) and Fig4.7 (d) show that profiles for the large-amplitude IAS of Slow-mode and Fast-mode are also rarefactive. But, amplitude is different.

ii). Effect of positron density

Taking the values of the plasma parameters of model e-i-p plasma, the profiles of small amplitude and large amplitude IAS for different values of positron density for Slow-mode and Fast-mode are shown in Figs. 4.8(a), 4.8(b), 4.8(c) and 4.8(d).

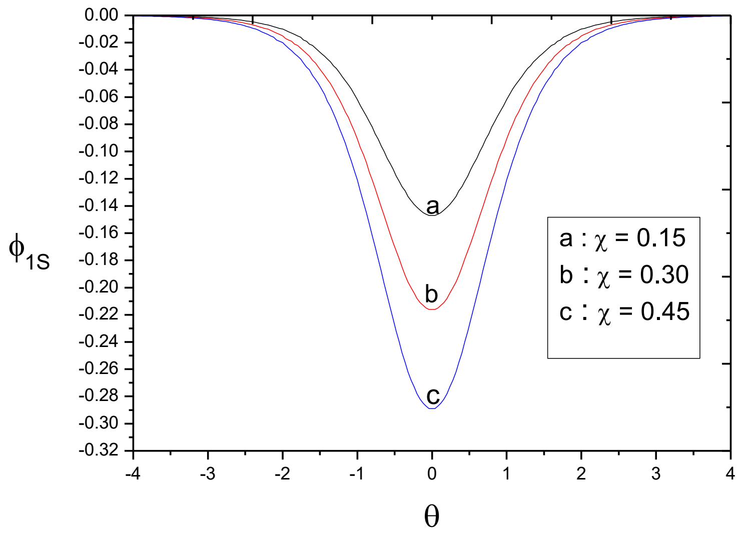

Fig.4.8 (a). Profiles of Small amplitude IAS of Slow-mode for different value of positron density (χ) in e-i-p plasma having v_{i0} = 1.4, σ_i =0.001, σ_p =0.5 and $p=1$. Graphs a, b and c represent χ = 0.15, 0.30 and 0.45 respectively.

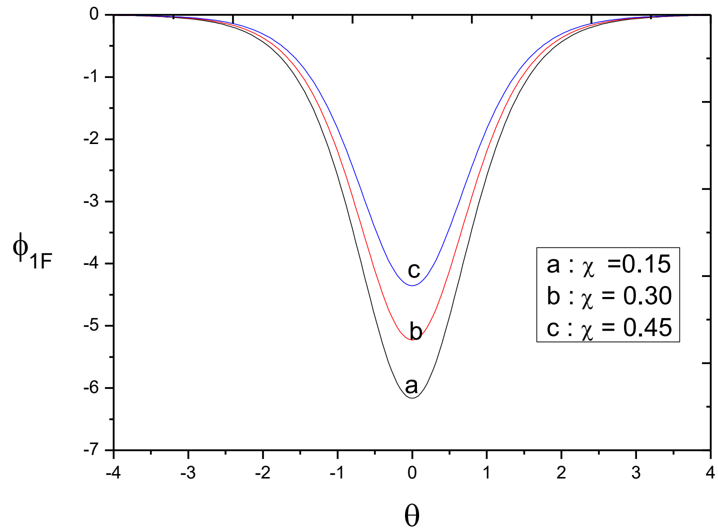

Fig.4.8 (b). Profiles of Small amplitude IAS of Fast-mode for different value of positron density (χ) in e-i-p plasma having v_{i0} = 1.4, σ_i =0.001, σ_p =0.5 and $p=1$. Graphs a, b and c represent χ = 0.15 ,0.30 and 0.45 respectively.

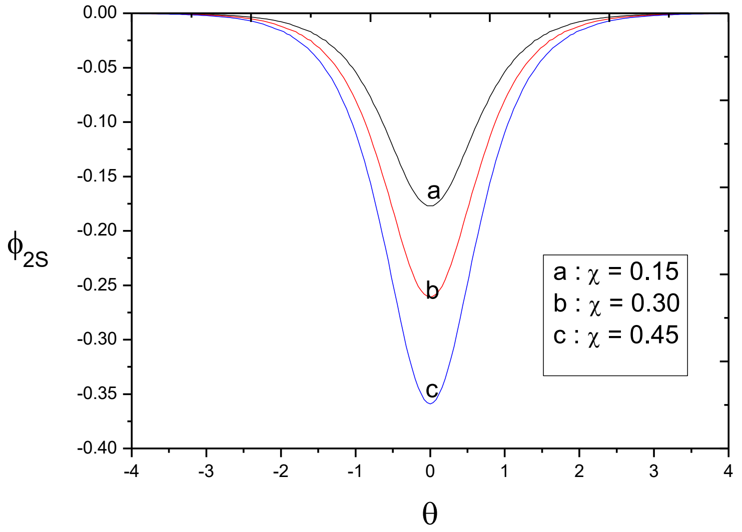

Fig.4.8(c). Profiles of Large amplitude IAS of Slow-mode for different value of positron density (χ) in e-i-p plasma having v_{i0} = 1.4, σ_i =0.001, σ_p =0.5 and $p=1$. Graphs a, b and c represent χ = 0.15 ,0.30 and 0.45 respectively.

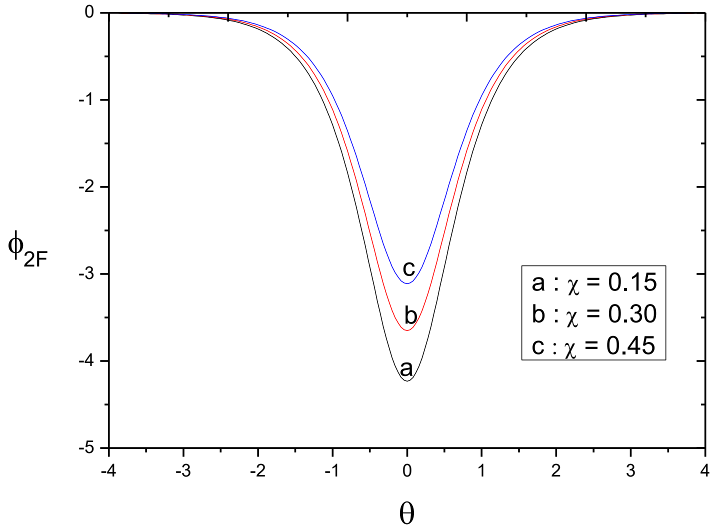

Fig.4.8(d). Profiles of Large amplitude IAS of Fast-mode for different value of positron density (χ) in e-i-p plasma having v_{i0} = 1.4, σ_i = 0.001, σ_p = 0.5 and $p=1$. Graphs a, b and c represent χ = 0.15 ,0.30 and 0.45 respectively.

It is seen from Fig.4.8(a) and Fig.4.8(b) that small amplitude IAS for Slow-mode and Fast-mode are rarefactive. For Slow-mode, the decrease of positron density (χ) increases the amplitude of IAS. For Fast-mode, the amplitude of IAS decreases with the increase of χ.

Again, Fig.4.8(c) and Fig 4.8(d) show that large amplitude IAS for Slow mode and Fast-mode are both rarefactive. For Slow-mode, the amplitude increases with the increase of χ. But, for Fast-mode the amplitude increases with the decrease of χ.

iii). Effect of positron temperature

Similarly, the profiles of small amplitude and large amplitude IAS in an e-i-p plasma for different values of positron temperature (σ_p) for Slow-mode and Fast-mode of the waves are shown in Figs. 4.9 (a), 4.9 (b), 4.9 (c) and 4.9 (d).

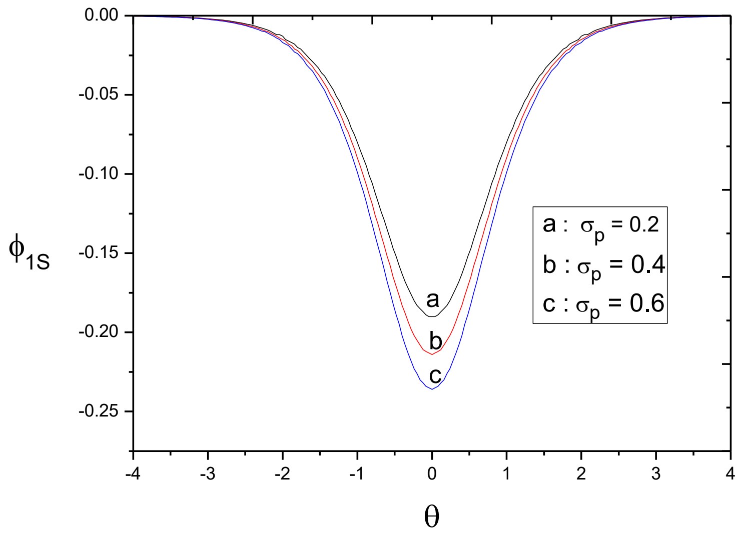

Fig. 4.9(a). Profiles of Small amplitude IAS profile of Slow mode for different value of positron temperature (σ_p) in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.001, χ = 0.4 and $p = 1$. Graphs a, b and c represent σ_p = 0.2, 0.4 and 0.6 respectively.

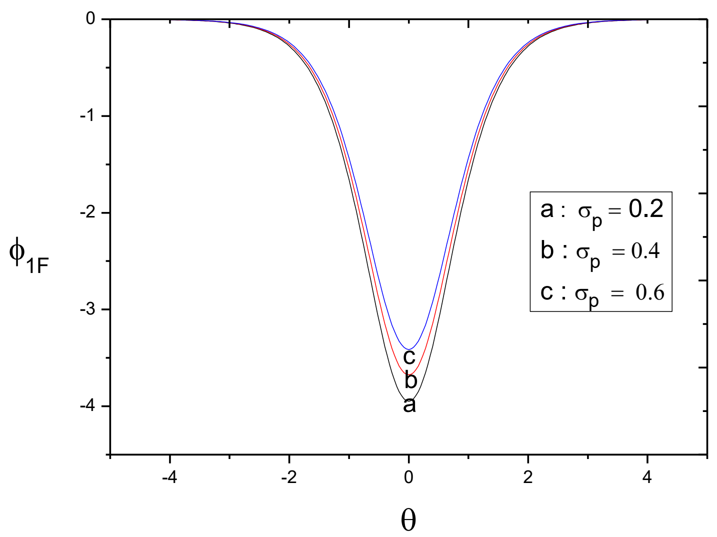

Fig. 4.9 (b). Profiles of Small amplitude IAS of Fast- mode for different value of positron temperature (σ_p) in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.01, χ = 0.5 and $p = 1$. Graphs a, b and c represent to σ_p = 0.1, 0.2 and 0.3 respectively.

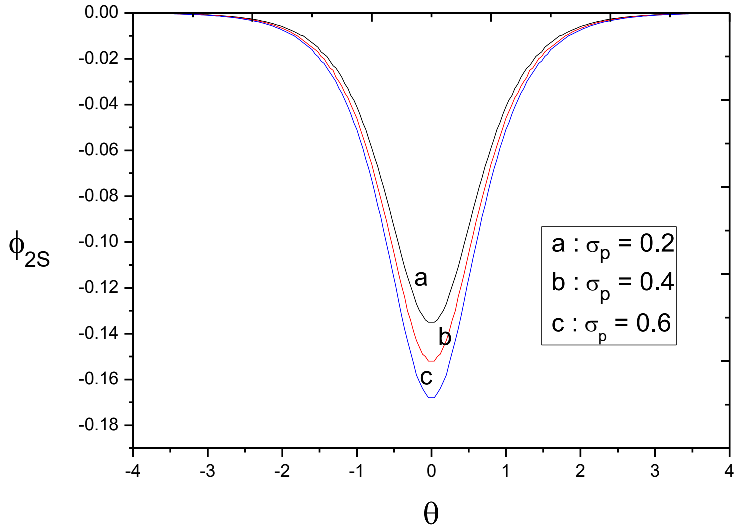

Fig.4.9(c). Profiles of Large amplitude IAS of Slow-mode for different value of positron temperature (σ_p) in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.01, χ = 0.5 and $p = 1$. Graphs a, b and c represent σ_p = 0.1, 0.2 and 0.3 respectively.

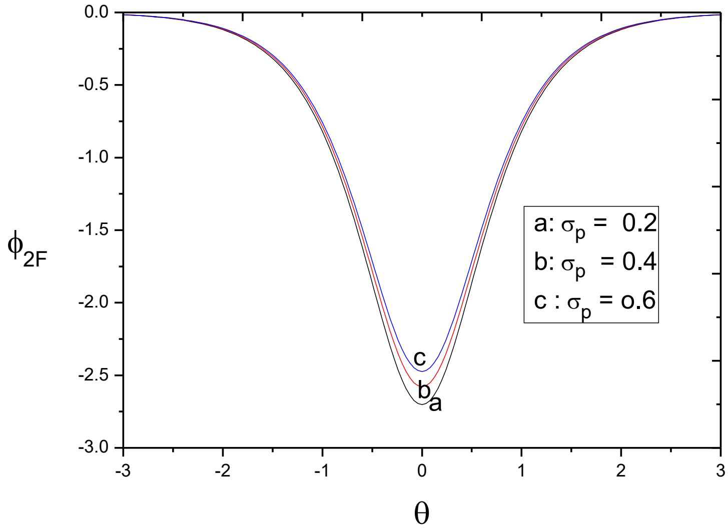

Fig.4.9(d). Profiles of Large amplitude IAS of Fast- mode for different value of positron temperature (σ_p) in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.01, χ = 0.5 and $p = 1$. Graphs a, b and c represent σ_p = 0.1, 0.2 and 0.3 respectively.

It is observed from Figs.4.9 (a) and 4.9(b) that small amplitude IAS for different values of positron temperature for Slow-mode and Fast-mode are rarefactive in the e-i-p plasma. For Slow-mode, the amplitude of IAS increases with increase of positron temperature (σ_p). But for Fast-mode, the amplitude of IAS decreases with the increase of σ_p. On the other hand,

Fig.4.9(c) and Fig.4.9 (d) show that large-amplitude IAS for Slow-mode and Fast-mode are also rarefactive in similar plasma. For Slow-mode the amplitude of large amplitude IAS increases with the increase of σ_p. But for fast-mode, it decreases with the increase of σ_p.

Similarly, the effects of ion temperature on the IAS for the Slow-mode and Fast-mode can be studied in e-i-p plasma having the different values of ion-stream. In this case, some interesting results would be obtained which may be important in the context of nonlinear wave processes in streaming plasma.

4.6.6. Amplitude of Ion-Acoustic Solitons for Slow-Mode and Fast- Mode

It is observed from the profiles Fig.4.7 (a) to Fig.4.9 (d) that the IAS for Slow-Mode and Fast- Mode are rarefactive in nature. The amplitudes of Slow-mode and Fast-mode are seen to be almost same, but close observation shows that these are different in magnitude. For this reason, we have numerically estimated i) small amplitude IAS for Slow-mode (φ_{01S}) and Fast-mode (φ_{01F}) using Eq.(4.29) and ii) Large amplitude IAS for Slow-mode (φ_{02S}) and Fast-mode (φ_{02F}) using Eqs.(4.35a) and (4.35b) for different values of the plasma parameters in e-i-p plasma. In Table-1, Table-2 and Table-3 we have shown the amplitudes of IAS for different values of ion stream velocity (v_{i0}), positron density (χ) and positron temperature (σ_p).

Table-1
Effect of Ion Stream (v_{i0}) on Amplitude of IAS
σ_i =0.001, σ_p =0.8, χ =0.8 and $p = 1$

v_{i0}	φ_{01S}	φ_{01F}	φ_{02S}	φ_{02F}
0.12	**-0.050**	**-0.215**	**-0.055**	**-0.154**
0.24	**-0.00922**	**-0.337**	**-0.00945**	**-0.243**
0.36	**-0.00133**	**-0.483**	**-0.00093**	**-0.351**
0.48	**-0.013**	**-0.651**	**-0.00911**	**-0.476**
0.60	**-0.058**	**-0.840**	**-0.041**	**-0.618**
0.72	**-0.131**	**-1.048**	**-0.094**	**-0.775**
0.84	**-0.231**	**-1.272**	**-0.166**	**-0.948**

Table-2
Effect of Positron Density (χ) on Amplitude of IAS
v_{i0} = 1.4, σ_i =0.001, σ_p =0.5 and $p = 1$

χ	φ_{01S}	φ_{01F}	φ_{02S}	φ_{02F}
0.150	**-0.248**	**-6.165**	**-0.177**	**-4.232**
0.300	**-0.364**	**-5.228**	**-0.261**	**-3.651**
0.450	**-0.498**	**-4.357**	**-0.359**	**-3.111**
0.600	**-0.644**	**-3.457**	**-0.469**	**-2.559**
0.750	**-0.774**	**-2.268**	**-0.577**	**-1.841**

Table-3
Effect of Positron Temperature (σ_p) on Amplitude of IAS
v_{i0} = 1.2, σ_i =0.01, χ =0.5 and $p = 1$

σ_p	φ_{01S}	φ_{01F}	φ_{02S}	φ_{02F}
0.2	**-0.190**	**-3.949**	**-0.135**	**-2.702**
0.4	**-0.214**	**-3.675**	**-0.152**	**-2.579**
0.6	**-0.236**	**-3.414**	**-0.168**	**-2.474**
0.8	**-0.256**	**-3.166**	**-0.183**	**-2.385**

4.7. SUMMARY AND CONCLUSION

In this present, the IAS has been theoretically studied in multicomponent plasma consisting of warm ions, isothermal electrons and positrons using the SP method neglecting Landau damping. The expressions of i) Sagdeev potential, ii) Phase velocity of fast- and slow- ion-acoustic mode, iii) Critical values of positron density, positron temperature, ion temperature and ion density for the excitation of IAS in e-i-p plasma are obtained. The solutions for the small amplitude and large amplitude IAS are obtained from the nonlinear equation derived from the Sagdeev potential. It is observed that both compressive and rarefactive IAS may be excited in the presence of positrons in e-i-p plasma having warm and drifting ions. It is seen that positron density, positron temperature, ion drift velocity and ion temperature have a considerable impact on the excitation of IAS in e-i-p plasma.

Interesting results which are found are: i) Small-amplitude IAS in the plasma may be compressive and rarefactive depending upon the values of the plasma parameters, but the large amplitude IAS are mainly compressive in the e-i-p plasma. The amplitude of small amplitude IAS is small for large value positron density.

The most important results are: Excitation of Slow- and Fast- IAS in e-i-p plasma in presence of ion stream. The Small amplitude and large amplitude of IAS of Slow-mode and Fast-modes are rarefactive different values of ion stream velocity, positron density, positron temperature and ion temperature, but the amplitudes of IAS are different depending on the values of plasma parameters of e-i-p plasma.

However, in the present investigation, the effect of Landau damping is neglected for the study of IAS in e-i-p plasma. It is expected that some more interesting results will be obtained for the IAS considering the effect of Landau damping in e-i-p plasma. It is to be noted that previous workers mainly have used the reductive perturbation method and the multiple scale method instead of using the Sagdeev potential method for the studies of IAS in Landau damped plasma. Because, it may be difficult to study nonlinear propagation of IAS considering the effect of Landau damping in a plasma using the Sagdeev potential method.

REFERENCES

[1]. N. L. Satalishvili, J. L. Javakeshvili and H. Kaya, Astrophys. Sp. Science., **250,** p**.**109 (1997)
[2].P.K.Shukla and L.Stenflo, Astrophys. Sp. Science , **209,** p. 323 (1993).
[3].S.Mahmood,A.Mustaq and H.Saleem, New J. Phys., **5,** p.28 (2003)
[4].H.Saleem, Phys. Plasmas, **13(3),** p.034503 (2006)
[5].W.Misner, K.S.Thorne and J.A.Wheeler , Gravitation,Freeman, San Francisco (1980).
[6].M.J.Rees, G.W.Gibbons,S.W. Hawkins and S. Siklas, (editors)The very earlier universe, Cambridge Universal Press (1983).
[7].Y.B.Zeldovich and I.Novikov, Relativistic Astrophysics, University of Chicago, Chicago, 1983.
[8].H.R.Millar and P.J. Wilte, Active Galactic Nuclei, Springer-Verlag, Berlin, p.202 (1987).
[9].P.A.Sturrock and K B Baker, Astrophys. J., **234**, p.612 (1979).
[10].F.C.Michel, Theory of Neutron Star Magneto-sphere, Chicago Univ. Press, Chicago, (1991).
[11].F.B.Rozzato, J.Plasma Phys,. **40,** p.289(1988)
[12].R. Pillay and R. Bharuthram, Astrophys. Sp. Science, **188,** p.592 (1992).
[13].S.I.Popel, S.V.Vladimirov and P.K.Shukla, Phys.Plasmas, **2,** p**.**716 (1995).
[14].P.K.Shukla, J.T.Mendonca and R.Bingham,Phys.Scripta.,**T113,** p. 133(2004).
[15]. H.Alinejad, S.Sovanian and J.Mahmoodi, Phys.Plasmas, **13,** p**.** 012304 (2006).
[16].S.Ghosh and R.Bharuthram, Astrophys.Sp. Science, **189,** p**.**121 (2008).
[17].W.Massod,N.Jehnab,A.M.Mirza and P.H.Sakanaka, Phys.Letts.,**A372,** p**.**4279 (2008).
[18].S.Ghosh and R.Bharuthram, Astrophys.Sp. Science, DOI **10**,1007/s 10519-0-10-04336 (2010).
[19].H. R .Pakzad, Phys.Letts., **A373,** p.847 (2009)
[20].H. R. Pakzad, Astrophys. Sp. Science., **331,** p.175 (2010)
[21].R.Z.Sagdeev, Rev. Plasma Phys edited by M A Leontovich, New York (Consultant Bureau), **4,** p. 23 (1966).
[22].H. K. Anderson, N. D’Angelo, P. Michelson and P. Nelson, Phys. Rev. Letts., **19,** p**.**149 (1967)

CHAPTER-5

ION-ACOUSTIC SOLITONS IN A SELF-GRAVITATING DUSTY PLASMA CONSISTING OF WEAK NONISOTHERMAL TWO-TEMPERATURE ELECTRONS

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5.1. INTRODUCTION

In recent years, there is considerable interest on the nonlinear propagation of waves in dusty plasma consisting of electron, ion and negatively charged dust particle. The dust particles in plasma has been found to be important because it helps to understand various linear and nonlinear phenomena in cometary tails , planetary rings , asteroids , magnetosphere , lower ionosphere , interstellar and circumstellar clouds , laboratory devices etc.[1-5]. Propagation of dust-acoustic wave (DAW) has been studied theoretically by Rao *et al.* [6], with the dust grains providing the inertia and the pressure of inertialess electrons and ions providing the restoring force. It is important to be noted that the dust particles in dusty plasma get negatively charged due to attachment of the background electrons and ions on the surface via collisions [7]. The charge on a dust particle does not remain fixed but depends on plasma properties, electron and ion currents flowing into or out of dust grains, photoemission etc. In fact the charge on the dust grain can be considered as an extra dynamical variable which controls the motion of dust grains in fluctuating electrostatic fields. But, for the study of wave propagation in dusty plasma, most of the authors have considered the effect of charge fluctuations on dust in the dusty plasma in an ad hoc fashion. The dust charge is treated as a dynamical variable which is related to the plasma density, potential, etc. Earlier authors have considered the continuity equations of electrons and ions without the sink term. As a consequence the charge conservation equation is not consistent with the charge variation equation. Considering the sink terms in continuity equation for the loss of electrons and ions due to attachment to the dust particles, the dynamical equation for charge fluctuation become consistent with the charge continuity equation.

In dusty plasma most interesting features are the solitary waves and the shock waves and a lot of works have been reported in last few years by various authors considering the ions with different parameters e.g. two-temperature electrons, nonisothermal electrons, nonthermal electrons and ions, super thermal electrons, magnetic field, inhomogeneity, finite geometry etc. for the applications in various fields. But, the self-gravitation of species of dusty plasma has not been taken into account in all those studies. In fact, the plasmas both in laboratory and in space have the gravitation field. In most of the studies of linear waves in collisional dusty plasma, gravitational field have been considered but the effects of gravitation for electrons, ions and neutral gas molecules are neglected. Misra *et al*. [8-10] and other authors [11,12] have theoretically investigated the solitary waves in gravitating dusty plasma from which it is seen that gravitational effect is important on the excitation of solitary waves in dusty plasmas. For the study of ion acoustic solitons (IAS) in a self-gravitating dusty plasma, Paul *et al* [13] have considered the sink terms in the continuity equations for both electrons and ions for the fluctuation of dust charges due to attachments of electrons and ions to the dust particles. They have shown that nonlinear excitation of waves follows a pair of coupled third order partial differential equations which is slightly different from usual case of coupled KdV system and the solitary wave structure is not possible in such self-gravitating dusty plasma. Since two-temperature electrons [14,15] have important roles on the excitation of solitary waves in plasma, Paul *et al*. [16] have studied the IAS in a self-gravitating dusty plasma with warm positive ions, isothermal two-temperature electrons and negatively charged dust particles using reductive perturbation method (RPM). They have assumed the sink term in the continuity equation of ions only for the dust charge fluctuation by the attachment of ions only to the dust particles. By deriving an uncoupled third order partial differential equation instead of coupled nonlinear equations they obtained a modified KdV (mKdV) equation in self-gravitating dusty plasma. They have discussed the effects of two-temperature electrons, gravity and dust charge fluctuations on the IAS waves in gravitating dusty plasma. Subsequently, using the RPM, Paul *et al.* [17] have studied the IAS in self-gravitating dusty plasma considering the nonisothermal electrons [18, 19] in the dusty plasma with fluctuating dust charges and drifting motion of the particles. They have obtained an uncoupled third order partial differential equation which yield mKdV equation from which the quasi-soliton solution of IAS has been obtained. Later, Paul *et al.* [20] have considered self-gravitating dusty plasma consisting of warm positive ions, nonisothermal two-temperature electrons and negatively charged dust particles. Finally, they have discussed the effects of nonisothermal two-temperature electrons, gravity and dust charge fluctuations on the IAS in self-gravitating dusty plasma.

However, the effects weak non-isothermalilty of two-temperature electrons on the IAS in self-gravitating dusty plasma have not yet been studied by any author. So, in this Chapter, we have studied the existence and characteristics of IAS in self-gravitating dusty plasma consisting of warm positive ions, weak non-isothermal two-temperature electrons and negatively charged dust particles with variable charge. Using the RPM, we have derived uncoupled third- order partial differential equations instead of coupled nonlinear equation for the usual nonisothermal and weak nonisothermal electrons in gravitating plasma. From this uncoupled nonlinear equation, the solution of IAS has been obtained and the effects of two-temperature electrons, gravity and dust charge fluctuations have been discussed.

5.2. BASIC EQUATIONS

An unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and non-isothermal two-temperature electrons are considered. It is assumed that the dust particles are mainly charged by the attachment of ions to dust grains. Initially, electrons are attached to dust particles at a faster rate than the ions because the electrons have lighter mass and faster motion than the ions. As a result the dust particles get negatively charged. Once the dust particles have enough negative charge the chance of an electron being attached to a negatively charged dust grain becomes lesser than the chance of an ion being attached to the dust grain. Ultimately, the charge fluctuation of dust grains is due to the attachment of positive ions only. Further, we consider that the self gravitating effect in dusty plasma influences both the linear and nonlinear modes of propagating ion acoustic waves. Under these assumptions, the normalized basic equations governing the plasma dynamics are [20]:

For ions*:*

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=-\beta_i n_i \tag{5.1}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial x}-\beta_i(v_i-v_d) \tag{5.2}$$

$$\frac{\partial p_i}{\partial t}+v_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial v_i}{\partial x}=0 \tag{5.3}$$

For dust particles:

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d v_d)=0 \tag{5.4}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}+\frac{\sigma_d}{\mu_d n_d}\frac{\partial p_d}{\partial x}=-\frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial x}-\beta_i(v_d-v_i) \tag{5.5}$$

$$\frac{\partial p_d}{\partial t}+v_d\frac{\partial p_d}{\partial x}+3p_d\frac{\partial v_d}{\partial x}=0 \tag{5.6}$$

Poisson's equations:

$$\frac{\partial^2 \phi}{\partial x^2}=n_{ec}+n_{eh}-n_i+Z_d n_d \tag{5.7}$$

$$\frac{\partial^2 \psi}{\partial x^2}=\frac{\omega_{gi}^2}{m_i\omega_i^2}\sum_{s=e,i,d} m_s n_s = X_i\sum_{s=i,d} m_s n_s \tag{5.8}$$

At equilibrium the charge neutrality condition is

$$n_i^{(0)}=n_e^{(0)}+Z_d n_d^{(0)}=n_{ec}^{(0)}+n_{eh}^{(0)}+Z_d n_d^{(0)} \tag{5.9}$$

the subscript $s = i$ represents for ions, $s = d$ for dust particles; m_s, n_s, v_s, q_s and p_s denote respectively the mass, number density , velocity, charge and scalar pressure of the s-th species of the plasma : n_{ec} ,n_{eh} are number density of electrons at low and high temperatures, Z_d is the number of electronic charge attached on the grains; $\mu_d = m_d/m_i$, $\sigma_i = T_i/T_{ef}$, $\sigma_d = T_d/T_{ef}$, $T_{ef}=T_{ec}T_{eh}/(n_{ec0}T_{eh}+n_{eh0}T_{ec})$, $X_i=\omega_{gi}^2/m_i\omega_i^2$, $\omega_{gi}^2=4\pi G m_i n_i$; $n_{i0}, n_{ec0}(n_{eh0})$ and n_{d0} are respectively the equilibrium number densities of ions, electrons and dust particles; G is the universal gravitational constant : φ and ψ are respectively the electrostatic and gravitational potentials; T_{ec} and T_{eh} are the low- and high- temperatures of electrons; T_{ef} is the temperature of free electrons; other symbols have their usual meanings. The parameter β_i is the attachment co-efficient (frequency) of ions to the dust particles. The right hand side of Eq. (5.1) is the sink term which describes the loss in the ion density due to attachment of ions to the dust particles. The attachment coefficient of ions may vary due to the change in ion current falling on the dust grains, and also due to the change in the densities of ions and dust grains. Since, β_i has the collision like behavior, the last terms in the right hand side of Eqs.(5.2) and (5.4) are also be like collisional effect between ions and dust particles.

In the above equations we have normalized distance x by Debye length $\lambda_D = (k_B T_e / 4\pi e^2 n_{i0})^{1/2}$, time t by the inverse of ion plasma frequency $\omega_i^{-1} = (m_i / 4\pi e^2 n_{i0})^{1/2}$, all pressures by ion plasma pressure $n_i^{(0)} K_B T_i$, φ by $K_B T_e /e$, ψ by $K_B T_e/m_i$, and all number densities by n_{i0}. $\sigma_i = T_i/T_{ef}$.

For the nonisothermal electrons at two different temperatures the number density is given by [21]

$$n_{ec} = \mu[1 + \frac{\varphi}{\mu+\nu\bar{\beta}} + \frac{4(1-\beta_l)}{3\pi^{1/2}}(\frac{\varphi}{\mu+\nu\bar{\beta}})^{3/2} + \frac{1}{2}(\frac{\varphi}{\mu+\nu\bar{\beta}})^2 - \frac{8(1-\beta_l^2)}{15\pi^{1/2}}(\frac{\varphi}{\mu+\nu\bar{\beta}})^{5/2} + \frac{1}{6}(\frac{\varphi}{\mu+\nu\bar{\beta}})^5 - \cdots] \tag{5.10}$$

$$n_{eh} = \nu[1 + \frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}} + \frac{4(1-\beta_h)}{3\pi^{1/2}}(\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}})^{3/2} + \frac{1}{2}(\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}})^2 - \frac{8(1-\beta_h^2)}{15\pi^{1/2}}(\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}})^{5/2} + \frac{1}{6}(\frac{\bar{\beta}\varphi}{\mu+\nu\bar{\beta}})^5 - \cdots] \tag{5.11}$$

where,

$$\bar{\beta} = T_{ec}/T_{eh}, \beta_l = T_{el}/T_{elt}, \beta_h = T_{eh}/T_{eht}, \mu = n_{ec0}, \nu = n_{eh0}, b_l = \frac{(1-\beta_l)}{\pi^{1/2}}, b_h = \frac{(1-\beta_h)}{\pi^{1/2}},$$
$$b_l^{(1)} = \frac{(1-\beta_l^2)}{\pi^{1/2}}, b_h^{(1)} = \frac{(1-\beta_h^2)}{\pi^{1/2}}, \mu = n_{ec}^{(0)}, \nu = n_{eh}^{(0)}.$$

and T_{et} is the temperature for the trapped electrons.

Therefore, from (5.10) and (5.11) we obtain

$$n_e = n_{ec} + n_{eh}$$
$$= 1 + \varphi - \frac{4}{3}(\frac{\mu b_l + \nu b_h \bar{\beta}}{\mu+\nu\bar{\beta}})^{3/2}\varphi^{3/2} + \frac{1}{2}\frac{(\mu+\nu\bar{\beta}^2)}{(\mu+\nu\bar{\beta})^2}\varphi^2 - \frac{8}{15}\frac{(\mu b_l^{(1)} + \nu b_h^{(1)}\bar{\beta}^{5/2})}{(\mu+\nu\bar{\beta})^{5/2}}\varphi^{5/2} + \cdots\cdots \tag{5.12}$$

5.3. DERIVATION OF KdV EQUATION AND ITS SOLUTION

In order to get an ion-acoustic wave equation in the form of a KdV equation, we introduce the following stretched variables, ξ and τ given by

$$\xi = \varepsilon^{1/4}(x - Vt), \qquad \tau = \varepsilon^{3/4} t \tag{5.13}$$

where, V is the linear phase velocity, ε is a dimensionless expansion parameter which is very small $\varepsilon \ll 1$ measuring the dispersion and nonlinear effects. Further, we assume the following perturbation expansion for the field variables:

$$\begin{aligned} P_s &= X_{s0} + \varepsilon P_{s1} + \varepsilon^{3/2} P_{s2} + \varepsilon^2 P_{s2} + \cdots\cdots \\ \beta_i &= \varepsilon^{3/4} \overline{\beta}_i \end{aligned} \tag{5.14}$$

Now, we use Eqs.(5.13) and 5.14) in Eqs. (5.1)-(5.8) and then equating the coefficients of $\varepsilon^{3/2}$ we obtain the equations consisting of $n_{i1}, v_{i1}, p_{i1}, n_{d1}, v_{d1}, p_{d1}$, and $v_{i2}, v_{d2}, p_{i2}, p_{d2}$. . Following standard mathematical procedure , the mKdV equation is derived for the electrostatic field in self-gravitating dusty plasma consisting non-isothermal two-temperature electrons:

$$\frac{\partial \phi_1}{\partial \tau} + C\phi_1^{1/2} \frac{\partial \phi_1}{\partial \xi} + \frac{1}{2} B \frac{\partial^3 \phi_1}{\partial \xi^3} + D\phi_1 = 0 \tag{5.15}$$

Where,

$$C = \frac{\left[\dfrac{2(\mu b_l + \nu\beta b_h)^{3/2}}{(\mu + \nu\overline{\beta})^{3/2}} \right]}{\left[\dfrac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\dfrac{V_1^2}{\lambda_1} + \dfrac{3a_1 n_{i0}}{\lambda_1} + 1) + \dfrac{Z_d n_{d0}}{V_2\lambda_2}(Z_d + \gamma_2^{1/2})(\dfrac{V_2^2}{\lambda_2} + \dfrac{3b_1 n_{d0}}{\lambda_2} + 1) \right]}$$

$$B = \frac{1/2}{\left[\dfrac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\dfrac{V_1^2}{\lambda_1} + \dfrac{3a_1 n_{i0}}{\lambda_1} + 1) + \dfrac{Z_d n_{d0}}{V_2\lambda_2}(Z_d + \gamma_1^{1/2})(\dfrac{V_2^2}{\lambda_2} + \dfrac{3b_1 n_{d0}}{\lambda_2} + 1) \right]}$$

$$D=\frac{\left[\frac{\beta_i n_{i0}}{\lambda_1}(V_1+\frac{Z_d V_2}{\lambda_2}+\frac{1}{V_1})+\frac{Z_d^2 n_{d0} V_1 \beta_i}{\lambda_2^2}-\frac{Z_d n_{d0} V_1 \gamma_1^{1/2}\beta_i}{\lambda_2^2}\right]}{\left[\frac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\frac{V_1^2}{\lambda_1}+\frac{3a_1 n_{i0}}{\lambda_1}+1)+\frac{Z_d n_{d0}}{V_2\lambda_2}(Z_d+\gamma_1^{1/2})(\frac{V_2^2}{\lambda_2}+\frac{3b_1 n_{d0}}{\lambda_2}+1)\right]}$$

It is seen that the dispersive term $\boldsymbol{B}$ of the mKdV Eq.(5.15) depends on the parameters of ions and dust particles in the gravitating plasma (i.e. $n_{i0}, n_{d0}, \mu_d, Z_d, \sigma_i, \sigma_d, \omega_{gi}, \omega_{gd}$) whereas the nonlinear term $\boldsymbol{C}$ depends on these parameters and also on the parameters of non-isothermal electrons (i.e. $\mu, \nu, \beta, b_l, b_h$). Since the coefficients $\boldsymbol{B}$ and $\boldsymbol{C}$ play crucial roles on the excitation of IAS , it is important to find the dependence of these coefficients on the amplitude, width and velocity of the IAS in the gravitating plasma. If $C=0$ i.e. when the nonlinear term in Eq.(5.15) vanishes at some critical values of i) dust density, or ii) gravitational effects for ions and dust particles, or iii) density (or temperature) of two-temperature electrons etc., the IAS will not exist in the self-gravitating dusty plasma. Similarly, for $\boldsymbol{B}$, the dispersive term in Eq.(5.15) , there will no excitation of the IAS in dusty plasma. Eq.(5.15) shows that there is one more term $D\varphi_1$ in the mKdV equation which consists of the attachment coefficient β_i and also the parameters of ions and dust particles in the gravitating plasma.

It is to be mentioned that an uncoupled KdV equation for the gravitational potential ψ can also be derived for the waves in self-gravitating dusty plasma using the standard mathematical method for electrostatic potential. Then one can examine the possibility of having soliton-like variation of the gravitational potential in dusty plasma.

Solving Eq. (5.15) the solution for IAS in a self-gravitating dusty plasma consisting of ions, charged dust particles and non-isothermal two-temperature electrons can be obtained for variable dust charges and also for constant dust charges. We first obtain the solution of IAS in gravitating dusty plasma neglecting the effect of charge fluctuation on the dust grain (i.e. $\beta_i=0$ and so D =0). The mKdV Eq.(5.15) is then reduced to

$$\frac{\partial\phi_1}{\partial\tau}+C\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+\frac{1}{2}B\frac{\partial^3\phi_1}{\partial\xi^3}=0 \tag{5.16}$$

For having the stationary solutions of equations (5.16) we introduce a new variable $\eta=\xi-\lambda\tau$ and integrate the resulting equation using the boundary condition,

$$i)\varphi_1\to 0, ii)\frac{d\varphi_1}{d\eta}\to 0, iii)\frac{d^2\varphi_1}{d\eta^2}\to 0 \quad \text{as } \eta\to\infty .$$

Thus we get the intermediate integral

$$\frac{d\varphi_1}{d\eta}=\left\{\frac{2\lambda}{\overline{R}}(\varphi_1)^2\left[1-\frac{8\overline{Q}}{15\lambda}(\varphi_1)\right]^{1/2}\right\}^{1/2} \tag{5.17}$$

and then from Eq.(5.17) we obtain the solution for IAS in self-gravitating dusty plasma given by :

$$\varphi_1=\varphi_0 Sech^4(\lambda/\delta) \tag{5.18}$$

where , $\varphi_0=\frac{225}{64}\frac{\lambda^2}{C^2}$ (is the amplitude), $\delta=[\frac{8B}{\lambda}]^{1/2}$ (is the width) and λ is the phase velocity of IAS. it is seen that both λ^2 and $\boldsymbol{C^2}$ are positive for any value of the plasma parameters. So the amplitude φ_0 is positive which indicates that only compressive IAS will be excited in the self-gravitating dusty plasma

When the variation of dust charges in dusty plasma is taken into account (i.e. $\beta_i\neq 0$, and so $D\neq 0$), the quasi-soliton solution for IAS is obtained from Eq.(5.15). The mKdV Eq. (5.15) has a quasi-soliton solution given by [22]

$$\varphi_1=\boldsymbol{a(\eta')\,sec\,h^4\,[(\frac{a(\eta')}{16})^{1/4}(\xi-\sqrt{a(\eta')}\eta')]} \tag{5.19}$$

The amplitude of IAS can be expressed as

$$\boldsymbol{a(\eta')=a(\eta_0')\,exp[-S(\eta'-\eta_0')]} \tag{5.20}$$

where η_0' corresponds to some initial value of the variable η'.

Obviously, if $\boldsymbol{D < 0}$ the amplitude of the quasi-soliton increases exponentially indicating instability. On the other hand if $\boldsymbol{D > 0}$ the amplitude of IAS decays exponentially with the increase in the independent variable η'.

5.4. EFFECT OF WEAK NONISOTHERMALITY OF TWO-TEMPERATURE ELECTRONS ON THE SOLITON SOLUTION

Now, we consider the case of dusty plasma having different effect of nonisothermality of two-temperature electrons. We assume that a small percentage of nonisothermality is present in the plasma i.e. the plasma has also a reasonable percentage of isothermal two temperature electrons. In this case, the above analysis for deriving the KdV equation will not hold good and recourse must be taken to the original governing Eqs. (5.1) to (5.8). This can be done by introducing the nonisothermal parameters b_l and b_h as small quantities in slight different forms; $b_{l,h}$ are assumed to be $b_{l,h} = \varepsilon^{1/2}\, b`_{l,h}$ where $b`_{l,h} > 0$. In this case, the scheme of perturbation expansion of the field variables and stretching of the space co-ordinates and time will be different from that considered earlier in Eq.(5.13). We have to use the scheme of Washimi and Taniuti [23] given by

$$\xi = \varepsilon^{1/2}(x - Vt) \text{ and } \tau = \varepsilon^{3/2} t \tag{5.21}$$

where, V is the linear phase velocity and ε is a smallness parameter measuring the dispersion and nonlinear effects. Further, the perturbation expansions for the field variables are :

$$\begin{aligned} P_s &= X_{s0} + \varepsilon P_{s1} + \varepsilon^2 P_{s2} + \varepsilon^3 P_{s2} + \cdots\cdots \\ \alpha_i &= \varepsilon^{5/2} \bar{\alpha}_i \end{aligned} \tag{5.22}$$

Now, to derive the mKdV equation we use Eqs.(5.21) and (5.22) in Eqs. (5.1) to (5.8) and then equating the coefficients of ε^2 we obtain the equations consisting of $n_{i1}, v_{i1}, p_{i1}, n_{d1}, v_{d1}, p_{d1}$, and $v_{i2}, v_{d2}, p_{i2}, p_{d2}$. Following the standard mathematical procedure ,we have derived the following modified KdV equation for the electrostatic field in self-gravitating dusty plasma consisting non-isothermal two-temperature electrons:

$$\frac{\partial \phi_1}{\partial \tau} + [A\phi_1 + C\phi_1^{1/2})\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + D\phi_1 = 0 \tag{5.23}$$

Where,

$$A=\frac{\left[\alpha_1\left\{1+\gamma_1^{1/2}\right\}^2-\frac{(\mu+\nu\bar{\beta}^2)}{(\mu+\nu\bar{\beta})^2}-Z_d\alpha_2\left\{Z_d+\mu_d\gamma_2^{1/2}\right\}^2\right]}{\left[\frac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\frac{V_1^2}{\lambda_1}+\frac{3a_1n_{i0}}{\lambda_1}+1)+\frac{Z_dn_{d0}}{V_2\lambda_2}(Z_d+\gamma_2^{1/2})(\frac{V_2^2}{\lambda_2}+\frac{3b_1n_{d0}}{\lambda_2}+1)\right]} \qquad (5.24)$$

The coefficients ***B, C*** and D are given above.

Neglecting the dust charge fluctuation (i.e. ***D*** = 0) we get the stationary solution of (5.23) with the boundary condition at $\chi\to 0$. We assume $\chi=\xi-U\tau$,U is the velocity of transformed co-ordinates. Using the technique of Davidson [24] and Das *et al*. [25] , we obtain the following intermediate equation

$$(\frac{\partial\varphi_1}{\partial\chi})^2=\frac{U}{B}(\varphi_1)^2(1-\frac{8C}{15}(\varphi_1)^{1/2}-\frac{A}{3U}\varphi_1) \qquad (5.25)$$

Now, we introduce $\phi=M^{-2}$ and the Eq. (5.25) is written as

$$-[\frac{U}{B}(M^2-\frac{8C}{15}M-\frac{A}{3U})]^{-1/2}dM=\pm d\chi \qquad (5.26)$$

Again integrating the equation and then putting back to into the equation of ϕ , we get the solution

$$\varphi_1=[\frac{4C}{15U}+(\frac{16C^2}{225U^2}+\frac{A}{3U})^{1/2}\cosh(\chi/\delta_2)]^{-2} \qquad (5.27)$$

where, $\delta_2=2(B/U)^{1/2}$ is the width of the solitary wave.

It is to be noted that the expression $(\frac{16C^2}{225U^2}+\frac{A}{3U})$ is positive if ***A*** is positive . But, this expression may not be positive if ***A*** is negative. The negative values of $(\frac{16C^2}{225U^2}+\frac{A}{3U})$ will give complex values of the amplitude which indicates the non-propagation of wave in the plasma. For a real solitary wave the expression $(\frac{16C^2}{225U^2}+\frac{A}{3U})$ should be positive. It is possible if $(\frac{16C^2}{225U^2}) > (\frac{A}{3U})$ for $\boldsymbol{A} < 0$. Moreover, the values of $\boldsymbol{A}$ and $\boldsymbol{C}$ give some idea about the solitary wave solutions. If C is very smaller than $\boldsymbol{A}$,the solution of IAS takes the form

$$\varphi_1 = \frac{3U}{A}[1-\frac{8C^2}{5\sqrt{3}AU}\sec h(\chi/\delta_2)-\frac{16C^2}{75AU}\{1-3\sec h^2(\chi/\delta_2)\}]\sec h^2(\chi/\delta_2) \tag{5.28}$$

If ***C*** is very small and if it is negligibly small then the solution becomes,

$$\varphi_1 = \frac{3U}{A}\sec h^2(\chi/\delta_2) \tag{5.29}$$

which is the value of the electrostatic potential of IAS in plasma having isothermal electrons?

Considering the charge fluctuation of dust particles in plasma i.e. $D \neq 0$ having isothermal two-temperature electrons, the solutions for the IAS is obtained by Paul *et al*. [20] given by

$$\varphi_{1S} = \varphi_0 \sec h^2\theta_S + D\tau \tag{5.30}$$

Where, $\varphi_0 = \frac{1}{3\zeta^2}$, $\theta_s = f(\zeta)$, ζ is a transformation parameter.

Moreover, in the case when ***A*** **i**s very small as compared to ***C*** the solution of IAS in plasma having no charge fluctuation becomes

$$\varphi_1 = \frac{225U^2}{64C^2}[1-\frac{75AU}{32C^2}\cosh(\chi/\delta_2)\times\sec h^2(\chi/2\delta_2)]\times\sec h^4(\chi/2\delta_2) \tag{5.31}$$

where the terms of $1/C^6$ are neglected.

As a special case , when ***A*** is negligibly small the above solution becomes

$$\varphi_1 = \frac{225U^2}{64C^2}\sec h^4(\chi/2\delta_2) \tag{5.32}$$

This is solution of the IAS in a plasma having non-isothermal electrons described by the mKdV Eq.(5.23) , neglecting the charge fluctuation parameter D for the attachment coefficients. Considering the fluctuation charge of dust particles through attachment coefficients, i.e. when $D \neq 0$, quasi-soliton solution of ion acoustic wave in gravitating plasma having nonisothermal electrons is given by Eq.(5.30).

5.5. RESULTS AND DISCUSSION

From the mKdV Eq.(5.23) and its solution Eq.(5.29) and Eq.(5.30) it is seen that the IAS will be excited in unmagnetized self-gravitating dusty plasma having nonisothermal two-temperature electrons for constant dust charges and also for variable dust charges. In a dusty plasma with constant dust charge, the amplitude of IAS is given by $\varphi_0 = (225/64)(\lambda^2/C^2)$ which shows the dependence of amplitude on i) the densities and temperatures of two-temperature electrons , ions and dust particles, ii) the masses of ions and dust particle and iii) the charge of dust particles. But, the width of IAS given by $\delta = (8B/\lambda)^{1/2}$ depends only on the parameters of ions and dust particles. The width of IAS does not depend on the density, temperature and nonisothermal parameter of electrons. On the other hand, it is seen that the attachment coefficients and the gravitational effects are involved in ***D*** in fourth term of mKdV Eq.(5.23) and these parameters will significantly influence the properties of IAS including its amplitude, speed, width as well as its stability. For the dust charge variation due to attachment coefficient of ions, the amplitude of IAS wave will grow or decay exponentially.

In dusty plasma, weak nonisothermal electrons have a different role on the excitation of IAS, In this case, the IAS has $\sec h^2-$ type profile instead of $\sec h^4-$type. The amplitude is given by $\varphi_0 = [\frac{4C}{15U} + (\frac{16C^2}{225U^2} + \frac{A}{3U})^{1/2}]^{-2}$ and width is $\delta = 2(B/U)^{1/2}$.

It is seen that amplitude of IAS depends on all the parameters in dusty plasma related to electrons, ions and dust particles including the gravitational effect. But, the electrons have no contribution on the width of IAS.

However, the plasma parameters of electrons, ions and dust particles are seen to be present in the expressions of amplitude and width of the IAS in a very complicated way in gravitating dusty plasma with ordinary-nonisothermal and weak-nonisothermal two-temperature electrons. It is very difficult to see the effect of this plasma parameter separately on the amplitude and width of the IAS. A close observation shows that for the increase of temperature and nonisothermal parameter of two-temperature electrons the amplitude of the IAS will be decreased and the width will be increased. On the other hand, the effects of gravitational forces of ions and dust particles on the amplitude and width of IAS are seen to be more significant than other parameters in the dusty plasma. The amplitude will be drastically reduced due to the gravitational effect, but the width will be increased for the gravitational forces of mainly due to dust particles. In fact, numerical analysis will give clear physical implications of the gravitational potential, dust density, ion temperature, and non-isothermal two-temperature electrons on the IAS in dusty plasma. Inclusion of drift motion of plasma leads to two distinct modes of wave propagation- one having comparatively high phase speed called as 'fast mode' and the other with lower phase speed called as 'slow mode'. For numerical analysis of amplitude and width of IAS in dusty plasma with drifting of ions and dust particles, one should carefully choice "fast-mode" or slow-mode" of the phase velocity for their investigation.

The attachment coefficients and the gravitational effects are involved in the mKdV equation; these parameters will influence the IAS properties including its amplitude, speed, width as well as its stability. The stability of solitary wave in gravitating dusty plasma is important for understanding various nonlinear wave processes in the plasma. It is found that stability of IAS depends on the sign of the coefficient $\boldsymbol{D}$ of the fourth term $D\phi$ in the mKdV equation. It is seen that the attachment coefficient, non-isothermal parameter of two-temperature electrons and gravitational effects are present into the factor $\boldsymbol{D}$ in a complicated way. The stability of the solitary structure will therefore be determined by the relative magnitudes of these effects.

5.6. SUMMARY AND CONCLUSION

In this Chapter, we have theoretically studied the IAS in self-gravitating dusty plasma consisting of positive ions, weak- nonisothermal two-temperature electrons and negatively charged dust particles with fluctuating charges and drifting ions by using the reductive perturbation method. Most important achievement in the paper is that instead of coupled nonlinear equations as obtained by earlier authors we have obtained an uncoupled nonlinear equation for the study of IAS in self-gravitating dusty plasma. However, we have not analyzed our theoretical results numerically for real dusty plasma of the astrophysical objects. Our analysis is advantageous for solving the nonlinear equation for getting the solution of solitary waves. The results obtained by previous authors for IAS in gravitating plasma with one component electrons can be easily recovered from our investigation.

In our study, it is seen that because of the extra electron the solitary wave in plasma shows a different role played as a result of the nonlinearity and the dispersive effect of the plasma. The multiple electrons in plasma exhibit a steepening tendency in solitary waves and therefore the IAS may not exist ultimately. Moreover, because of the appearance of large amplitude ion acoustic waves move with faster velocity, causing the IAS to break into many more IAS. These phenomena are exhibited in the isothermal plasma, whereas the nonisothermal plasma does not have any such singularity. However, the plasma with a small percentage of nonisothermality may introduce such singularity. A close observation indicates that for the increase of temperature and nonisothermal parameter of two-temperature electrons the amplitude of IAS will be decreased and the width of it will be increased. On the other hand, the effects of gravitational forces of ions and dust particles on the amplitude and width of IAS are seen to be more significant than other parameters in the dusty plasma. The amplitude of IAS will be drastically reduced due to the gravitational effect, but its width will be increased for the gravitational forces of mainly due to dust particles.

Instead of dusty plasma one may consider gravitational effect in heavy-ion plasma [26-29] and study the heavy-ion IAS and shock structures for constant charge of heavy-ions. It is to be remembered that the dynamics of dusty plasma and heavy ion plasma are completely different. The charge and the mass of dusts in dusty plasma are very large than the heavy ions in dust-less electron-ion plasma. So, the gravitational effects of heavy ions on the solitary waves and double layers (or shock waves) would be insignificant.

Some Special Remarks:

In dusty plasma both dust- acoustic and ion-acoustic solitary waves are excited in presence of drift motion. It is true that drift motion of dust particles are slower than the positive ions in dusty plasma. But, the motion of dust particles has important roles on linear and nonlinear propagation of waves in dusty plasma because both fast-mode and slow- mode are generated due to drift motion.

In this Chapter, we have considered unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and non-isothermal two-temperature electrons. We have assumed that the dust particles are mainly charged by the attachment of ions to dust grains. Initially, electrons are attached to dust particles at a faster rate than the ions because the electrons have lighter mass and faster motion than the ions. As a result the dust particles get negatively charged. Once the dust particles have enough negative charge the chance of an electron being attached to the negatively charged dust grain becomes very much smaller than the chance of an ion being attached to the dust grain. Ultimately, the charge fluctuation of dust grains is due to the attachment of positive ions only. But, the dust particles remain negatively charged which is varying due to attachment of ions to the dust particles. After a long period of time, the dust particles become neutral in charge and finally become positively charged due to continuous attachment of positive ions. In present paper, we have considered the case when the charge of the dust particles are negative before going to be neutral due to attachment of positive ions. However, when the dust particles become positively charged the mathematical analysis for the study of ion acoustic solitary waves would be same.

In dusty plasma, it is assumed that the electron- current and ion- current are the causes of charge fluctuation of dust particles [30]. The variation of dust charge is given by

$$\frac{dQ_{d1}}{dt}+\eta Q_{d1}=\left|I_{e0}\right|\left[\frac{n_{i1}}{n_{i0}}-\frac{n_{e1}}{n_{e0}}\right]$$

where, $\eta=(e\left|I_{e0}\right|/C)[1/T_e+1/\omega_0]$, C is the capacitance of dust grain,

$\omega_0=T_i-e\phi_{f0}$, T_i is the ion temperature and $e\phi_{f0}$ is the equilibrium floating potential.

Since, in this Chapter, we have assumed that all electrons are already attached to dust particles i.e. there is no free electron for attachment and only ions are being attached to dust particles causing dust charge variation.

So, I_{e0} =0, η =0 and $\frac{dQ_{d1}}{dt}$=0.

Therefore, attachment of ions is the only source for dust charge fluctuation. So, inclusion of attachment coefficient of ions in the continuity equations and the momentum equations is justified. Since the electrons are inertia- less, the question of inclusion of sink- term in the continuity equation of electron does not arise.

REFERENCES

[1]. G. K. Goertz, Rev. Geophys., **27**, p. 271 (1989)

[2]. D. A. Mendis and M. Rosenberg Ann. Rev. Astron. Astrophys.. **32,**p.419 (1994)

[3]. F. Verheest, Space Sci. Rev., **77** , p.267 (1996)

[4] .P K. Shukla and A. A. Mamun, Introduction to Dusty Plasma Physics (Bristol: Institute of Physics Publishing) (2002)

[5]. S. K. Maharaj, S. R. Pillay, R. Bharuthram, R. V. Reddy, S. V. Singh, and G. S. Lakhina, J. Plasma Phys. ,**72**, p.43 (2006)

[6]. N. N. Rao, P. K. Shukla and M. Y. Yu, Planet Space Sci., **38,** p.543 (1990)

[7]. M. R. Jana, A. Sen and P. K .Kaw, Phys. Rev., **E 48,** p.3930 (1993)

[8]. A. P. Misra, A. R . Chowdhury and K. R.Chowdhury, Phys. Letts. , **A 323**, 110 (2004).

[9] .A. P. Misra, K. R. Chowdhury and A. R. Chowdhury, Phys. Sripta, **69,** 44 (2005).9

[10] .A. P . Misra and A. R . Chowdhury, Euro Phys. J. , **D37**, 105 (2006).

[11]. S. Burman and A. Roychowdhury, Chaos, Solitons and Fractals,**13**, 973 (2002).

[12]. F. Verheest and P. K. Shukla, Phys. Scr. , **55**, 83 (1997).

[13]. S N Paul, K Roychowdhury, S. Burman, A. Roychowdhury and B. Paul, Czech. J. Phys., **56**, 1453 (2006).

[14]. B. N. Goswami and B. Buti, Phys. Letts., **A57**, p. 149 (1976).

[15] .P . N. Murthy, S. C . Tagare, and P . S. Abrol, Can. J. Phys., **62** , p. 45 (1984)

[16]. S. N. Paul, G. Pakira, B. Paul and B. Ghosh, WASET, Proceedings of International Conference ICFPP held at Bali, Indonesia on 26-28 October, (2011).

[17]. S. N. Paul, G. Pakira, B. Paul and B. Ghosh, ACTA PHYSICA POLONICA **A, 122**, p.116 (2012)

[18]. G. C. Das and J. Sarma, Phys. Plasmas, **5**, p.3918 (1998)

[19]. O. Rahman, M. D. I. Bhuyan, M. Haidar and J. Islam, International J. Astron. and Astrophys.,**4**, p.119 (2014)

[20]. S. N. Paul, A. Chatterjee and Indrani Paul , Indian J. Phys., **91**, p. 101(2017)

[21]. S. R. Majumdar, S. N. Paul and K. P. Das, J.Plasma Phys, **64(3**), 297 (2000).

[22]. W. S. Duan and Z. B. Zhao, Phys. Plasmas , **6**, 3484 (1999).

[23]. H. Washimi and T. Taniuti, Phys. Rev. Letts. , **17** 966 (1966)

[24] .R. C. Davidson, Methods in Nonlinear Plasma Theor**y.** New York, Academic, (1972)

[25]. G. C. Das, B. Karmakar and S. N. Paul, Phys. Fluids, **29,** 2192 (1986).[26] M.G. Shah, M.M. Rahman, M.R. Hossen and A.A. Mamun, Comm.Theo. Phys. , **64 (2)**, p. 208 (2015).

[27]. M. G. Shah ,M. M. Rahman,M. R. Hossen, A. A. Mamun, M. G. Shah , Plasma Physics Reports,**42 (2),** p.168 (2016).

[28]. S. A. Ema, M. R. Hossen, and A. A. Mamun, Phys. Plasmas, **22**, p.092108 (2015).

[29]. S. A. Ema, M. R. Hossen, A. A. Mamun , Contrib. Plasma Phys**. 55 (** 8), p.596 (2015).

[30]. J. R .Bhat and B. P. Pandey, Phys. Rev.E , **50**, p. 3980 (1994)

CHAPTER-6

DRESSED SOLITON IN A DENSE DUSTY PLASMA HAVING ULTRA-RELATIVISTIC DEGENERATE TWO-TEMPERATURE ELECTRONS

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6.1. INTRODUCTION

Ion-acoustic solitons (IAS) have been studied by various authors considering different types of plasma (negative ion plasma, positron plasma, quantum plasma, dusty plasma etc.) with different parameters (ion temperature, nonisothermal electrons, two-temperature electrons, nonthermal electrons, super thermal electrons, Tsallis distributed electrons etc.) to fit their results in various physical situations in laboratory and astrophysical plasma. However, two-temperature electrons have special kind of effects in the excitation of solitons in the plasma. A small percentage of the cooler component of electrons leads to effects qualitatively different from those obtained for a plasma having only one-temperature electron species. Goswami and Buti [1] and some other authors [2-8] have investigated the IAS and other aspects of the wave in two-temperature electron plasma and obtained important results which are applicable to various physical situations.

However, it is found from the experimental observations of **IAS** that theoretically predicted values of the amplitude, width, and velocity of the solitary waves do not obey the experimental results [9-11]. To remove the discrepancy between the theoretical and experimental results, a number of authors studied and analyzed the inclusion of higher order perturbation corrections into the KdV soliton, and the resulting solution has been termed 'dressed soliton'. Ichikawa Matsuhasi, and Konno [12] were the first to examine such higher-order effects by the reductive perturbation method for IAS in a plasma consisting of cold ions. Considering the temperature of ions in a plasma, Lai [13] investigated the higher-order effect on the IAS. Kodama and Taniuti [14] have reconsidered this problem and have shown by the method of renormalization how to eliminate the secular terms appearing in higher-order terms of the expansion. Later, Tewari and Misra [15], El-Labani *et al.* [16] and other authors [17-19] have investigated the propagation of higher-order IAS i.e. dressed solitons in different types of plasma.

But, in relativistically degenerate and ultra-relativistic dense plasma, propagation of waves must be studied following the works of Chandrashekhar [20,21]. The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the nonrelativistic limit, and $P_e \propto n_e^{4/3}$ for the ultrarelativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Shah *et al.* [22] have undertaken investigation on the effect of trapping on the formation of solitary structures in relativistic degenerate (RD) plasmas. They have used the relativistic Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. Masood and Eliasson [23] have studied electrostatic solitary waves in quantum plasma with relativistically degenerate electrons and cold ions. The inertia is given by the ion mass while the restoring force is provided by the relativistic electron degeneracy pressure, and the dispersion is due to the deviation from charge neutrality. Later, Chandra *et al.*[24] have studied electron-acoustic solitary waves in a relativistically degenerate quantum plasma with two-temperature electrons using the QHD model and degenerate electron equation of state $(P_e \propto n_e^{4/3})$. They have shown that degeneracy parameter significantly influences the conditions of formation and properties of solitary structures. Mamun and Shukla [25] have investigated arbitrary amplitude solitary waves and double layers assuming ultra-cold electron fluid, inertial ultra-cold ion fluid, and negatively charged static dust by the pseudo-potential approach. Later, Esfandyari-Kalejahi *et al*. [26] have studied arbitrary amplitude IAS using Sagdeev-Potential approach in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons and numerically investigated the matching criteria of existence of such solitary waves. Later, Roy *et al.* [27] investigated the nonlinear propagation of waves (specially solitary waves) in an ultra-relativistic degenerate (URD) dense plasma containing ultra-relativistic degenerate electrons and positrons, cold, mobile, inertial ions, and negatively charged static dust by the reductive perturbation method. They have derived the linear dispersion relation and the KdV equation whose numerical solutions are analyzed to identify the basic features of electrostatic solitary structures that may form in such degenerate dense plasma. The existence of solitary structures has been also verified by employing the pseudo-potential method.

From above discussions, it is seen that two-temperature electrons have important role to the solitary waves and dressed solitons in plasma. Moreover, ultra-relativistic electrons have significant effect on nonlinear wave processes in plasma. But, no investigation has yet been reported on the dressed solitons in ultra-relativistic dense plasma with degenerate electrons having equation of state (pressure) $P_e \propto n_e^{\frac{4}{3}}$. So, in this Chapter, we have studied the dressed solitons of ion acoustic waves in a dense plasma with cold ions and URD two-temperature electrons with static negatively charged dust particles. The profiles of dressed solitons are drawn and discussed for different values of density and temperature of URD electrons. Moreover, solution for the dressed solitons at critical state is also obtained and discussed graphically. It is seen that degenerate two-temperature electrons has considerable impact on the structure of dressed solitons in URD two-temperature electron plasma.

6.2. BASIC EQUATIONS

We consider four component plasma consisting of inertia less URD two-temperature electrons, cold inertial ion fluid, and negatively charged static dust. The degenerate pressure of electrons has been expressed in terms of density by using the ultra-relativistic limit of Chandrasekhar [20, 21]. The nonlinear dynamics of the electrostatic wave in such URD dense plasma is described by the following equations:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0 \tag{6.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \varphi}{\partial x} \tag{6.2}$$

$$n_{ec} \frac{\partial \varphi}{\partial x} - \frac{3}{4} \beta_{ec} \frac{\partial n_{ec}^{4/3}}{\partial x} = 0 \tag{6.3}$$

$$n_{eh} \frac{\partial \varphi}{\partial x} - \frac{3}{4} \beta_{eh} \frac{\partial n_{eh}^{4/3}}{\partial x} = 0 \tag{6.4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \alpha_{ec} n_{ec} + \alpha_{eh} n_{eh} - n_i + \alpha_d \tag{6.5}$$

where, s = e, i denote for electron and ion; velocity u_s is normalized by ion-acoustic speed $C_i = (m_e c^2 / e)^{1/2}$, density n_s is normalized by the equilibrium ion density, $n_e = n_{ec} + n_{eh}$, n_{ec} and n_{eh} are the densities of electrons at low and high temperature, the electrostatic potential ϕ is normalized by $m_e c^2/e$, space variable x and time variable t are normalized by $(m_e c^2/4\pi n_{i0} e^2)^{1/2}$ and ion plasma period $(\omega_{pi})^{-1}$; $\omega_{pi} = (4\pi n_{i0} e^2 / m_i)^{1/2}$;

$$\left.\begin{array}{l}\alpha_{ec} = \lambda_c n_{ec0}^{1/3}, \alpha_{eh} = \lambda_c n_{eh0}^{1/3}, \lambda_c = hc / m_e, \\ \beta_{ec} = K_0 \alpha_{ec}, \beta_{eh} = K_0 \alpha_{eh}, K_0 = (\hbar / m_e c), \\ \alpha_{ec} = (n_{ec0} / n_{i0}), \alpha_{eh} = (n_{eh0} / n_{i0}), \alpha_d = (Z_d n_{d0} / n_{i0}).\end{array}\right\} \tag{6.6}$$

n_{ec0} , n_{eh0} and n_{i0} are the equilibrium densities of electrons at low and high temperature , and positive ions; Z_d is number of negative charges of dust particles. Other parameters have their usual meanings.

The charge neutrality condition is

$$\alpha_{ec} + \alpha_{ec} + \alpha_d = 1, \text{ i.e. } \alpha_{ec} + \alpha_{eh} = 1 - \alpha_d \tag{6.7}$$

In absence of dust particles, α_d =0, i.e. $\alpha_{ec} + \alpha_{eh} = 1$

For a solitary wave solution we assume that the dependent variables depend on a single independent variable $\xi = x - Vt$, where V is the velocity of ion-acoustic wave. Then from the equations (6.1) - (6.3) we obtain after using the boundary conditions $n \to 1, \varphi \to 0$ as $\xi \to \pm\infty$

$$n_i = \frac{1}{\sqrt{1 - \frac{2\varphi}{V^2}}} \; , \; n_{ec} = \left(1 + \frac{\varphi}{3\beta_{ec}}\right)^3, n_{eh} = \left(1 + \frac{\varphi}{3\beta_{eh}}\right)^3 \tag{6.8}$$

where, β_{ec} and β_{eh} are the temperature degenerate electrons ar low and high temperatures.

Now using (6.8) in (6.4) we get

$$\frac{\partial^2 \varphi}{\partial \xi^2} = \Delta_1 \varphi + \Delta_2 \varphi^2 + \Delta_3 \varphi^3 \tag{6. 9}$$

Where

$$\Delta_1 = [\frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}} - \frac{1}{V^2}] \tag{6.10a}$$

$$\Delta_2 = [\frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2} - \frac{3}{2V^4}] \tag{6.10b}$$

$$\Delta_3 = [\frac{\alpha_{ec}}{27\beta_{ec}^3} + \frac{\alpha_{eh}}{27\beta_{eh}^3} - \frac{5}{2V^6}] \tag{6.10c}$$

An integral of (6. 9) satisfying the conditions $d\varphi/dX \to 0, \varphi \to 0$ as $\xi \to \pm\infty$ is

$$\frac{\varepsilon}{2}\left(\frac{d\varphi}{dX}\right)^2 = \frac{1}{2}\Delta_1 \varphi^2 + \frac{1}{3}\Delta_2 \varphi^3 + \frac{1}{4}\Delta_3 \varphi^4 \tag{6.11}$$

where we have stretched the ξ coordinate according to the relation

$$X = \in^{1/2} \xi \tag{6.12}$$

where ε is a small parameter , ε < 1. It is a measure of the weakness of dispersion.

We now make the following perturbation expansions for φ and V.

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + \ldots, \tag{6.13a}$$

$$V = V_0 + \varepsilon V^{(1)} + \in^2 V^{(2)} + \ldots \tag{6.13b}$$

Using with the expansion of V given by (6.13b), Δ_1, Δ_2 and Δ_3 of Eq.(6.9) become

$$\left.\begin{aligned} \Delta_1 &= \delta_1^{(0)} + \varepsilon\delta_1^{(1)} + \varepsilon^2\delta_1^{(2)} + \varepsilon^3\delta_1^{(3)} + \cdots\cdots \\ \Delta_2 &= \delta_2^{(0)} + \varepsilon\delta_2^{(1)} + \varepsilon^2\delta_2^{(2)} + \cdots\cdots\cdots \\ \Delta_3 &= \delta_3^{(0)} + \varepsilon\delta_3^{(1)} + \cdots\cdots \end{aligned}\right\} \tag{6.14}$$

Where

$$\left.\begin{aligned} \delta_1^{(0)} &= 0 \\ \delta_1^{(1)} &= 2\gamma_1^2 V_0 V^{(1)} \\ \delta_1^{(2)} &= 2\gamma_1^2 V_0 V^{(2)} + \gamma_1^2(1-4\gamma_1 V_0^2)(V^{(1)})^2 \\ \delta_1^{(3)} &= 2\gamma_1^2 V_0 V^{(3)} + 2\gamma_1^3(1-4\gamma_1 V_0^2)V^{(1)}V^{(2)} + 4\gamma_1^3 V_0(2\gamma_1^2 V_0^2 - 1)(V^{(1)})^3 \\ \delta_2^{(0)} &= \gamma_2 - \frac{3}{2}\gamma_1^2 \\ \delta_2^{(1)} &= 6\gamma_1^3 V_0 V^{(1)} \\ \delta_2^{(2)} &= 6\,\gamma_1^3 V_0 V^{(2)} + 3\gamma_1^3(1-6\gamma_1 V_0^2)(V^{(1)})^2 \\ \delta_3^{(0)} &= \gamma_3 - \frac{5}{2}\gamma_1^3 \\ \delta_3^{(1)} &= 15\gamma_1^4 V_0 (V^{(1)})^2 \\ \gamma_1 &= \frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}}, \gamma_2 = \frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2}, \gamma_3 = \frac{\alpha_{ec}}{27\beta_{ec}^3} + \frac{\alpha_{eh}}{27\beta_{eh}^3} \end{aligned}\right\} \tag{6.15}$$

Substituting the expansions (6.15) in (6.11) we get a sequence of equations for the $\varphi^{(i)}$'s and the equation for $\varphi^{(i)}$ at each order becomes a first-order inhomogeneous differential equation for $i > 1$.

For linear dispersion relation, $(\Delta_1)_0 = \partial_1^{(0)} = 0$.

Therefore,

$$\frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}} = \frac{1}{V_0^2} = \gamma_1, \tag{6.16a}$$

$$\text{i.e. } V_0 == \sqrt{\frac{\beta_{ec}\beta_{eh}}{(\alpha_{ec}\beta_{eh} + \alpha_{eh}\beta_{ec})}}\ ,\ \gamma_1 = V_0^{-2} \tag{6.16b}$$

6.3. SOLITARY WAVE SOLUTION

6.3.1. First-Order Equation and its Solution

In the first-order i.e. in the lowest order which is at the order of ε^3 we obtain the following equation for $\varphi^{(1)}$.

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2} + \frac{1}{3}\delta_2^{(0)}\varphi^{(1)^3} \tag{6.17}$$

The solution can be written as

$$\frac{d\varphi^{(1)}}{dX} = \pm\sqrt{\frac{2}{3}}\varphi^{(1)}[3V_0 - \varphi^{(1)}]^{\frac{1}{2}} \tag{6.18}$$

The solution of which is the first-order KdV soliton

$$\varphi^{(1)} = f_1 \sec h^2 \eta \tag{6.19}$$

Where f_1 is the first-order amplitude,

$$f_1 = -\frac{6V_0^{-5}}{(2\gamma_2 - 3V_0^{-4})}V^{(1)} \tag{6.20a}$$

$$\eta = \left(\frac{V_0^{-3}V^{(1)}}{2}\right)^{\frac{1}{2}} X \tag{6.20b}$$

$\phi^{(1)}$ vanishes at infinity and it attains a maximum value at $\eta = 0$ giving the amplitude of first-order solitary wave f_1, $f_1 = \phi_{01}(say)$.

$$V^{(1)} = \frac{(2\gamma_2 - 3V_0^{-4})V_0^5 f_1}{6} \tag{6.21}$$

The first-order width D_1 of the solitary wave is

$$D_1 = \left(\frac{2}{V_0^{-3}V^{(1)}}\right)^{\frac{1}{2}} \eta_1 \tag{6.22}$$

where η_1 is the positive root of the equation , $\tanh^2 \eta_1 = 0.58$.

The Mach number correct up to first-order

$$M_1 = 1 + \frac{V^{(1)}}{V_0} = 1 + \frac{(2\gamma_2 - 3V_0^{-4})V_0^4 f_1}{6} \tag{6.23}$$

It is important to note that the perturbed part $V^{(1)}$ of the expansion for V in (6.13b) remain unspecified and hence the amplitude of the soliton, f_1, can be changed by varying $V^{(1)}$.

6.3.2. Second-Order Eqation and its Solution

At the second-order we get the following equation for $\varphi^{(2)}$ using (6.16) and (6.22)

$$\frac{d\varphi^{(1)}}{dX}\frac{d\varphi^{(2)}}{dX}-[\partial_1^{(1)}\varphi^{(1)}+\partial_2^{(1)}\varphi^{(1)^2}]\varphi^{(2)}=\frac{1}{2}\delta_1^{(2)}\varphi^{(1)^2}+\frac{1}{3}\delta_2^{(1)}\varphi^{(1)^3}+\frac{1}{4}\delta_3^{(0)}\varphi^{(1)^4} \tag{6.24}$$

Differentiating (6.24) with respect to $\varphi^{(1)}$ we get

$$\frac{d^2\varphi^{(1)}}{dX^2}=\delta_1^{(1)}\varphi^{(1)}+\delta_2^{(0)}\varphi^{(1)^2} \tag{6.25}$$

By the use of this relation and by introducing the independent variables given by (6.22) the Eq.(6.24) can be written as

$$\frac{d\varphi^{(2)}}{d\eta}\frac{\varphi_{\eta\eta}^{(1)}}{\varphi_\eta^{(1)}}\varphi^{(2)}=\frac{\partial_1^{(2)}}{V_0V^{(1)}}\frac{\varphi_1^{(1)^2}}{\varphi_\eta^{(1)}}+\frac{2\delta_2^{(1)}}{3V_0V^{(1)}}\frac{\varphi_1^{(1)^3}}{\varphi_\eta^{(1)}}+\frac{\delta_3^{(0)}}{2V_0V^{(1)}}\frac{\varphi_1^{(1)^4}}{\varphi_\eta^{(1)}} \tag{6.26}$$

Multiplying both sides of (6.26) by the integrating factor $1/\varphi_\eta^{(1)}$ and then integrating we obtain the following solution $\varphi^{(2)}$ that vanishes at infinity and that attains a maximum at $\eta=0$.

$$\varphi^{(2)}=\frac{6V_0V^{(1)}\,\delta_1^{(2)}}{4V_0\;V^{(1)}}\eta\sec h^2\eta\tan h\eta+\;6V_0V^{(1)}\left(\frac{\delta_1^{(2)}}{4V_0V^{(1)}}+\frac{\delta_2^{(1)}}{2}+\frac{9\delta_3^{(0)}\,V_0V^{(1)}}{4}\right)\sec h^2\eta$$
$$-\frac{27\delta_3^{(0)}\,V_0^{\;2}V^{(1)^2}}{4}\sec h^4\eta \tag{6.27}$$

Owing to the presence of the first term in the above expression, the $\varphi^{(2)}/\varphi^{(1)}$ does not remain finite at $\eta\rightarrow\pm\infty$. So for a uniformly valid expansion of ϕ the coefficient of this term must be equal to zero. Therefore we get $\delta_1^{(2)}=0$

which gives

$$V^{(2)}=-\frac{3V_0V^{(1)^2}}{2} \tag{6.28}$$

Removing the secular term, the solution of (6.27) is obtained as

. .

$$\varphi^{(2)}=\left[\,f_2+\,f_3\tan h^2\eta\right]\sec h^2\eta \tag{6.29}$$

Where

$$f_1 = -\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}} = -\frac{6V_0^{-5}}{(2\gamma_2 - 3V_0^{-4})}V^{(1)}$$

$$f_2 = \frac{2f_1^2}{\delta_1^{(1)}}\left[\frac{1}{3}\delta_2^{(1)} + \frac{1}{2}f_1\delta_3^{(0)}\right] = \frac{36}{V_0^6(2\gamma_2 - 3V_0^{-4})^2}\left[2 + \frac{3}{2(2\gamma_2 - 3V_0^{-4})}(2\gamma_3 - 5V_0^{-6}\right]V_0^2(V^{(1)})^2 \tag{6.30}$$

$$f_3 = -\frac{f_1^3}{2}\frac{\delta_3^{(0)}}{\delta_1^{(1)}} = -\frac{f_1^3}{2}\frac{(\gamma_3 - \frac{5}{2}V_0^{-6})}{2V_0^{-3}V^{(1)}} = -\frac{f_1^3}{8}\frac{V_0^3(2\gamma_3 - 5V_0^{-6})}{V^{(1)}}$$

The amplitude of second-order correction term of (6.29) is

$$f_2 = \frac{36}{V_0^6(2\gamma_2 - 3V_0^{-4})^2}\left[2 + \frac{3}{2(2\gamma_2 - 3V_0^{-4})}(2\gamma_3 - 5V_0^{-6}\right]V_0^2(V^{(1)})^2 \tag{6.30a}$$

The second-order width D_2 of the solitary wave (6.29) is

$$D_2 = \left(\sqrt{\frac{2V_0^3}{V^{(1)}}}\right)\eta_2 \tag{6.31}$$

where η_2 is the positive root of the equation

$$(f_3 / f_2)\tanh^4\eta_2 - [1 + 2(f_3 / f_2)]\tanh^2\eta_2 + [(f_3 / f_2) + 0.58] = 0 \tag{6.32}$$

f_2 and f_3 are given above.

6.3.3. Corrected Higher-Order Solution up to Second-Order (Dressed Soliton)

Using (6.19) and (6.29) the solitary waves correct up to second-order become

$$\Phi_2 = \varphi^{(1)} + \varphi^{(2)} = f_4(1 + f_5\tanh^2\eta)\sec h^2\eta \tag{6.33}$$

where,

$$f_4 = f_1 + f_2 \text{ and } f_5 = f_3 / f_4 \tag{6.34a}$$

$f_4 = f_1 + f_2$, is the amplitude of corrected second-order solitary waves ,

$$f_4 = \frac{6V^{(1)}}{V_0^7(2\gamma_2 - 3V_0^{-4})}\left[1 + \frac{6V_0^{-2}}{(2\gamma_2 - 3V_0^{-4})}\left\{2 + \frac{3}{(2\gamma_2 - 3V_0^{-4})}(\gamma_3 - \frac{5}{2}V_0^{-6})\right\}V_0V^{(1)}\right] \tag{6.34b}$$

The Mach number of second-order correction term is

$$M_2 = \frac{V^{(1)^2}}{2V_0^2} \tag{6.35}$$

The Mach number of dressed solitary waves is

$$M = 1 + \frac{V^{(1)}}{V_0} + \frac{V^{(1)^2}}{2V_0^2}, \tag{6.36}$$

where, $V^{(1)}$ is given by (6.21)

6.4. DRESSED SOLITONS AT CRITICAL CASE

In critical case, the nonlinear coefficient vanishes, i.e. $\delta_2^{(0)} = 0$. Under this condition the solitary wave will not be excited in plasma. But near critical value of phase velocity V_0 (i.e. when $\delta_2^{(0)} = 0$) ,solitary wave will be excited with *sechη* profile instead of usual *sech²η* profile. To study the solitary wave near critical state $\delta_2^{(0)} \approx 0$ the following perturbation expansion for ϕ should be used,

$$\varphi = \varepsilon^{1/2}\varphi^{(1)} + \varepsilon\varphi^{(2)} + \varepsilon^{3/2}\varphi^{(3)} + \cdots\cdots \tag{6.37}$$

Substituting (6.12), (6.13b) and (6.37) in (6.9) and equating the coefficients of ε^2 from both sides we get

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2} + \delta_2^{(0)}\varphi^{(1)^2}\varphi^{(2)} + \frac{1}{4}\delta_3^{(0)}\varphi^{(1)^4} \tag{6.38}$$

Near critical situation when $\delta_2^{(0)} \approx 0$, Eq.(6.38) is reduced to

$$\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \delta_1^{(1)}\varphi^{(1)^2} + \frac{1}{2}\delta_3^{(0)}\varphi^{(1)^4} \tag{6.39}$$

Eq.(6.39) has the solution for solitary waves in the form

$$\varphi_{1C} = A_1 \sec h\eta \tag{6.40}$$

where, A_1 is amplitude of first-order solitary wave,

$$A_1^2 = -\frac{2\delta_1^{(1)}}{\delta_3^{(0)}} = -\frac{4V_0^{-3}}{(\gamma_3 - 2.5V_0^{-6})}V^{(1)} \tag{6.41}$$

$$\eta = \frac{1}{2}(\sqrt{\delta_1^{(1)}})X \quad , \ \delta_1^{(1)} = 2V_0^{-3}V^{(1)}$$

In critical case, $\delta_2^{(0)} = 0$; so, from (6.15) we obtain

$$\gamma_2 - \frac{3}{2}\gamma_1^2 = 0 \tag{6.42}$$

where,

$$\gamma_1 = \frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}},\ \gamma_2 = \frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2}$$

Using the values of γ_1 and γ_2 in (6.42) , we obtain the following quadratic equation for the critical value of density of ultra-relativistic degenerate electrons at $\delta_2^{(0)} = 0,$

$$\alpha_{ec}^2 - [\frac{(\beta_{eh} + 9\alpha_{eh}\beta_{ec})}{3\beta_{eh}}]\alpha_{ec} - [\frac{(2 - 3\alpha_{eh})\alpha_{eh}\beta_{ec}^2}{2\beta_{eh}^2}] = 0 \tag{6.43}$$

From (6.43) we obtain two values of the density of degenerate electrons given by

$$\alpha_{ec1} = B + (B^2 + 4C)^{1/2} \tag{6.44a}$$

$$\alpha_{ec2} = B - (B^2 + 4C)^{1/2} \tag{6.44b}$$

where,

$$B = [\frac{(\beta_{eh} + 9\alpha_{eh}\beta_{ec})}{3\beta_{eh}}],\ C = \frac{(2 - 3\alpha_{eh})\alpha_{eh}\beta_{ec}^2}{2\beta_{eh}^2}$$

From (6.44a) and (6.44b) the values of critical density α_{ec1} and α_{ec2} of ultrarelativistic degenerate electron at low temperature may be obtained, if other parameters, $\alpha_{eh}, \beta_{ec}, \beta_{eh}$ are known.

Similarly, at critical state the values of the temperatures β_{ec} of URD electrons for the excitation of solitary waves may be obtained from the quadratic equation of β_{ec} using (6.42).

Now, to find the higher-order correction in soliton at critical state of second-order Eq.(6.24), we equate the co-efficient €$^{5/2}$ and obtain the equation

$$\frac{d\varphi^{(1)}}{dX}\frac{d\varphi^{(2)}}{dX} = \partial_1^{(1)}\varphi^{(1)}\varphi^{(2)} + \partial_2^{(0)}\varphi^{(1)}\varphi^{(2)^2} + \partial_2^{(0)}\varphi^{(1)^2}\varphi^{(3)} + \frac{1}{3}\delta_2^{(1)}\varphi^{(1)^3} + \delta_3^{(0)}\varphi^{(1)^3}\varphi^{(2)} \tag{6.45}$$

Assuming $\delta_2^{(0)} = 0$ we solve Eq.(6.45) and obtain the second-order correction term $\varphi^{(2)}$ given by

$$\varphi_{2C} = [A_2 \sec h\eta + \tanh \eta] \sec h\eta \tag{6.46}$$

where, A_2 is amplitude of second-order correction term,

$$A_2 = -\frac{2}{3}\frac{\delta_2^{(1)}}{\delta_3^{(0)}} = -\frac{4\gamma_1^3 V_0 V^{(1)}}{(\gamma_3 - 5\gamma_1^3/2)} = -\frac{4V_0^{-5}V^{(1)}}{(\gamma_3 - 2.5V_0^{-6})} \tag{6.47}$$

Using (6.40) and (6.46) the dressed soliton at critical state ($\delta_2^{(0)} = 0$) is obtained as

$$\Phi_{2C} = \varphi_{1C} + \varphi_{2C} = [A_1 + A_2 \sec h\eta + \tanh \eta] \sec h\eta \tag{6.48}$$

6.5. RESULTS AND DISCUSSION

In this Chapter, we have studied the dressed solitons of ion acoustic waves in dense dusty plasma having URD two-temperature electrons considering higher-order nonlinear and dispersive effects by carrying out calculations up to second-order. The secularity conditions at each order gives corrections to the velocity of the solitary wave at the corresponding higher-order. It is shown that higher-order nonlinear and dispersion terms have significant contributions to the formation of dressed solitons in the plasma under consideration. To find the dressed soliton we have first obtained the first-order KdV soliton, then second-order correction to the KdV soliton and finally the corrected second-order solitons. From (6.19), (6.29) and (6.33) it is seen that density and temperature of URD two-temperature electrons have significant contribution to excite of the dressed solitons.

6.5.1 Profiles of Dressed Solitons

Considering a model URD plasma, we have numerically estimated the respective amplitudes and have drawn the profiles of KdV soliton, second-order correction and finally the dressed solitons in the plasma.The effects of plasma parameters on the solitary waves are graphically shown in Figs.(6.1)-(6.4).

To understand the nature of dressed solitons in URD two-temperature electron plasma, the profiles are drawn for different values of density and temperature of degenerate electrons.. In Fig. 6.1 to Fig. 6.4, the structure of KdV solitons are shown with variation of density and temperature of the URD electrons (dimensionless) for fixed values of other plasma parameters.

i). Effect of α_{ec}.

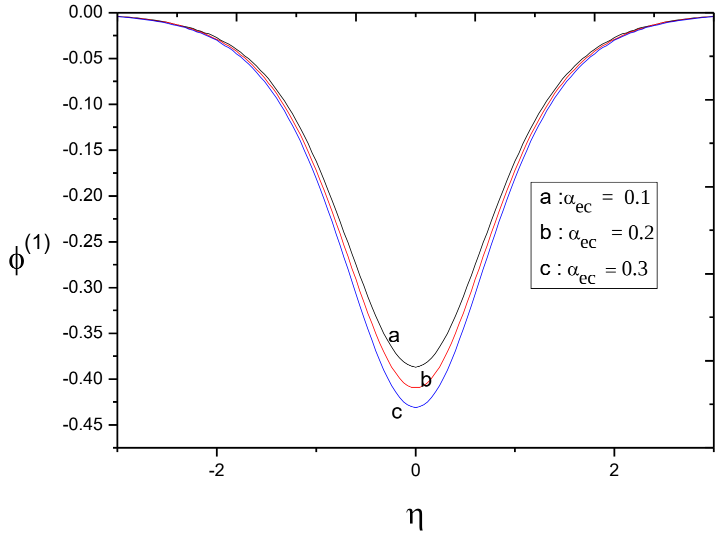

Fig. 6.1(a). The profiles of KdV soliton $(\varphi^{(1)})$ for different values of degenerate electron density (α_{ec}) for fixed values of the plasma parameters β_{ec} =0.1, β_{eh} =0.5, α_d =0.01; Graphs a , b and c represent α_{ec} = 0.1, 0.2 and 0.3 respectively.

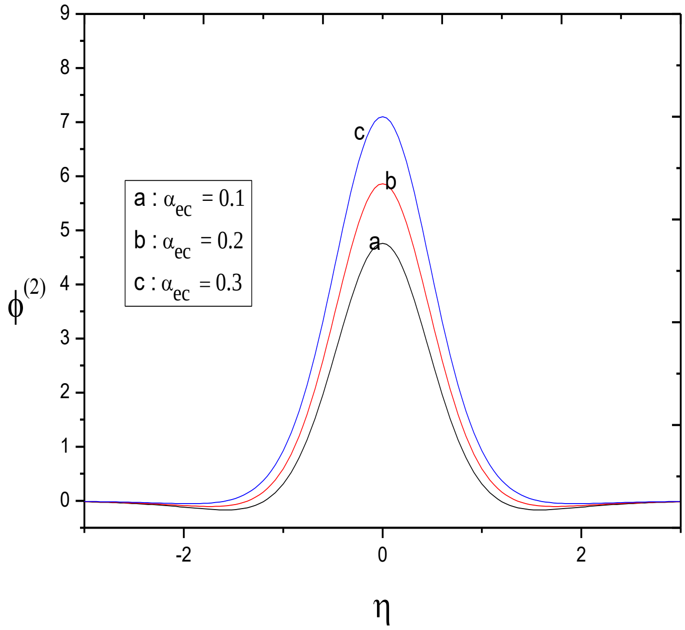

Fig.6.1(b). The profiles of second-order correction $(\varphi^{(2)})$ for different values of degenerate electron density (α_{ec}) for fixed values of the plasma parameters β_{ec} =0.1, β_{eh} =0.5, α_d =0.01; Graphs a, b and c represent α_{ec} = 0.1, 0.2 and 0.3 respectively.

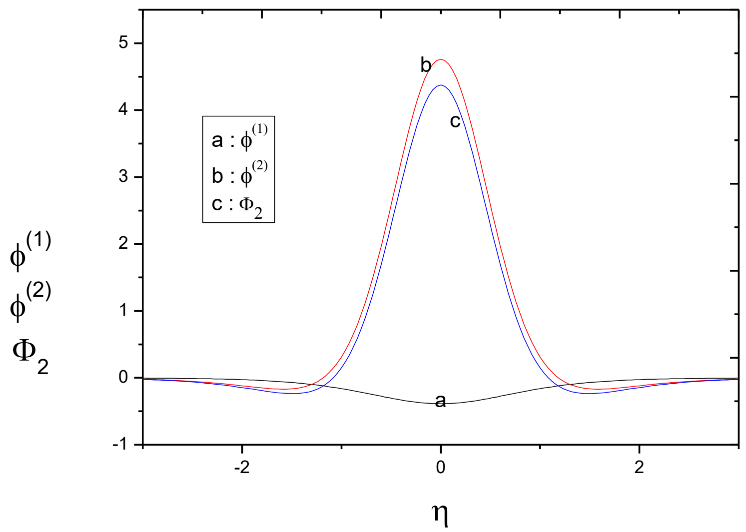

Fig. 6.1(c). The profiles of $\varphi^{(1)}$, $\varphi^{(2)}$ and Φ_2 for degenerate electron density α_{ec} =0.1 for fixed values of the plasma parameters β_{ec} =0.1, β_{eh} =0.5, α_d =0.01; Graphs a, b and c represent the KdV soliton ($\varphi^{(1)}$), second-order correction ($\varphi^{(2)}$) and dressed soliton ($\Phi_2 = \varphi^{(1)} + \varphi^{(2)}$) .

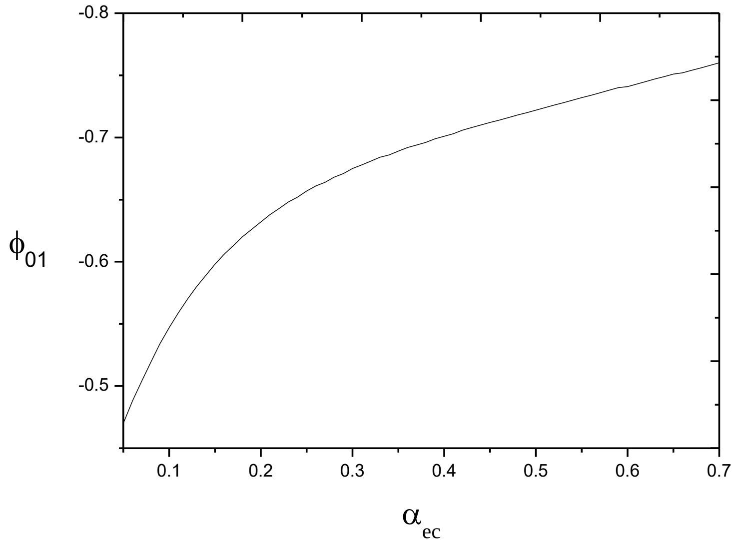

Fig. 6.1(d) . Variation of amplitude of KdV soliton (ϕ_{01}) with the density of degenerate electron (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5, α_d =0.01.

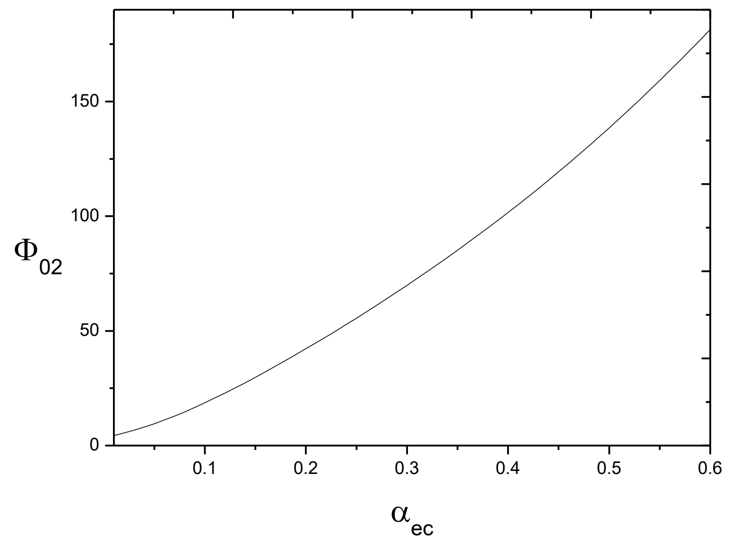

Fig.6.1(e). Variation of amplitude of dressed soliton (Φ_{02}) with density of degenerate electron (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5, α_d =0.01.

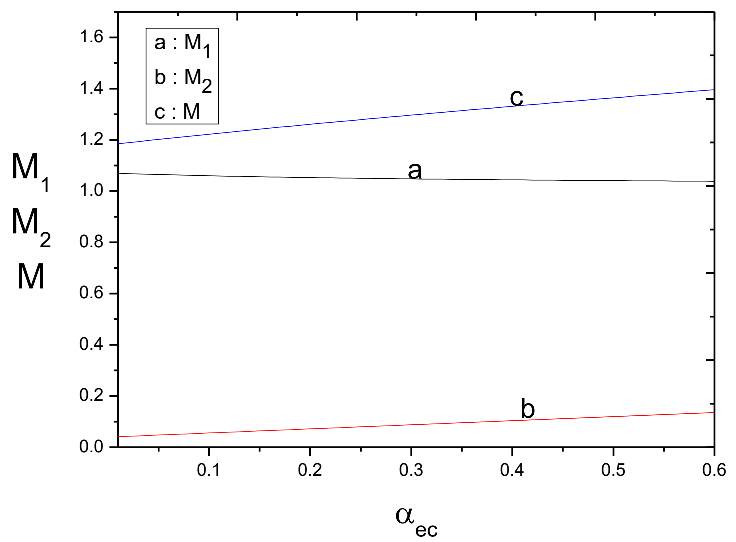

Fig. 6.1(f) Variation of Mach numbers of M_1, M_2 and M with density of degenerate electrons (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5, α_d =0.01 ; Graphs a, b and c represent the Mach number of the KdV soliton (M_1), second-order correction of soliton (M_2) and dressed soliton ($M = M_1 + M_2$) respectively.

In Fig. 6.1(a), the profile of KdV soliton ($\varphi^{(1)}$) is shown for different values of degenerate electron density (α_{ec}) for fixed values of the plasma parameters β_{ec} =0.1, β_{eh} =0.5, α_d =0.01. It is observed that rarefactive solitary waves would be excited in the plasma and the amplitude will increase with the increase of α_{ec} . In Fig.6.1(b), the structure of solitary waves for second-order correction ($\varphi^{(2)}$) is shown for the same plasma. It is observed that ($\varphi^{(2)}$) in soliton is compressive in nature and the amplitude is large for large values of the density of degenerate electrons. The Fig. 6.1(c) shows the profiles of the dressed solitons (Φ_2) for degenerate electron density α_{ec} =0.1, for fixed values of the plasma parameters β_{ec}=0.1, β_{eh} =0.5, α_d =0.01. It is seen that the dressed soliton is compressive in nature and the amplitude of dressed soliton is much larger than that of the KdV soliton.The variation of amplitude of KdV soliton and dreseed solitons in URD two temperature electrons are shown in Fig. 6.1(d) and Fig.6.1(e). It is observed that for the increase of α_{ec} the amplitude (ϕ_{01}) of first-order rarefactive KdV solitons and the amplitude (Φ_{02}) of compressive dressed solitons are increasing. Fig. 6.1(f) shows the variation of Mach number (M_1) of KdV soliton, Mach number (M_2) of second-order correction of soliton and Mach number (M) of dressed soliton are described for different values of α_{ec} in URD plasma for fixed values of the plasma parameters β_{ec}=0.1, β_{eh}=0.5 and α_d =0.01. It is observed that the Mach number (M) of dressed soliton is greater than that of KdV soliton (M_1).

ii). Effect of β_{ec}

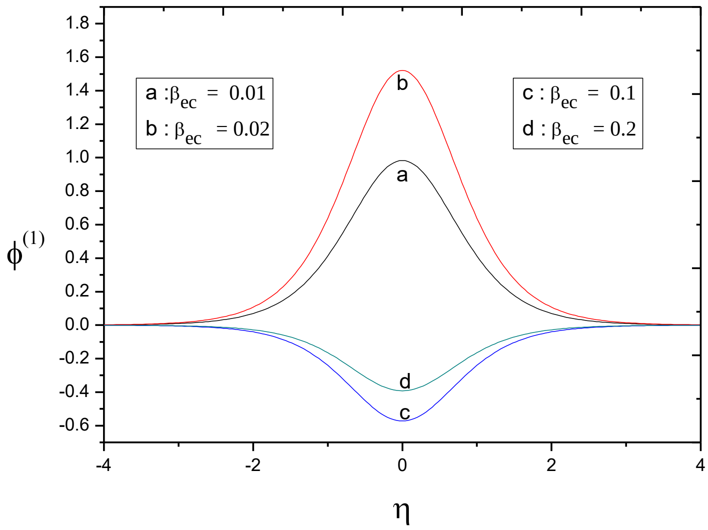

Fig. 6.2(a). The profiles of KdV soliton ($\varphi^{(1)}$) for different values of temperature (low) of degenerate electron (β_{ec}) for fixed values of the plasma parameters α_{ec} =0.1, β_{eh} =0.6, α_d =0.01; Graphs a, b, c and d represent β_{ec} = 0.01, 0.02 , 0.1 and 0.2 respectively.

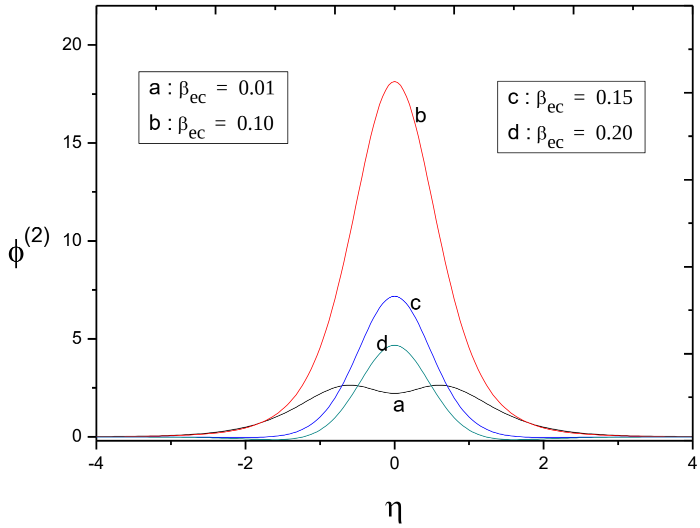

Fig. 6.2(b). The profiles of the second-order correction $(\varphi^{(2)})$ for different values of temperature (low) of degenerate electrons (β_{ec}) for fixed values of the plasma parameters α_{ec} =0.1, β_{eh} =0.6, α_d =0.01; Graphs a, b, c and d represent β_{ec} = 0.01, 0.10 , 0.15 and 0.20 respectively.

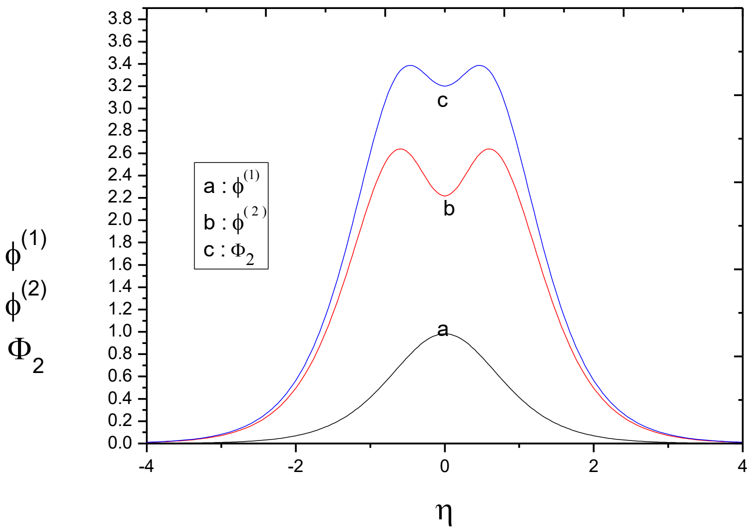

Fig. 6.2(c). The profiles of $\varphi^{(1)}$, $\varphi^{(2)}$ and Φ_2 for the temperature (low) of degenerate electrons β_{ec} =0.01 for fixed values of α_{ec} =0.1, β_{eh} =0.6 and α_d =0.01; Graphs a, b and c represent the KdV soliton $(\varphi^{(1)})$, second-order correction $(\varphi^{(2)})$ and the dressed soliton $(\Phi_2 = \varphi^{(1)} + \varphi^{(2)})$.

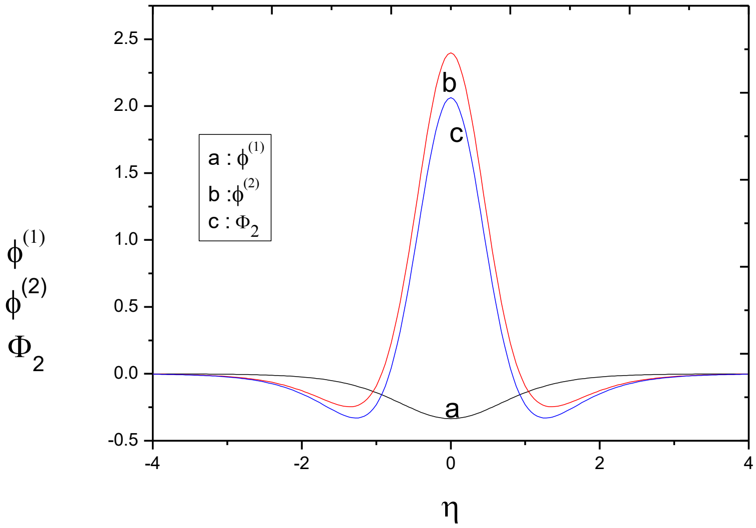

Fig. 6.2(d). The profiles of of $\varphi^{(1)}$, $\varphi^{(2)}$ and Φ_2 for the degenerate electron temperature β_{ec} = 0.5 for fixed values of α_{ec} =0.1, β_{eh} =0.6, α_d =0.01 ; Graphs a, b and c represent KdV soliton $(\varphi^{(1)})$, second-order correction $(\varphi^{(2)})$ and dressed soliton $(\Phi_2 = \varphi^{(1)} + \varphi^{(2)})$.

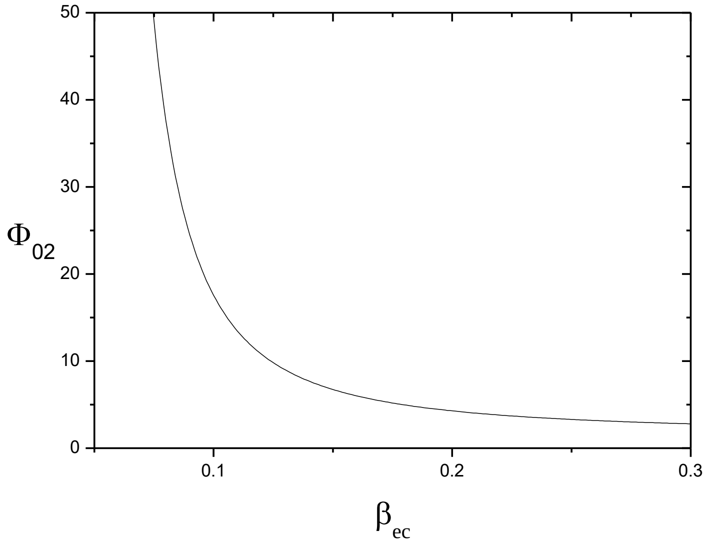

Fig. 6.2(e) . Variation of amplitude (Φ_{02}) of dressed soliton for different values of temperature of degenerate electron (β_{ec}) for fixed values of α_{ec} =0.1, β_{eh} =0.6 and α_d =0.01.

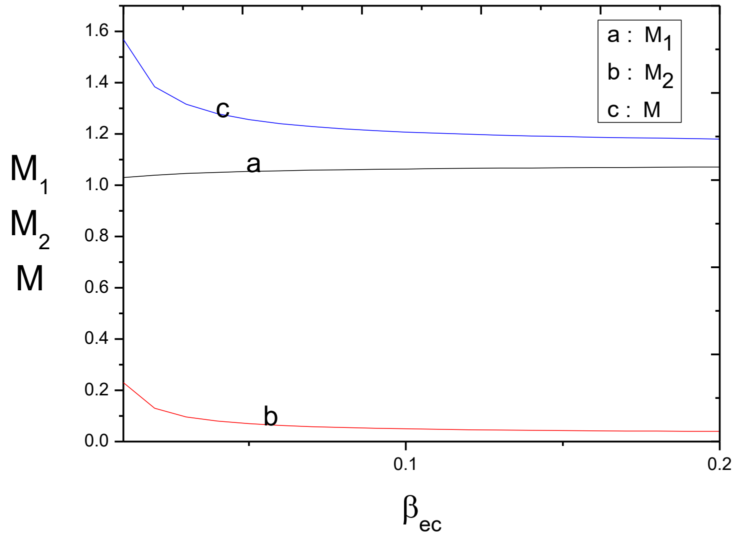

Fig. 6.2(f). Variation of Mach numbers of M_1, M_2 and M with temperature of degenerate electrons (β_{ec}) for fixed values of α_{ec} = 0.1, β_{eh} =0.6, α_d =0.01 ; Graphs a, b and c represent the Mach number of the KdV soliton (M_1), second- order correction of soliton (M_2) and dressed soliton $(M = M_1 + M_2)$ respectively.

The effect of temperature (β_{ec}) of URD electrons on the KdV soliton is shown in Fig. 6.2(a) in the plasma for fixed values of othern parameters α_{ec} =0.1, β_{eh} =0.6, α_d =0.01. It is observed that both compressive and rarefactive solitary waves may be excited in the plasma depending upon the values of β_{ec}. For β_{ec} = 0.01 and 0.02 the solitary wave will be compressive and the amplitude is large for large value of β_{ec}, but β_{ec} = 0.1 and 0.2 the rarefactive solitary wave will be excited and the amplitude is increased with decrease of β_{ec}. The profiles of second-order correction $\varphi^{(2)}$ are shown in Fig. 6.2(b) for different values temperature of degenerate electrons β_{ec} for fixed values of the plasma parameters α_{ec} =0.1, β_{eh} =0.6 are , α_d =0.01. It is observed that second-order corrections ($\varphi^{(2)}$) in soliton are compressive in nature and the amplitude depends on the values of $\beta_{ec.}$ At low value of $\beta_{ec} = 0.01$ it is compressive with wave-like behaviour.

The profiles of dressed solitons are shown in Fig. 6.2(c) and 6.2(d) in URD plasma for the values of β_{ec} = 0.01 and β_{ec} = 0.5 for fixed values of α_{ec} =0.1, β_{eh} =0.6 and α_d =0.01. It is observed that structure of dressed soliton is more interesting for low values of $\beta_{ec} = 0.01$ than large values of $\beta_{ec} = 0.5$.

The variation of amplitude of dressed solitons in URD plasma having two temperature electrons are shown in Fig.6.2(e). It is observed that for the increase of β_{ec} the amplitudes of compressive dressed solitons are increasing for low value β_{ec} and decreasing for large values β_{ec}. Fig. 6.2(f) shows the variation of the Mach number of KdV soliton (M_1), the Mach number of second-order correction (M_2) and the Mach number of dressed soliton $(M = M_1 + M_2)$ with temperature of degenerate electrons (β_{ec}) for fixed values of α_{ec} = 0.1, β_{eh} =0.6, α_d =0.01. It is observed that Mach number of dressed soliton (M) is greater than that KdV soliton (M_1).

6.5.2: Profiles of Dressed Soliton at Critical State

i). Effect of α_{ec} :

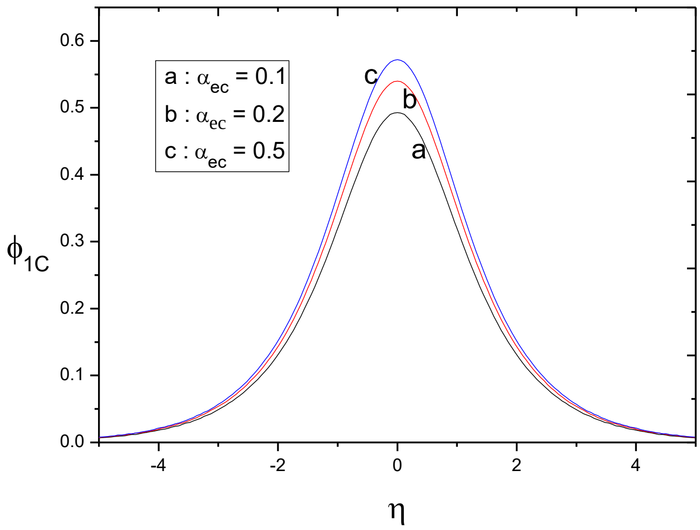

Fig.6.3(a). The profiles of first-order soliton (ϕ_{1C}) at critical state of degenerate electron density (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5 and α_d =0.01; Graphs a, b and c represent α_{ec} = 0.1, 0.2 and 0.5 respectively.

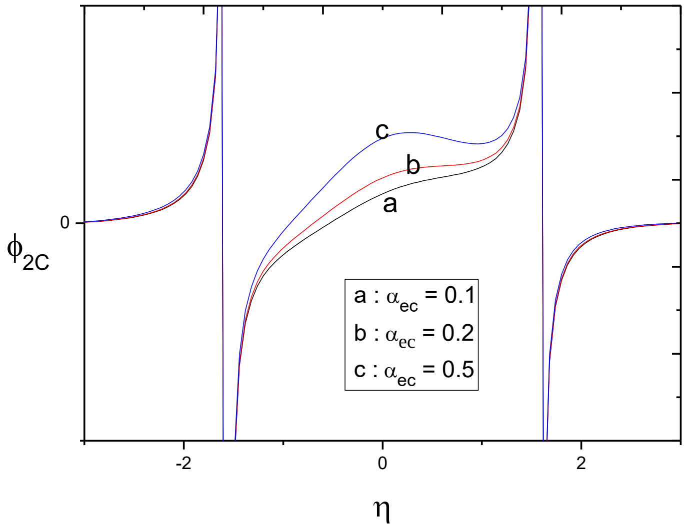

Fig. 6.3(b) The profiles of second- order correction in soliton (ϕ_{2C}) at critical state of degenerate electron density (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5 and α_d =0.01; Graphs a, b and c represent α_{ec} = 0.1, 0.2 and 0.5 respectively.

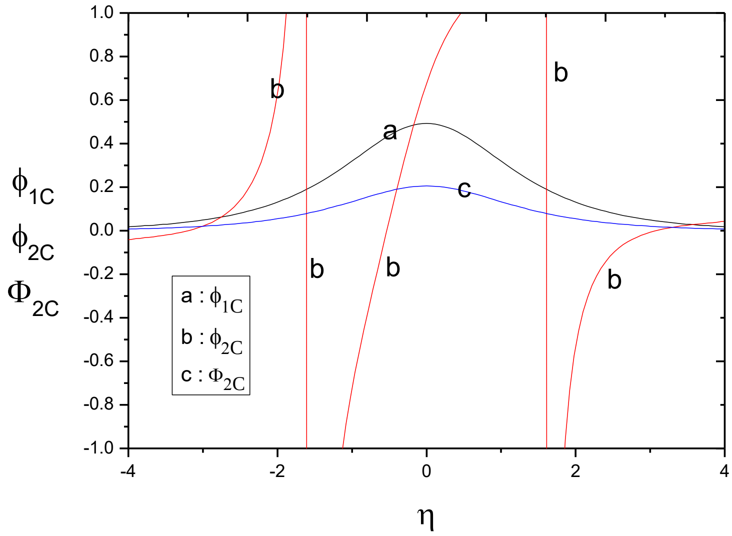

Fig.6.3(c). The profiles of ϕ_{1C}, ϕ_{2C} and Φ_{2C} of the dressed soliton at critical state of degenerate electron density α_{ec} = 0.1 for fixed values of β_{ec} =0.1, β_{eh} =0.5 and α_d =0.01 ; Graphs a, b and c represent the KdV soliton (ϕ_{1C}), second-order correction(ϕ_{2C}) and dressed soliton (Φ_{2C} = ϕ_{1C} + ϕ_{2C}) respectively.

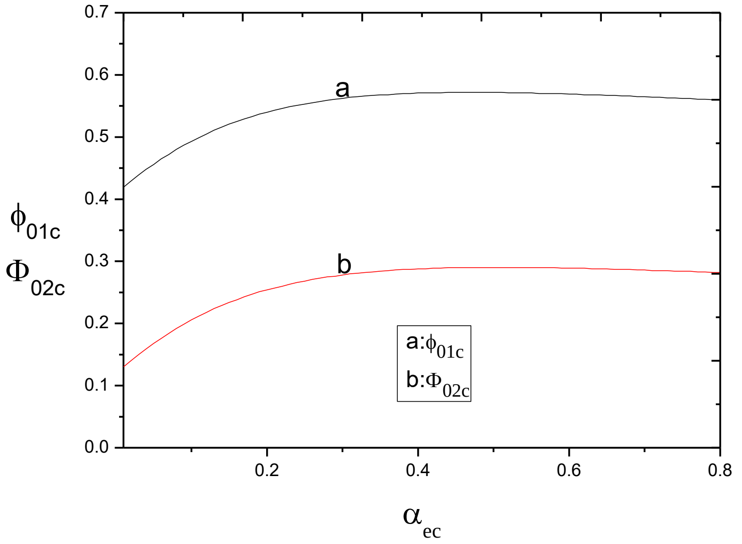

Fig. 6.3(d). Variation of amplitude of first-order soliton (ϕ_{01c}) and dressed solitons (Φ_{02c}) at critical state for different values of degenerate electron density (α_{ec}) for fixed values of β_{ec} =0.1, β_{eh} =0.5 and α_d =0.01. Graphs a and b represent (ϕ_{01c}) and (Φ_{02c}) respectively.

At critical state, the nonlinear coefficient vanishes, i.e. $\delta_2^{(0)} = 0$, the profiles of first-order solitary wave (ϕ_{1C}) are ahown in Fig. 6.3(a) for different values of α_{ec} in ultra-relativistic degenerate two-temperature electrons for fixed values of β_{ec} =0.1, β_{eh} =0.5, α_d =0.01. It is observed that at critical state the first-order solitary waves is compressive and the amplitude increases with the increase of α_{ec} . The second-order correction (ϕ_{2C}) are shown in Fig. 6.3(b) for different values of α_{ec} for fixed values of the parameters of same plasma. It is observed that second-order corrections (ϕ_{2C}) in soliton may be compressive and rarective in nature and the amplitude is large for large values of α_{ec}. At critical state , the profiles of dressed soliton in ultra-relativistic two temperature electrons is shown in Fig. 6.3(c). It is seen that amplitude of dressed soliton is lower than the first-order soliton.

Amplitude of first-order soliton and dressed soliton in ultra-relativistic two temperature electrons is shown in Fig. 6.3(d). It is observed that amplitude of dressed soliton is lower than the first-order soliton.

ii). Effect of β_{ec} :

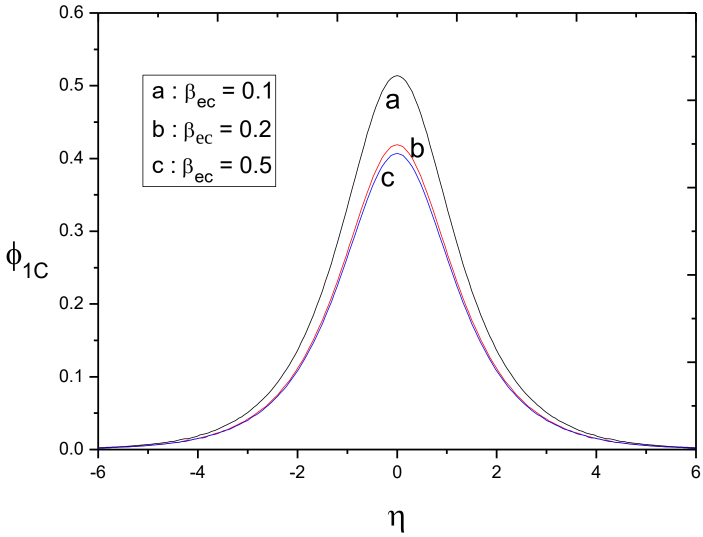

Fig.6.4(a). The profiles of first-order soliton (ϕ_{1C}) at critical state of degenerate electron temperature (β_{ec}) for fixed values of α_{ec} =0.1, β_{eh} =0.5 and α_d =0.01; Graphs a, b and c reprent β_{ec} = 0.1, 0.2 and 0.5 rspectively.

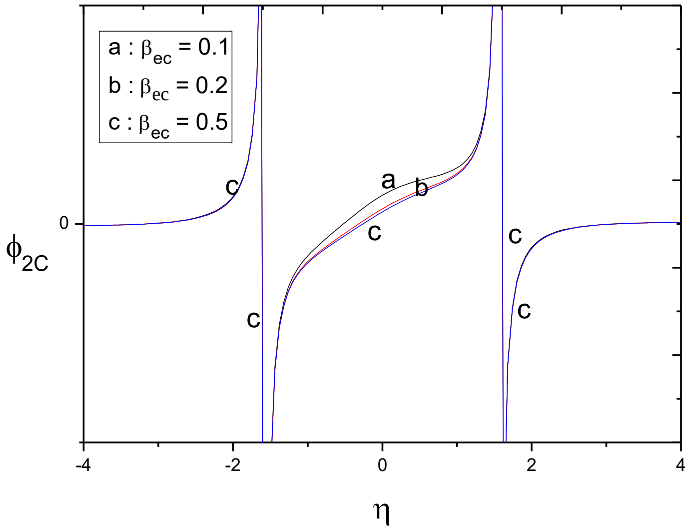

Fig. 6.4(b). The profiles of second-order of soliton (ϕ_{2C}) at critical state of degenerate electron temperature (β_{ec}) for fixed values of α_{ec} =0.1, β_{eh} =0.5 and α_d =0.01; Graphs a, b and c represent β_{ec} = 0.1, 0.2 and 0.5 respectively.

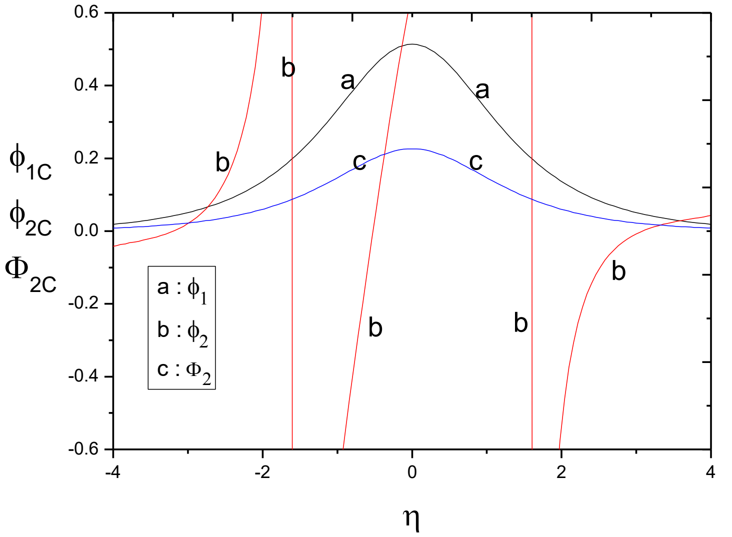

Fig.6.4(c). The profiles of ϕ_{1C}, ϕ_{2C} and Φ_{2C} at critical state of degenerate electron temperature $\beta_{ec} = 0.1$ for fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.6$ and $\alpha_d = 0.01$; Graphs a, b and c represent the KdV soliton (ϕ_{1C}), second-order correction(ϕ_{2C}) and dressed soliton ($\Phi_{2C} = \phi_{1C} + \phi_{2C}$) respectively.

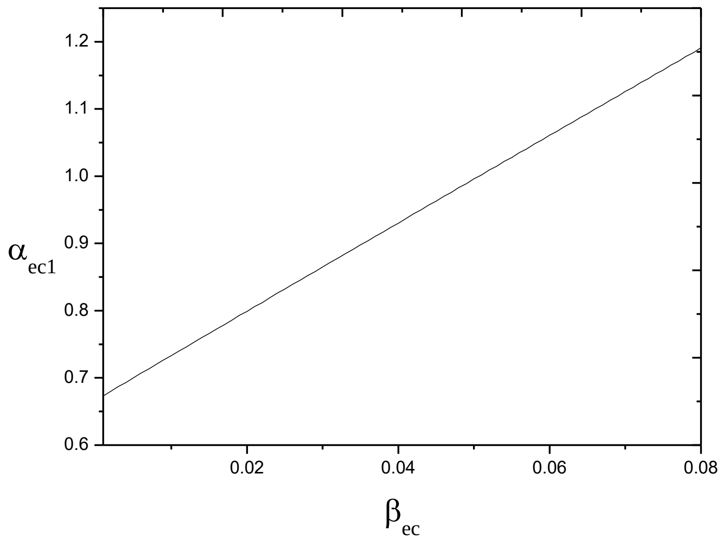

Fig. 6.4(d). Variation of critical value of electron density α_{ec1} with electron temperature β_{ec} in ultra-relativistic degenerate plasma for fixed values of β_{eh} =0.6 and α_d =0.01.

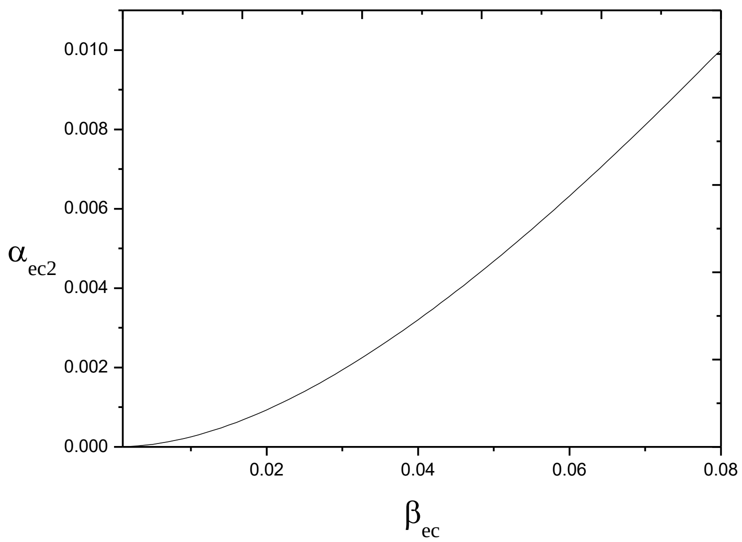

Fig. 6.4(e). Variation of critical value of electron density α_{ec2} with electron temperature β_{ec} in ultra-relativistic degenerate plasma for fixed values of β_{eh} =0.6 and α_d =0.01.

The temperature β_{ec} of ultrarelativistic degenerate electrons also has important role the nature of the dressed solitons at critical state. Let us first see the effect of β_{ec} on the first-order solitary waves. In Fig. 6.4(a), the structures of first-order solitary waves are shown for different values of β_{ec} in plasma for fixed values of α_{ec} =0.1, β_{eh} =0.5 and α_d =0.01. It is seen that solitary waves are compressive in nature and the amplitude decreases with increase of β_{ec}. At critical state, the higher-order corrections (ϕ_{2C}) in solitary waves for different values of β_{ec} are shown in Fig. 6.4(b) in same plasma. It is observed that the correction term ϕ_{2C} may be comressive and rarefactive. More over, the amplitude of ϕ_{2C} is small for large values of β_{ec}. The profiles of ϕ_{1C}, ϕ_{2C} and Φ_{2C} at critical state of degenerate electron temperature are shown in Fig.6.4(c) for β_{ec} = 0.1 with fixed values of α_{ec} =0.1, β_{eh} =0.6 and α_d =0.01. It is seen that the amplitude of dressed soliton is lower than the first-order soliton. In this regard, it is necessary to state that solitons will be excited in the ultra-relativistic plasma at critical state ($\delta_2^{(0)} = 0$), when the density α_{ec} will have two critical values α_{ec1} and α_{ec2} given by Eq.(6.44a) and Eq.(6.44b). The variations of α_{ec1} and α_{ec2} with β_{ec} for fixed values of β_{eh} =0.6 and α_d =0.01 are shown in Fig. 6.4(d) and Fig. 6.4(e).. It is seen that both α_{ec1} and α_{ec2} are increased with the increase of β_{ec}.

6.6. SUMMARY AND CONCLUSION

In this Chapter, the problem of dressed ion acoustic soliton in a dusty plasma composed of URD two-temperatyre electrons, cold ions and negatively charged dust particles has been solved .Starting from an integrated form of the system of governing equations in terms of the pseudo-potential considering higher-order nonlinear and dispersive effects the dressed ion acoustic solitary waves and double layers using the approach based on the expansion of the Sagdeev potential up to the third-order and the standard reductive perturbation method. Most important and significant results in this Chapter on dressed soliton of ion acoustic waves in cold ion and dusty plasma having URD two-temperature electrons are obtained as i). The rarefactive KdV solitary waves would be excited for different values of α_{ec} in the plasma and the amplitude will increase with the increase of α_{ec}. The second-order correction in soliton are compressive in nature for different values of α_{ec} and the amplitude is large for large values of α_{ec}. The dressed soliton is compressive in nature for different values of α_{ec}. The amplitude of dressed soliton is much larger than that of the KdV soliton; ii). Both compressive and rarefactive KdV solitary waves would be excited in the plasma depending upon the values of β_{ec}. For β_{ec} = 0.01 and 0.02 the solitary wave will be compressive and the amplitude is large for large value of β_{ec}; but for β_{ec} = 0.1 and 0.2 the rarefactive solitary wave will be excited and the amplitude is increased with increase of β_{ec}.

The second-order corrections are compressive and the structure is wavy for low value of β_{ec}. The structure of dressed soliton is more interesting for low increase of β_{ec}. It is compressive and wavy in nature. The amplitude of dressed soliton is higher than that of the KdV soliton. For large values of β_{ec} the dressed soliton is compressive and its amplitude is lower than that of the KdV soliton;

iii). At critical state the first-order solitary waves is compressive and the amplitude increases with the increase of α_{ec}. The dressed solitons are also compressive and its amplitude is lower than that of the first-order solitons. At critical state the temperature β_{ec} of URD electrons also have important role on the nature of the solitons. It is seen that first-order solitary waves are compressive and the amplitude decreases with increase of β_{ec}. The dressed soliton is also compressive and the amplitude of dressed soliton is lower than that of the first-order soliton.

It is important to be mentioned that most of the authors have studied the dressed of ion-acoustic waves using the renormalization procedure in the reductive perturbation method. But, in our present analysis we have used the integral form of governing equations in terms of pseudo potential. The advantage of this method over reductive perturbation method is that instead of solving a second-order inhomogeneous differential equation at each order one has to solve a first-order inhomogeneous equation at each order except at the first. The dressed solitons at critical state has also been investigated. Previous authors have studied the dressed solitons in plasma but they have not considered the situation at critical state. It is to be mentioned that the density of charged dust particles has important role on the excitation of dressed solitons in plasma. In our investigation, we have considered the density of dust particles is low and much smaller than the density of degenerate two-temperature electrons. In dense dusty plasma, the dust particles would play different role to the KdV solitons and dressed solitons in URD plasma having single temperature electrons. The space plasmas are known to have multispecies composition with two temperatures of electrons. So, our results on the dressed solitons presented may be useful for the study of nonlinear wave phenomena in many space plasma environments where the effects of URD electrons are important.

REFERENCES

[1]. B. N. Goswami and B. Buti, Phys. Letts., **57A,** p**.**149 (1976)

[2]. K. Nishihara and M. Tajiri, J. Phys.Soc.Japan, **50,** p 4047 (1981)

[3]. K. K .Ghosh, B. Paul, Chandra Das and S. N. Paul, J. Phys. A: Math. Theor., **41,** p 335501 (2008)

[4]. T. K. Baluku, M .A. Hellberg and F. Verheest, Euro. Phys. Letts., **91,** p 15001(2010)

[5]. O. R. Rufai, R .Bharuthram, S. V. Singh, and G. S. Lakhina, Phys. Plasmas, **19,** p 122308 (2012)

[6]. T. Akhter, M. M. Hossain, A. A. Mamun, Chin. Phys. B , **22** (7), p 075201 (2013)

[7]. Shalini and N. S. Saini, Phys. Plasmas, **21**, p 102901 (2014)

[8]. F. Verheest and M. A. Hellberg, Phys. Plasmas **22,** p 072303 (2015)

[9]. H. Ikezi, R. J. Tayler and D. R .Baker, Phys. Rev. Letts., **25 ,** p 11 (1970)

[10]. M. Q. Tran, Physica Scripta, **20,** p 317 (1979)

[11]. K. E. Lonngren, Plasma Phys., **25,** p 943 (1983)

[12]. Y. H. Ichikawa, T. Matsuhasi and K. Konno, J. Phys. Soc. Japan, **41,** p 1382 (1976).

[13]. C. S. Lai, Can. J. Phys., **57,** p. 490 (1979).

[14] .Y. Kodama and T. Taniuti , J. Phys. Soc. Japan , **45**, p.298 (1978).

[15]. R. S. Tiwari and M. K. Mishra , Phys.Plasmas, **13**, p.062112 (2006).

[16]. S. K. El-Labany, W. F. El-Taibany, O. M . El-Abbasy, Chaos Solitons Fractals, **33** p.813 (2007).

[17]. K. Roy , P. Chatterjee and S. Kundu, Advances in Space Research ,**50** p.1288 (2012)

[18]. R. Amour, L. A. Gougam and M. Tribeche, Physica A, **436** , p.856 (2015)

[19]. H. G. Abdelwahed, E. K. El-Shewy, M. A. Zahran, and S. A. Elwakil Phys. Plasmas **23,** p.022306 (2016)

[20]. S. Chandrasekhar, Philosophical Magazine, **11,** p.592 (1931).

[21]. S. Chandrasekhar, Mon. Not. R. Astron. Soc., **170,** p.405 (1935).

[22]. H. A. Shah, W. Masood, M.N.S. Qureshi and N. L. Tsintsadze, Phys. Plasmas, **18**, p.102306 (2011).

[23] W. Masood and B. Eliasson, Phys. Plasmas, **18**, p.034503 (2011)

[24]. S. Chandra, B. Ghosh and S.N. Paul, Astrophys. Space Sci. **343** p.213 **(**2013)

[25]. A. A. Mamun and P. K. Shukla, Phys. Plasmas, **17,** p.104504 **(**2010)

[26]. A. Esfandyari-Kalejahi , M. Akbari-Moghanjoughi and E. Saberian, Plasma and Fusion Research: Regular Articles **5,** p.045 **(**2010).

[27]. N. Roy , M. S. Zobaer and A. A .Mamun, Journal of Modern Physics, **3,** p.850 **(**2012),

CHAPTER-7

EFFECT OF ION BEAM ON THE ION-ACOUSTIC SOLITONS IN UNMAGNETIZED QUANTUM PLASMA

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7.1. INTRODUCTION

In recent years, researchers are very much interested to study propagation of waves in quantum plasma because quantum effects become important in a variety of environments. Such as in intense laser-solid density plasma interaction experiments, in dense astrophysical and cosmological environments, in ultra small electronic devices and in metal nanostructures. The recent developments in nanoscience [1], biophotonics [2], fiber optics [3], quantum confinement [4], ultra-cold plasmas [5] etc. the study has gained momentum in a totally new direction. The quantum effects are also important in laboratory produced ultra-dense plasmas [6] and laser based inertial fusion experiments [7] as well as in the interior of Jovian planets, neutron stars [8], and white dwarfs [9] etc. where the density is very high. The quantum plasmas show numerous nonlinearities which are absent in classical plasmas. This has led to the investigation of waves in quantum plasma more important than ever. There have been many modes of waves in plasmas like ion-acoustic waves, electron-acoustic waves, dust-acoustic waves, dust ion acoustic waves, electron plasma waves, shock waves etc. Using the Quantum Hydrodynamic (QHD) model, Haas *et al*. [10] have studied the importance of the role of quantum diffraction in linear and nonlinear regime. They adopted an equation of state pertaining to a zero-temperature Fermi gas for the electrons by disregarding pressure effects for the ions. By an appropriate rescaling of the variables, they identified a nondimensional parameter H, proportional to quantum diffraction effects. The system is shown to support linear waves, which in the limit of small H resemble the classical ion-acoustic waves. In the weakly nonlinear limit, the quantum plasma is shown to support waves described by a deformed Korteweg-deVries (KdV) equation which depends in a nontrivial way on the quantum parameter H. In the fully nonlinear regime, the system also admits traveling waves which can exhibit periodic patterns. The works of Haas *et al*. [10] have encouraged large number of authors to study nonlinear propagation of acoustic waves (e.g. solitary waves, modulation instability etc) in quantum plasma. Ali and Shukla [11] have considered one-dimensional QHD model for a three species quantum plasma to derive a KdV equation incorporating quantum corrections and studied the nonlinear properties of dust acoustic solitary waves. They examined the quantum mechanical effects numerically both on the profiles of the amplitude and the width of dust acoustic solitary waves. It is found that the amplitude remains constant but the width shrinks for different values of a dimensionless electron quantum parameter H . Later, Misra and Bhowmik [12] have considered a nonplanar spherical geometry to study the nonlinear properties of ion-acoustic waves (IAW) in electron–ion quantum plasma with the effects of quantum corrections. Using the QHD model and standard reductive perturbation method they derived Kadomtsev–Petviashvili (KP) equation having variable coefficient and examined numerically the importance of quantum mechanical effects on the compressive and rarefactive solitons. It is found that H plays a significant role in the formation of compressive and rarefactive solitons. A critical value of H is also found which depends on the phase velocity of the wave and the ion to electron Fermi temperature ratio, for which the soliton formation ceases to exist. Subsequently, Mushtaq and Khan [13] have studied ion acoustic solitary wave with weakly transverse perturbations in quantum electron-positron-ion plasma using QHD model.

Akbari-Moghanjoughi [14] has investigated large-amplitude ion-acoustic solitons (IAS) in a degenerate dense electron-positron-ion plasma considering the ion-temperature as well as electron/positron degeneracy effects. He has shown that the ion-temperature effects play an important role in the existence criteria and allowed Mach-number range in such plasmas. They have also pointed out the fundamental difference in the existence of supersonic IAS propagations between degenerate plasmas with nonrelativistic and ultra- relativistic electrons and positrons.

Chandra *et el.* [15] have studied the linear and nonlinear propagation of electron plasma waves in two component unmagnetized dense quantum plasma with streaming of ions. Later, Chandra *et al.* [16] have theoretically studied the linear and nonlinear propagation of electron plasma wave in a two component unmagnetized dense quantum plasma with streaming of ions both analytically and numerically using one-dimensional quantum hydrodynamic model. It has been shown that quantum effect modifies the linear dispersion character of the electron plasma waves in presence of streaming motion and makes the possibility of two distinct modes. They also have derived the KdV equation and have shown that both compressive and rarefactive solitary waves would be excited in the model plasma on some critical values of the quantum diffraction parameter. Recently, Dip *et al.* [17] have studied higher -order nonlinearity of the electro-acoustic waves (EAW), specifically ion-acoustic waves (IAW) in an unmagnetized, collision less, quantum electron-positron-ion plasma. They have derived the modified KdV equation. The plasma system is supposed to be formed of positively charged inertial heavy ions, inertia less electrons and positrons. To analyze the solitary waves (SWs) and the standard Gardner (SG) equation to analyze the higher-order SWs as well as double layers (DLs).

However, propagation of IAW and EAW in quantum (degenerate) plasma having electron-beam or ion-beam would give fascinating results which are not studied yet critically, though many authors studied the wave propagation in non-degenerate / classical plasma [18- 22]. So, in this Chapter, we have studied the IAS in unmagnetized quantum plasma in presence of an ion beam using the one dimensional quantum hydrodynamic model. Using the reductive perturbation technique, the KdV equation has been derived and the solution of IAS is obtained. The theoretical results have been analyzed numerically and graphically for different values of ion beam density, ion beam velocity and ion beam temperature. It is seen that formation and structure of the IAS are significantly affected by the ion beam in quantum plasma.

7.2. BASIC EQUATIONS

We consider the quantum plasma is unmagnetized and collisionless and it consists of positive ions, beam- ions and inertialess electrons. All species of the plasma are assumed to follow the Fermi–Dirac statistics. The positive ions and positive beam ions are considered to be singly ionized. The IAW is a low frequency wave, in which the restoring force comes from the pressure of inertialess electrons and the ion masses provide the driving force to maintain the wave. The electrons are considered inertialess by assuming that the phase velocity of IAW is much less than the Fermi velocity of electrons and much greater than the Fermi velocities of positive and beam ions. . We assume that the plasma particles behave as a one-dimensional Fermi gas at zero temperature and therefore the pressure law is:

$$p_j = \frac{m_j V_{Fj}^2}{3n_{j0}^2} n_j^3 \tag{7.1}$$

where $j = e$ for electrons, $j = i$ for ions and $j = b$ for beam ions; m_j is the mass; $V_{Fj} = \sqrt{2K_B T_{Fj} / m_j}$ is the Fermi speed, T_{Fj} is the Fermi temperature and K_B is the Boltzmann constant; n_j is the number density with the equilibrium value n_{j0} . $\omega_{pe} = \sqrt{4\pi n_0 e^2 / m_e}$ is the electron plasma oscillation frequency and V_{Fe} is the Fermi thermal speed of electrons.

The normalized equations governing the plasma dynamics are:

$$0 = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2 \sqrt{n_e}}{\partial x^2}\right] \tag{7.2}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{7.3}$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}\right) u_i = -\frac{\partial \phi}{\partial x} - \sigma_i n_i \frac{\partial n_i}{\partial x} \tag{7.4}$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b u_b)}{\partial x} = 0 \tag{7.5}$$

$$\left(\frac{\partial}{\partial t} + u_b \frac{\partial}{\partial x}\right) u_b = -\mu \frac{\partial \phi}{\partial x} - \mu \sigma_b n_b \frac{\partial n_b}{\partial x} \tag{7.6}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \chi n_e - n_i - (1 - \chi_e) n_b \tag{7.7}$$

It is to be mentioned that the following normalization has been used in the above equations:

$$x \to x\omega_{pi} / V_{Fi},\ t \to t\omega_{pi},\ \varphi \to e\varphi / 2k_B T_{Fj},\ n_j \to n_j / n_0 \text{ and } u_j \to u_j / V_{Fe}$$

and $H = \hbar\omega_{pe} / 2\,K_B T_{Fe}$ is a non-dimensional parameter. The quantum parameter is proportional to the quantum diffraction and is equal to the ratio between the plasma energy $\hbar\omega_{pe}$ (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy $K_B T_{Fe}$; $\chi_e = n_{e0} / n_{i0}$ is the ratio of unperturbed electron density and ion density; $\mu = m_i / m_b$ is the ratio of positive ion and beam-ion masses and $\sigma_{i,b} = T_{Fi,b} / T_{Fe}$ is the ratio of ion (beam) Fermi temperature to electron Fermi temperature.

7.3. THE KdV EQUATION

In order to investigate the behavior of ion-acoustic we make the following perturbation expansions for the field quantities $n_e, n_i, n_b, u_e, u_i, u_b$ and φ about their equilibrium values:

$$\begin{bmatrix} n_e \\ u_e \\ n_i \\ u_i \\ n_b \\ u_b \\ \phi \end{bmatrix} = \begin{bmatrix} n_{e0} \\ 0 \\ n_{i0} \\ 0 \\ n_{b0} \\ u_{b0} \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{e1} \\ u_{e1} \\ n_{i1} \\ u_{i1} \\ n_{b1} \\ u_{b1} \\ \phi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{e2} \\ u_{e2} \\ n_{i2} \\ u_{i2} \\ n_{b2} \\ u_{b2} \\ \phi_2 \end{bmatrix} + \cdots\cdots \tag{7.8}$$

where, $n_{b0} = 1 - \chi_e$ and $n_{i0} = 1$ which are obtained from the equilibrium condition of densities after normalization.

To get the desired KdV equation describing the nonlinear behavior of IAW we use the standard reductive perturbation technique. We introduce the usual stretching of the space and time variables:

$$\xi = \varepsilon^{1/2}(x - V_p t) \quad \text{and} \quad \tau = \varepsilon^{3/2} t \tag{7.9}$$

where V_p is the phase speed normalized by electron Fermi speed V_{Fi} and ε is a smallness parameter measuring the dispersion and nonlinear effects.

Using (7. 9), the Eqs. (7.2) - (7.7) are written in terms of the stretched coordinates ξ and τ and then the perturbation expansions (7.8) is substituted. Solving the lowest order equations in ε (i.e., $\varepsilon^{3/2}$) we obtain the linear long wave phase speed V_p in terms of plasma parameters after using the boundary conditions:

$$\chi_e - \frac{1}{V_p{}^2 - \sigma_i} - \frac{(1-\chi_e)\mu}{[(V_p - u_{b0})^2 - \sigma_b (1-\chi_e)^2]} = 0 \tag{7.10}$$

From Eq. (7.10) the phase speed of IAW in quantum plasma in presence of ion-beam can be studied analytically and numerically and the effects of plasma parameters on the phase speed would be understood .

Now, to derive the KdV equation we using the stretching co-ordinates given by Eq. (7.9). Taking the terms of $\varepsilon^{5/2}$ we obtain from Eq.(7.2) - Eq.(7.7),we obtain the equations consisting of $n_{e1}, n_{i1}, n_{b1}, \mathrm{u}_{i1}, u_{b1}, \varphi_1$ and $n_{e2}, n_{i2}, n_{b2}, \mathrm{u}_{e2}, \mathrm{u}_{i2}, u_{b2}, \varphi_2$. Now, following standard mathematical technique we obtain the following KdV equation in quantum plasma having ion-beams,

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{7.11}$$

where

$$A = \frac{\chi_e \mu(\sigma_i - V_p^2)(V_p^2 - \sigma_i)^2 (V_b{}^2 - \mu\sigma_b)^2}{(V_p^2 - \sigma_i)(V_b^2 - \sigma_b)[2\chi_e \mu V_p(\sigma_i - V_p^2 - \frac{1}{\chi_e})(V_p^2 - \sigma_i) - 2\mu V_p(V_b^2 - \mu\sigma_b)]}$$

$$- \frac{\mu(\sigma_i + 3V_p^2)(V\,_b^2 - \mu^2\sigma_b) + \chi_e \mu^2(\mu\sigma_b + 3V_b^2)(\sigma_i - V_p^2 - \frac{1}{\chi_e})(V_p^2 - \sigma_i)^2}{(V_p^2 - \sigma_i)(V_b^2 - \mu\sigma_b)[2\chi_e \mu V_p(\sigma_i - V_p^2 - \frac{1}{\chi_e})(V_p^2 - \sigma_i) - 2\mu V_p(V_b^2 - \mu\sigma_b)]}$$

$$B = \frac{(H^2 - 4)(V_p^2 - \sigma_i)^2 (V_b^2 - \mu\sigma_b)}{4[2\chi_e V_p(\sigma_i - V_p^2 - \frac{1}{\chi_e})(V_p^2 - \sigma_i) - 2V_p(V_b^2 - \mu\sigma_b)}$$

where, φ_1 is written as φ, $V_b = V_p - u_{b0}$ and the parameters $\mu, \sigma_i, \sigma_b, \chi_e, u_{b0}$, H and V_p are defined earlier .

It is known that solitary wave structure is formed due to the balance between the dispersive effect and nonlinear effect. Relative strength of these two effects determines the characteristic of such solitary wave structure. The coefficients *A* of nonlinear term and coefficient *B* of dispersive term thus play a crucial role in determining the solitary wave structure.

To find the steady state solution of the KdV Eq.(7.11), the independent variables ξ and τ are transformed into one variable $\eta = \xi - U\tau$ where U is the normalized constant speed of the wave frame. Applying the boundary conditions: as $\eta \to \pm\infty$; i) $\varphi_1 \to 0$, ii) $\frac{d\varphi_1}{d\eta} \to 0$, iii) $\frac{d^2\varphi_1}{d\eta^2} \to 0$ the solution is obtained as

$$\varphi_1 = \varphi_0 \sec h^2\theta \tag{7.12}$$

where the amplitude φ_0 and width δ_1 of the solitary waves are given by:

$$\varphi_0 = \frac{3U}{A}, \quad \delta_1 = \sqrt{\frac{4B}{U}}, \quad \theta = \left(\frac{\eta}{\delta_1}\right) \tag{7.13}$$

For positive value of A the solitary wave will be compressive and the negative value of A will generate the rarefactive solitary wave. Since, A is independent of H the amplitude of the solitary wave will remain same for any value of H. On the other hand, the width of the solitary wave depend on H and other plasma parameters $\sigma_i, \sigma_b, n_{b0}, u_{b0}, \mu$.

7.4. RESULTS AND DISCUSSION

7.4.1. Variation of the Coefficients of Nonlinear Term and Dispersive Term

It is seen from the KdV equation (7.11) that the coefficient A of the nonlinear term depends on the plasma parameters i.e. $\mu, \sigma_i, \sigma_b, \chi_e, u_{b0}$ but the quantum diffraction parameter H has no effect on A; whereas the coefficient B of dispersive term depends on the plasma parameters including quantum diffraction parameter H. Thus the nonlinear and dispersive terms get modified by the ion beam, quantum diffraction effect and the quantum statistical effect through electron-ion Fermi temperatures. The nonlinear coefficient A has been numerically estimated for the quantum plasma for plasma parameters related to ion beam and are graphically shown in Fig. 7.1(a), Fig. 7.1(b) and Fig. 7.1(c).

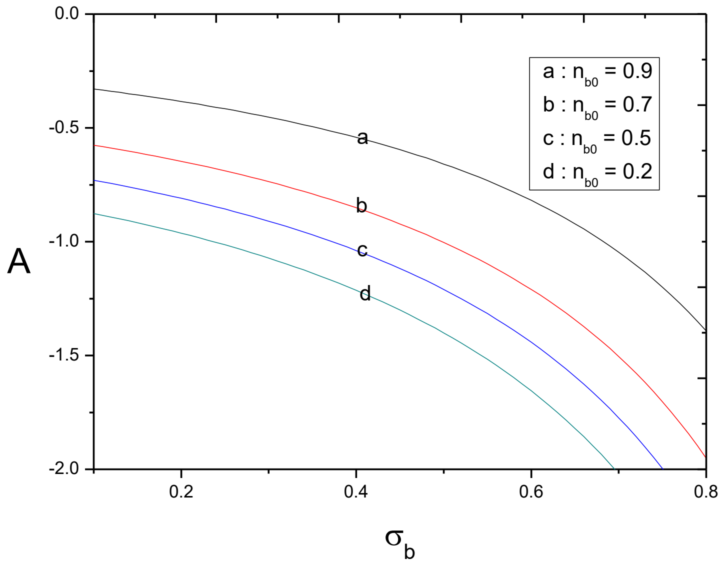

Fig. 7.1(a) -Variation of Nonlinear term (A) with ion beam temperature (σ_b) for different value of ion beam density (n_{b0}) in quantum plasma having the plasma parameters σ_i = 0.01, u_{b0} = 0.5, V_p =1.6, μ =1 and $H = 0.1$. Graphs a, b c and d represent n_{b0}, n_{b0} = 0.9, 0.7 , 0.5 and 0.2 respectively.

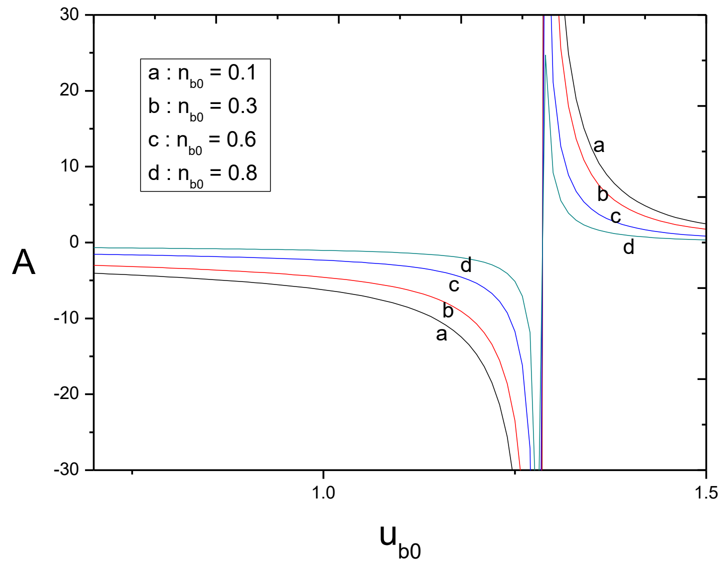

Fig.7.1 (b). Variation of nonlinear term with ion beam velocity (u_{b0}) for different ion beam density (n_{b0}) in quantum plasma having the plasma parameters V_p =1.6, σ_i =0.1, σ_b =0.1, μ =1 and $H = 0.1$. Graphs a, b c and d represents n_{b0} = 0.1, 0.3, 0.6 and 0.8 respectively.

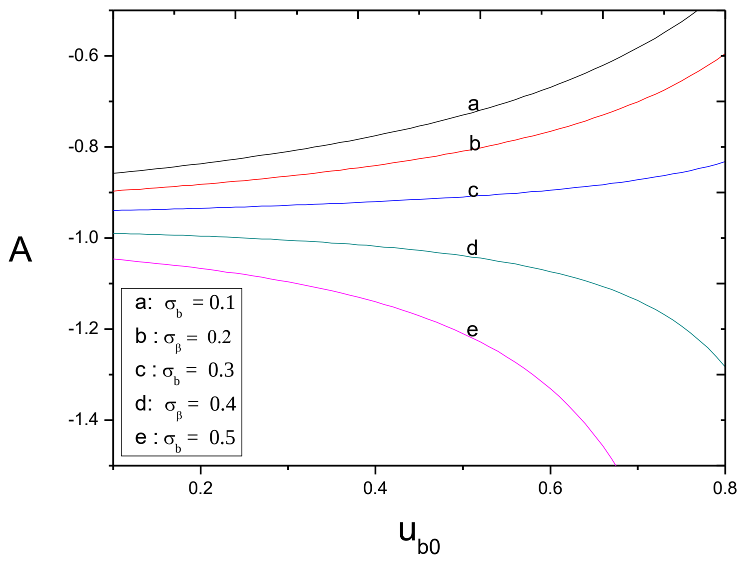

Fig.7.1(c). Variation of nonlinear term with ion beam velocity ((u_{b0}) for different ion-beam temperature (σ_b) in quantum plasma having the plasma parameters σ_i = 0.01, n_{b0} = 0.5, V_p =1.6, μ =1 and $H = 0.1$. Graphs a, b, c, d and e represent σ_b = 0.1, 0.2, 0.3, 0.4 and 0.5 respectively.

It is seen from Fig. 7.1(a) that nonlinear term is negative and it decreases with the increase of ion beam temperature but it is increased with increase of ion beam density. Similarly, the variation of nonlinear term with ion beam velocity for different values of ion beam density is shown in Fig. 7.1(b) in a quantum plasma with fixed values of other parameters as H = 0.1. V_p =1.6, σ_i =0.1, σ_b =0.1, μ =1. It is interesting to see that that the nonlinear term is negative and it decreases up to certain value of ion beam velocity ($u_{b0} \approx 1.2$) and then it is suddenly increased to a maximum positive value and finally it starts to decrease with increase of ion beam velocity keeping positive value. Moreover, the nonlinear term is seen to increase with increase of ion beam density. The variation of nonlinear term with ion beam velocity for different ion beam temperature can be understood from Fig.7.1(c) for quantum plasma having fixed values of σ_i = 0.01, n_{b0} = 0.5, $H = 0.1$ V_p =1.6, μ =1. It is observed that nonlinear term is positive and it increases with increase of beam velocity when ion beam temperature σ_b = 0.1 to 0.3. But, the nonlinear term becomes negative and it is decreased with increase of ion beam velocity for ion beam temperature σ_b = 0.4 and 0.5.

Similarly, the dispersive coefficient (B) is numerically estimated for a quantum plasma and its variation with the quantum diffraction parameter H and the parameters related to ion beam are graphically shown in Figs.7.2(a), 7.2(b) and 7.2(c).

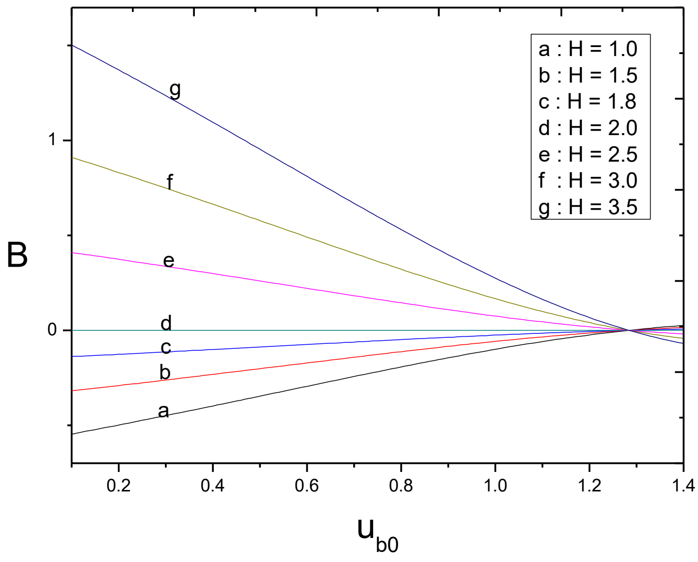

Fig.7.2 (a). Variation of the dispersive term (B) with ion beam velocity (u_{b0}) for different values of quantum diffraction parameter (H) in quantum plasma having the plasma parameters σ_i = 0.1, σ_b =0.1, u_{b0} = 0.5, n_{b0} = 0.5, V_p =1.6 and μ =1. Graphs a, b, c, d, e, f and g represent H = 1.0, 1.5, 1.8, 2.0, 2.5, 3.0 and 3.5 respectively.

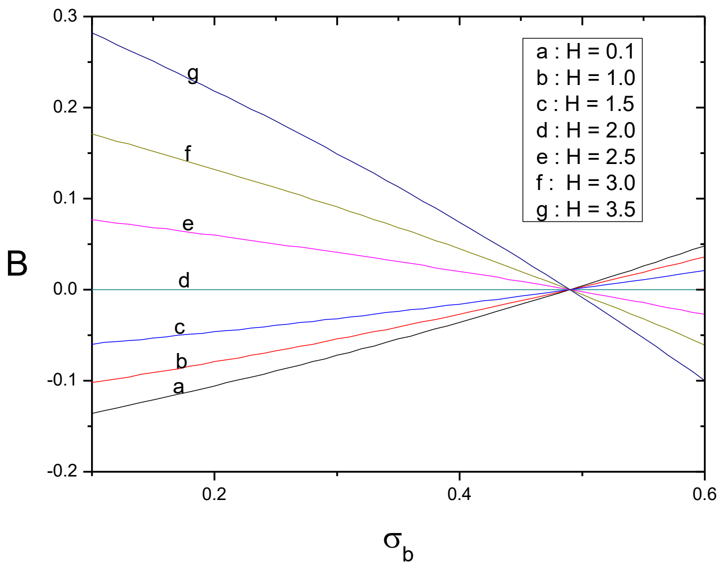

Fig.7.2 (b). Variation of the dispersive term (B) with ion beam temperature (σ_b) for different values of quantum diffraction parameter (H) in quantum plasma having the plasma parameters σ_i = 0.01, u_{b0} = 0.5, n_{b0} = 0.5, V_p =1.6 and μ =1. Graphs a, b, c, d, e, f and g represent H = 0.1, 1.0, 1.5, 2.0, 2.5, 3.0 and 3.5 respectively.

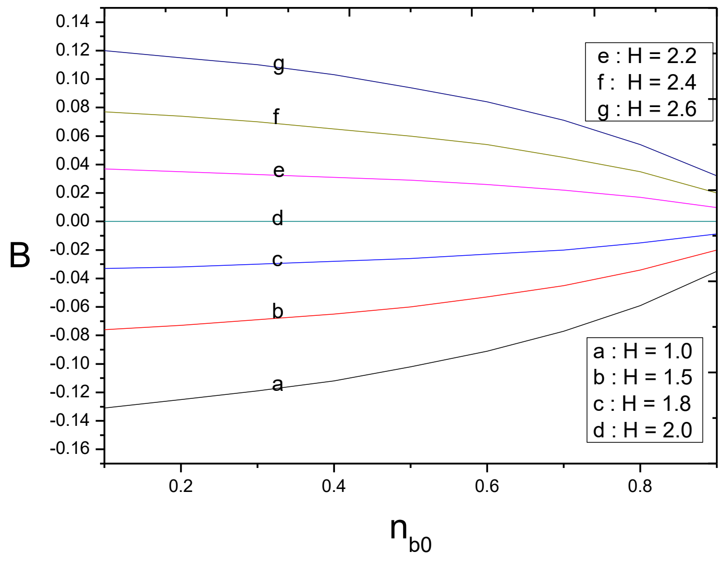

Fig.7.2(c). Variation of the dispersive term with ion beam density (n_{b0}) for different values of quantum diffraction parameter (H) in quantum plasma having the plasma parameters having σ_i =0.01, σ_b =0.1, u_{b0} =0.5, V_p =1.6 and μ =1. Graphs a, b, c, d, e, f and g represent H = 1.0, 1.5, 1.8, 2.0, 2.2, 2.4 and 2.6 respectively.

It is seen from Fig. 7.2(a) that the dispersive coefficient (B) is significantly affected by the quantum diffraction parameter (H). The dispersive coefficient is negative and it increases with ion beam velocity (u_{b0}) for H < 2 and B = 0 for H =2. But, B is positive and it decreases with ion beam velocity for H > 2.

In Fig. 7.2(b), the variation of B with ion beam temperature for different values of H is shown for a quantum plasma having fixed values of σ_i = 0.01, u_{b0} = 0.5, n_{b0} = 0.5, V_p =1.6 and μ =1. In this case also B is negative and it increases with ion beam temperature for H < 2 and B = 0 for H =2. But, B becomes positive and it decreases with ion beam temperature for H > 2.

The variation of B with n_{b0} and H is shown in Fig.7.2(c) in a quantum plasma having fixed values of σ_i = 0.01, σ_b =0.1, n_{b0} = 0.5, V_p =1.6 and μ =1. It is seen that B is negative for H < 2 and it increases with the increase of n_{b0}. B = 0 when H =2. For H > 2 the diffraction coefficient B is positive and it decreases with the increase of n_{b0}.

7.4.2 Profiles of Ion-Acoustic Solitons

To understand the nature of IAS the profiles are drawn in quantum plasma having an ion beam for different values of velocity, density and temperature of the ion beam.

i). Effect of ion beam velocity.

In Fig. 7.3(a) the structure of IAS are shown with variation of u_{b0} in the quantum plasma having parameters (dimensionless) $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

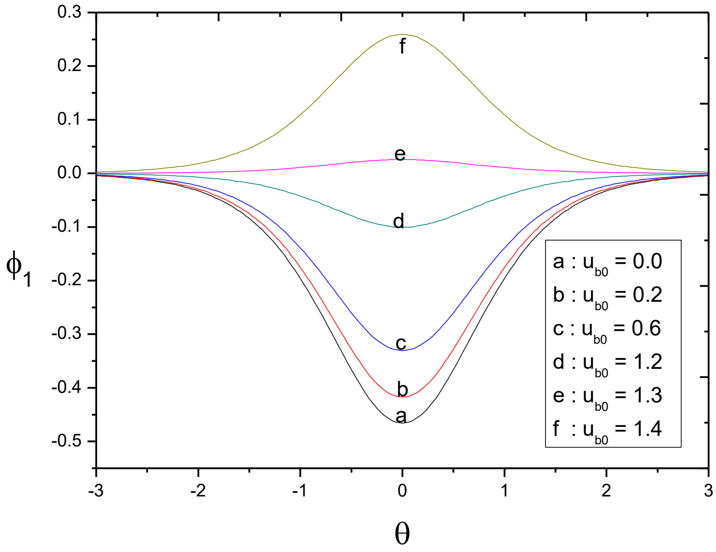

Fig.7.3(a) .The profiles of IAS (ϕ_1) in a quantum plasma with variation of ion-beam velocity (u_{b0}) having plasma parameters are $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ and U = 0.1 . Graphs a, b, c, d, e and f represent u_{b0} = 0.0, 0.2, 0.6, 1.2, 1.3 and 1.4 respectively.

ii).Effect of ion beam density

In Fig. 7.3(b) the structure of IAS are shown with variation of n_{b0} in the quantum plasma having parameters (dimensionless) parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

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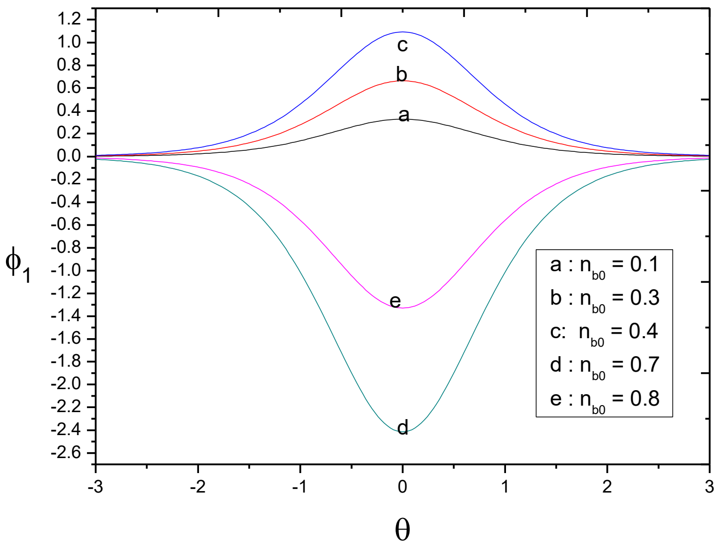

Fig.7.3 (b) .The profiles of IAS (ϕ_1) in a quantum plasma with variation of ion-beam density (n_{b0}). The plasma parameters σ_i = 0.01, σ_b =0.1, u_{b0} = 0.5, V_p =1.6, μ =1 and U = 0.1 Graphs a, b, c, d and e represent n_{b0} = 0.1, 0.3, 0.4, 0.7 and 0.8 respectively.

iii).Effect of ion beam temperature

In Fig. 7.3(c) the structure of IAS are shown with variation of σ_b in the quantum plasma having parameters (dimensionless) parameters a σ_i = 0.01, u_{b0} = 0.5, n_{b0} = 0.5, V_p =1.6, μ =1.

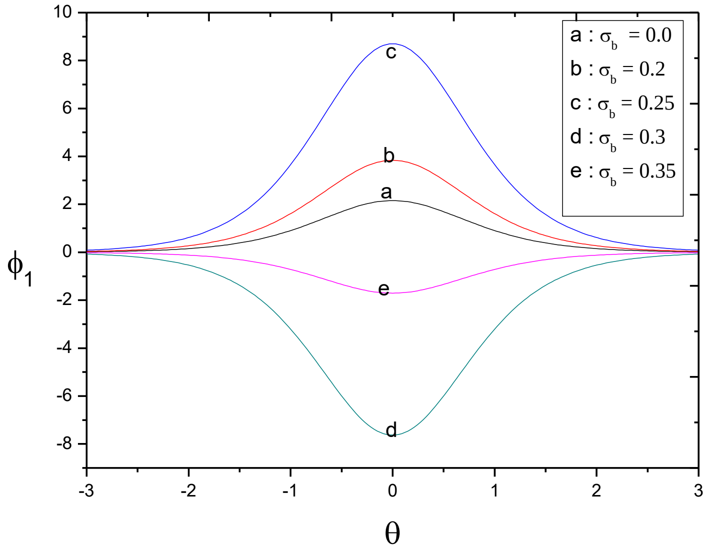

Fig.7.3(c) .The profiles of IAS (ϕ_1) in a quantum plasma with variation of ion-beam temperature (σ_b). The plasma parameters σ_i = 0.01, u_{b0} = 0.5, n_{b0} = 0.5, V_p =1.6, μ =1 and U =0.1. Graphs a, b, c, d and e represent σ_b = 0.0, 0.2, 0.25, 0.3 and 0.35 respectively.

It is observed form Fig. 7.3(a) that both compressive and rarefactive ion- acoustic solitons may be excited in the quantum plasma depending upon the values of u_{b0}. For u_{b0} = 0 to 1.2 the solitary wave will be rarefactive and the amplitude will decrease with the increase of u_{b0}, but for u_{b0} = 1.3 and 1.4 the compressive IAS will be excited and the amplitude will increase with increase of u_{b0}.

Fig.7.3 (b) shows that the values of n_{b0} decide to excite the compressive or rarefactive IAS. For low values of n_{b0} (= 01, 0.3 and 0.4) the IAS are compressive and the amplitudes increase with the increase of n_{b0}, but for large values of n_{b0} (0.7 and 0.8) the IAS will be rarefactive in nature and the amplitudes decrease with the increase of n_{b0}.

It is seen from Fig.7.3(c) that both compressive and rarefactive IAS may be excited in the quantum plasma and these are dependent on the values of σ_b. For σ_b = 0.0, 0.2 and 0.25 the IAS are compressive and the amplitudes increase with the increase of σ_b. But when σ_b =0.3 and 0.35 the rarefactive IAS are excited and the amplitudes decrease with the increase of σ_b.

7.4.3 Profiles of Width of Ion-Acoustic Solitons

Since the width is very important for the study of IAS in quantum plasma we have drawn the profiles of width for different values of ion beam parameters. From Eq. (7.12) we see the width of IAS depends on H and the plasma parameters $\sigma_i, \sigma_b, n_{b0}, u_{b0}$.

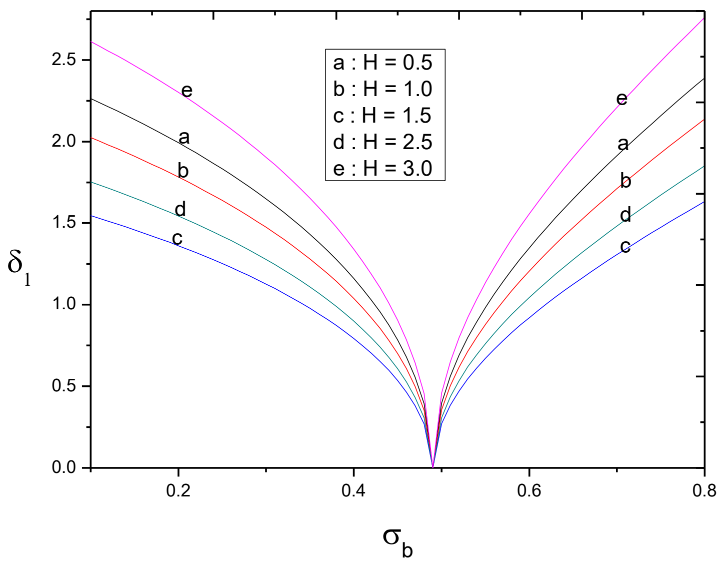

Fig.7.4 (a). Variation of width of IAS with ion beam temperature (σ_b) for different values of quantum diffraction parameter (H) in quantum plasma having σ_i =0.01, n_{b0} =0.5, u_{b0} =0.5, V_p =1.2 and μ =1. Graphs a, b, c, d and e represent H = 0.5, 1.0, 1.5, 2.5 and 3.0 respectively.

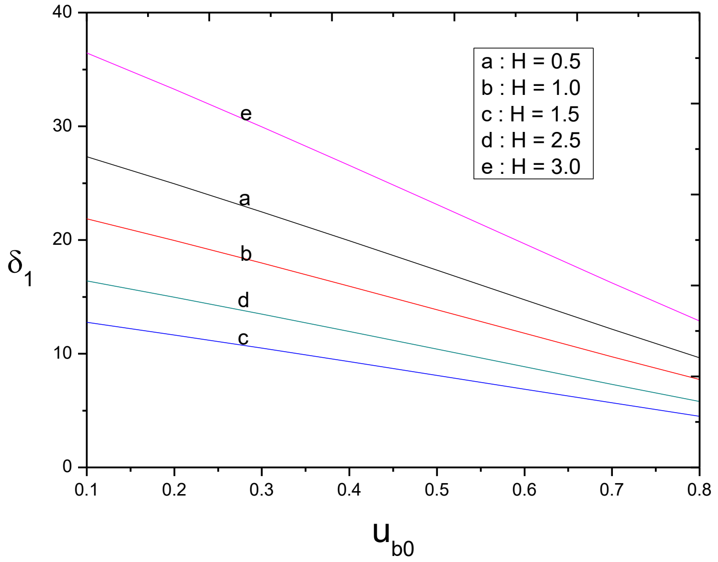

Fig.7.4 (b). Variation of width of IAS with ion beam velocity (u_{b0}) for different values of quantum diffraction parameter (H) are depicted for a quantum plasma having fixed values of σ_i = 0.1, σ_b =0.1, n_{b0} = 0.5, V_p =1.2 and μ =1. Graphs a, b, c, d and e represent H = 0.5, 1.0, 1.5, 2.5 and 3.0 respectively.

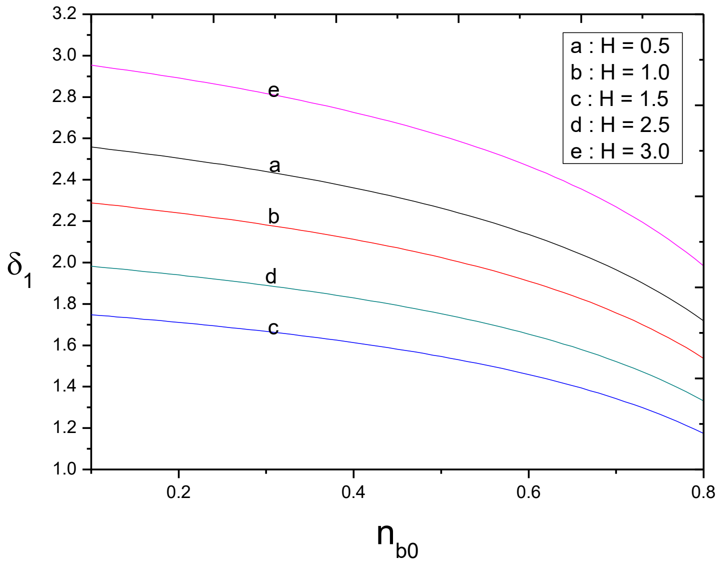

Fig.7.4(c).Variation of width of IAS with ion beam density (n_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma having the plasma parameters σ_i = 0.01, σ_b =0.1, u_{b0} = 0.5, V_p =1.2 and μ =1 . Graphs a, b, c, d and e represent H = 0.5, 1.0, 1.5, 2.5 and 3.0 respectively.

The variation of width of IAS with ion beam temperature (σ_b) in Fig.7.4 (a) for different values of H is very interesting. The width starts to decrease with the increase of σ_b and it becomes zero at certain value of σ_b ($\sigma_b \approx 0.5$). Then it is increased with the increase of σ_b. The widths are also decreased with the increase of H when H < 2. But the widths are increased with the increase of H when H > 2.

On the other hand, it is seen from Fig. 7.4(b) that the widths are decreased with the increase of u_{b0} and it is also depended on the values of quantum diffraction parameter (H). Similarly, from Fig. 7.4(c) it is observed that the widths are decreased with the increase of n_{b0} and it is depended on the values of quantum diffraction parameter (H).

It is important to point out that IAS will not exist when the co-efficient of nonlinear term (A) of the KdV Eq. (7.11) vanishes for some critical values of the plasma parameters $\mu, \sigma_i, \sigma_b, \chi, u_{b0}$ and H. Assuming $A \approx 0$ we obtain an equation

$$\chi_e[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)] = (\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2) \tag{7.14}$$

From Eq.(7.14) the critical values of the plasma parameters can be obtained for non-existence of the solitary waves in the quantum plasma. The critical value of ion beam density (N_{bc}) is

$$N_{bC} = 1 - \frac{[(\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2)]}{[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)]} \tag{7.15}$$

Again, for some critical values H, V_p and V_b the IAS will be non-dispersive if the coefficient $B \approx 0$ in KdV Eq.(7.11). In this case, $H = 2$ or $V_p = \sqrt{\sigma_i}$, or $V_b = \sqrt{\mu\sigma_b}$.

The variation of critical ion beam density (N_{bC}) with ion beam velocity and ion beam temperature is shown in Fig.7.5.

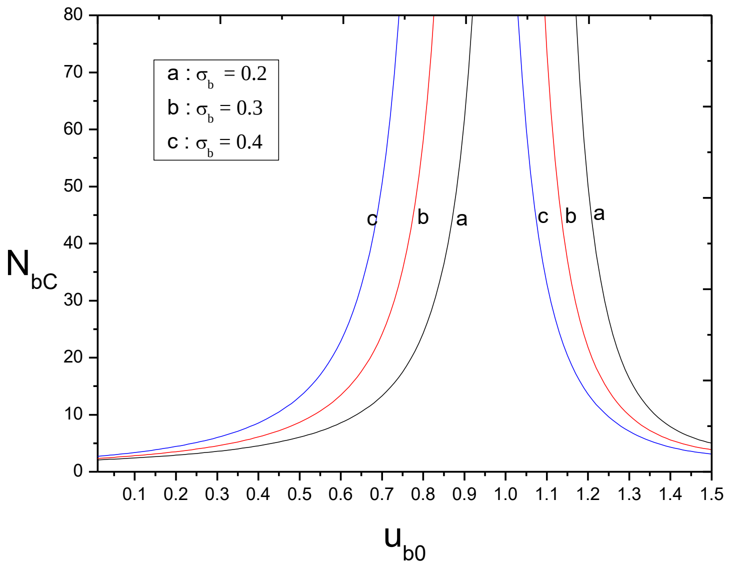

Fig.7.5. Variation of critical value of ion beam density (N_{bC}) with ion beam velocity (u_{b0}) for different ion beam temperature (σ_b) of IAS in quantum plasma having V_p =1.6, σ_i =0.1 and μ =1. Graphs a, b and c represent σ_b = 0.2, 0.3 and 0.4 respectively.

It is observed that the critical value of ion beam density N_{bC} does not depend on H and it increases with the increase of u_{b0} and at certain value of u_{b0} it obtains the maximum value. Then N_{bC} decreases with the increase of u_{b0}, N_{bC} is small for large values of u_{b0}.

7.5. SUMMARY AND CONCLUSION

In this chapter, we have used the one-dimensional quantum hydrodynamic (QHD) model to investigate IAS in dense quantum plasma in presence of ion beam. For the study of nonlinear behavior of IAS, KdV equation has been derived by using the reductive perturbation technique. It is seen that formation and structure of solitary waves are significantly affected by the presence of ion beam in quantum plasma. Our results may be useful for understanding the beam-plasma interactions and formation of nonlinear wave structures in dense quantum plasma. The model includes quantum statistical effect through the equation of state and quantum diffraction effect through the parameter *H*. It is known that quantum effects in plasma become important when thermal de Broglie wavelength becomes much larger than the average inter-particle distance. In most practical situations, quantum effects for the ions may be neglected because of their heavier mass than the electrons.

In our analysis, the explicit form of IAS in quantum plasma is obtained from the KdV equation and the structure of IAS is graphically discussed for different values of density, velocity and temperature of ion beam.

In this Chapter, it is seen that:

i). Both the compressive and rarefactive IAS may be excited in the quantum plasma depending upon the values of ion beam parameters but the quantum diffraction has no role on it.

ii). The compressive IAS will be excited for an ion beam velocity u_{b0} = 1.3 and 1.4 and amplitude will increase with the increase of u_{b0}; but IAS will be rarefactive for the ion beam velocity u_{b0} = 0 to 1.2 and the amplitude will decrease with the increase of u_{b0}.

iii). For low values of ion beam density (n_{b0} = 0.1, 0.3 and 0.4) the IAS will be compressive and the amplitudes are increased with increase of beam density; but for large values of n_{b0} (0.7 , 0.8) the IAS will be rarefactive and amplitude decreases with the increase of beam density.

iv). The compressive IAS will be excited for the ion beam temperature σ_b = 0.0, 0.2 , 0.25 and the amplitude increases with the increase of σ_b; but for σ_b =0.3 and 0.35 the IAS will be rarefactive and the amplitude decreases with the increase of σ_b.

v). The quantum diffraction parameter H plays important roles on the width of IAS in the quantum plasma. For $H < 2$ and $H > 2$ the width may be increased or decreased depending upon the values of ion beam parameters.

However, with reference to laboratory and space plasma, interesting results would be obtained on the propagation of electrostatic waves in bounded system like plasma-filled waveguides. The boundaries in such a system may add additional effects. The wave with no dispersion in unbounded system may become dispersive in bounded system; and the stable wave of an unbounded system may become unstable in bounded geometry. In relativistically degenerate plasma, nonlinear propagation of electro-static solitary solitons are by Mamun and Shukla [23] and others [24,25] Later, Chandra *et al*.[26] have considered two-temperature electrons in a relativistically degenerate plasma and have theoretically investigated the electro-acoustic solitary waves in the plasma. It has been shown that the relativistic degeneracy parameter significantly influences the conditions of formation and properties of solitary waves. Recently, Hossen and Mamun [27] have studied theoretically and numerically the nonlinear propagation of modified electro-acoustic waves in an unmagnetized, collision less, relativistic degenerate quantum plasma through the derivation of nonplanar KdV equation which admits a wave solution for the solitary wave profile.

REFERENCES

[1].G.Manfredi, P.A.Hervieux, Y.Yin and N. Crouseilles, Collective Electron Dynamics in Metallic and Semiconductor Nanostructures, in: Advances in the atomic-scale modeling of nanosystems and nanostructured materials (2009).Eds. C. Massobrio, H. Bu-lou, and C. Goyenex (Springer, Heidelberg), p. 1.

[2]. W.L. Barnes, A. Dereux and T.W. Ebbesen, Nature (London), **424**, p.824 (2003).

[3]. G. P.Agrawal , Nonlinear Fiber Optic (Academic, San Diego) Chaps. 2-5.4 (1995)..

[4]. L.K. Ang, ,T. Kwan, Tan and Y.Y.Lau, Phys. Rev. Letts., **91**, p.208303 (2003).

[5]. T. C. Killian, Science , **316** , p.705 (2007).

[6]. D. Kremp, Th. Bornath, M. Bonitz and M. Schlanges, Phys. Rev. E, **60**, p.4725 (1999).

[7]. H. Azech, FIREX Project, Plasma Phys. Control. Fusion, **48 B**, p.267 (2006).

[8]. S .I. Shapiro and S.A. Teukolsky, Black Holes, White Dwarfs and Neutron Stars, (John Wiley & Sons, New York) (1983).

[9]. E. Z. Madelung , Zeitschrift für Physik, **40,** p**.**322 (1926).

[10]. H. Haas, L. G. Garcia and J. Goedert, Phys. Plasmas, **10**, p.3858(2003).

[11]. S. Ali and P.K. Shukla, Phys. Plasmas, **13**, p.129902 (2006).

[12]. A. P. Misra and C. Bhowmik, Phys. Letts. **A 369**, 90 (2007).

[13]. A. Mushtaq and S.A. Khan, Phys. Plasmas, **14**, p.052307 (2007).

[14]. M. Akbari-Moghanjoughi, Phys. Plasmas, **17** , p.082315 (2010).

[15]. S. Chandra, S.N. Paul and B. Ghosh, Indian J. Pure and Appl. Phys., **50,** p.314 (2012).

[16]. S. Chandra, S.N.Paul and B. Ghosh, Astrophys. Sp. Science, **343** , p.213 (2013).

[17]. P.R.Dip, M.A.Hossen, M.Salahuddin and A.A. Mamun, Eur. Phys. Journal, **D71,** p.52 (2017).

[18]. T. Yajima and M. Wadati, J. Phys. Soc. Japan, **59**, p.237 (1990).

[19]. K.K.Mondal, S. N. Paul and A. Roychowdhury, IEEE Trans. Plasma Sc. **26** , p**.**987 (1998).

[20]. G .P. Zank and J.F. McKenzie, J. Plasma Phys., **39**, p.183 (2009).

[21]. E.Saberian, A. Esfandyari-Kalejahi, A. Rastkar-Ebrahimzadeh and M. Afsari-Ghazi, Phys. Plasmas, **20**, p.032307 (2013).

[22]. A. Danehkar, Plasma Phys. & Contr. Fusion, **60**, p.065010 (2018).

[23]. A.A. Mamun and P.K. Shukla, Phys. Plasmas **17**, p.104504 (2010).

[24]. W. Masood and B. Eliasson, Phys. Plasmas, **18**, p.034503 (2011).

[25]. M. Akbari-Moghanjoughi, Astrophys. Sp. Science, **332**, p.187 (2011).

[26]. S. Chandra, S.N.Paul and B. Ghosh, Astrophys. Sp. Science, **343** , p.213 (2013).

[27] M.R.Hossen and A.A.Mamun, J. Korean Phys. Society, **65**, p.2045 (2014).

CHAPTER-8

MODULATION INSTABILITY AND ENVELOPE SOLITONS OF HIGH FREQUENCY SURFACE WAVES ON WARM PLASMA HALF-SPACE

[This Chapter is based on the publication: Canadian Journal of Physics, Vol. 90, pp. 291-297 (2012)]

8.1. INTRODUCTION

In recent year's considerable interest has been aroused in the study of the non-linear behavior of surface waves in bounded plasmas [1-16] this is due to the fact that the surface wave modes promise a wide range of applications in many fields such as the plasma heating by large amplitude waves, laser-plasma interactions, plasma diagnostics, and microelectronics and in nanotechnology. Nonlinear behavior of surface waves on a cold plasma half-space has been studied by a number of authors. Its frequency spectrum extends from ω = 0 to $\omega = \omega_e / \sqrt{2}$. These waves have both electrostatic and electromagnetic characters with the former dominating at the upper limit of the frequency spectrum [17]. Gradov and Stenflo [18] considered electrostatic non-dispersive surface waves on a cold homogeneous plasma. On the other hand, Yu and Zhelyakov [19] included electromagnetic effects to provide finite dispersion at a frequency range where surface waves are usually treated electro-statically. Recently, Ghosh *et al.* [20] have made a detailed electromagnetic treatment of the nonlinear behavior of surface waves on a cold plasma half-space throughout the whole spectrum of its existence. Inclusion of thermal effect broadens the surface waves frequency spectrum [21] and makes the electrostatic surface waves dispersive even for homogeneous plasma. It can provide finite dispersion necessary for the formation of surface wave solitons. So, it remains interesting to make detailed analysis regarding the nonlinear behavior of surface waves taking into account of the thermal motion of electrons. In the present paper, we make detailed analysis of the nonlinear behavior of high frequency surface waves on a plasma half-space taking into account the thermal motion of electrons. By the method of multiple scales [22] we derive a nonlinear Schrodinger equation whose nonlinear term is found to be complex. From this equation instability criteria are found out and it is shown that high frequency surface waves on a warm plasma half-space are modulationally unstable. The instability growth rate is found to increase with increase of wave number and temperature. The quadratic non-linearities in the system equations are found to generate second harmonics a part of which decays, as well, exponentially away from the surface while the other part carry a fraction of the surface wave energy into the bulk of plasma. So, the instability will have to grow in competition with the continuous energy extraction by the second harmonics.

8.2. BASIC EQUATIONS

We are interested in the nonlinear behavior of surface waves at the upper frequency range of their existence where they may be treated as electrostatic [17]. So the basis equations governing the propagation of such high frequency electro static waves in hot electron plasma are the following:

$$\frac{\partial n}{\partial t} + \mathrm{N}_0 \vec{\nabla}.\vec{u} = -\vec{\nabla}.(\overrightarrow{nu}) \tag{8.1}$$

$$\frac{\partial \vec{u}}{\partial t} - \frac{e}{m_e}\vec{\nabla}\varphi + \frac{V_e^{\,2}}{N_0 + n}\vec{\nabla}n = -(\vec{u}.\vec{\nabla})\vec{u} \tag{8.2}$$

$$\nabla^2 \phi = 4\pi e n \tag{8.3}$$

where n is the perturbation in the uniform density N_0 of the electrons, $\vec{u}$ is the fluid velocity of electrons, $V_e^{\,2}$ is the mean square thermal velocity, φ is the electric potential and other symbols have their usual meanings. Ions are assumed to form a uniform neutralizing background (which is correct when $\omega >> \omega_p$). We also assume that $\omega >> kV_e$. The plasma is assumed to occupy the half space x > 0 and bounded by a vacuum. For vacuum (x < 0) we use the well known Laplace's equation

$$\overrightarrow{\nabla^2}\varphi_v = 0 \tag{8.4}$$

where the subscript v correspond to vacuum.

The boundary conditions to be used are the continuity of electric potential and normal electric displacement across the plasma-vacuum interface. As we are considering warm plasma, the normal component of the fluid velocity of electron may be assumed to be zero on the interface [23].

8.3. NONLINEAR ANALYSIS

Following the usual procedure [24, 25] we make the following Fourier expansion of the field quantities as

$$W = \varepsilon^2 W_0 + \sum_{s}^{\infty} \varepsilon^s [W_s \exp(is\psi) + W_s^* \exp(-is\psi)] \tag{8.5}$$

where $\psi = kz - \omega t$ (ω and k are respectively linear frequency and wave number); ε is a smallness parameter; W_0, W_s, W_s^* are functions of x; $\xi = \varepsilon(z - C_g t)$; $\tau = \varepsilon^2 t$, $C_g = d\omega / dk$ (the group velocity); and W stands for the field quantities n, u_x, u_z, φ and φ_v .

Substituting this expansion of Eq.(8.5) in Eqs. (8.1) - (8.4) and equating on both sides the coefficients of $\exp(i\psi), \exp(2i\psi)$ and terms independent of ψ we get three sets of equations which we call I, II and III respectively. To solve these equations we make the following perturbation expansion for the field quantities W_1, W_2, W_0 which are denoted by $W_j\,(j = 1, 2, 0)$.

$$W_j = W_j^{(1)} + \varepsilon W_j^{(2)} + \varepsilon^2 W_j^{(3)} + \cdots\cdots \tag{8.6}$$

Solving the lowest order equations obtained from the set-I after substituting Eq. (8.6) we get

$$\varphi^{(1)} = \frac{\alpha}{\gamma(\omega_e^2 - \omega^2)}[-\omega_e^2 k e^{-\gamma x} + \gamma\omega^2 e^{-kx}]$$

$$n_1^{(1)} = -\frac{\alpha e N_0 k(\gamma^2 - k^2)}{\gamma m_e(\omega_e^2 - \omega^2)} e^{-\gamma x}$$

$$u_{x1}^{(1)} = \frac{i\alpha e\omega k}{m_e(\omega_e^2 - \omega^2)}[e^{-\gamma x} - e^{-kx}]$$

$$u_{z1}^{(1)} = \frac{\alpha e\omega k}{m_e\gamma(\omega_e^2 - \omega^2)}[k e^{-\gamma x} - \gamma e^{-kx}]$$

$$\varphi_v^{(1)} = \alpha e^{kx} \tag{8.7}$$

where ω and k satisfy the following linear dispersion relation

$$\gamma(2\omega^2 - \omega_e^2) = \omega_e^2 k \tag{8.8}$$

where, $\gamma^2 = k^2 + (\omega_e^2 - \omega^2)/V_e^2$ and ω_e is the electron plasma frequency. α Appearing in Eq.(8.7) represents the amplitude of the wave field and is a function of ξ, τ . We seek an evolution equation for α .

At low temperatures Eq. (8.8) gives approximately,

$$\omega = \frac{\omega_e}{\sqrt{2}}\left(1 + \frac{kV_e}{\sqrt{2}\omega_e}\right) \tag{8.9}$$

It clearly shows that the effect of thermal motion of the electrons is to broaden the frequency spectrum of high frequency surface waves in semi-infinite plasma.

Similarly, solving the lowest order equations obtained from the sets II and III we get the following solutions for the second harmonic and zero- th harmonic quantities respectively,

$$\varphi_2^{(1)} = \alpha^2[C_0 e^{i\gamma_2 x} + \sum_{j=1}^{3} C_j e^{-p_j x}]$$

$$n_2^{(1)} = \frac{\alpha^2}{4\pi e}[-(\gamma_2^2 + 4k^2)C_0 e^{i\gamma_2 x} + \sum_{j=2,3} C_j (p_j^2 - 4k^2) e^{-p_j x}]$$

$$u_{x2}^{(1)} = -\frac{\alpha^2 e}{2\omega m_e}[\frac{4\omega^2\gamma_2}{\omega_e^2} C_0 e^{i\gamma_2 x} + i\sum_{j=1}^{3} p_j D_j e^{-p_j x}]$$

$$u_{z2}^{(1)} = -\frac{\alpha^2 e}{2\omega m_e}[\frac{8\omega^2 k}{\omega_e^2} C_0 e^{i\gamma_2 x} + 2k\sum_{j=1}^{3} D_j e^{-p_j x}] \qquad (8.10)$$

And

$$\varphi_0^{(1)} = \alpha\alpha^* a_2 k^2 \sum_{j=1}^{4} A_j e^{-p_j x}$$

$$n_0^{(1)} = \frac{\alpha\alpha^* a_2 k^2}{4\pi e}\sum_{j=1}^{4} p_j^2 A_j e^{-p_j x}$$

$$u_{x0}^{(1)} = 0$$

$$u_{z0}^{(1)} = \frac{\alpha\alpha^* a_2 e k^2 \gamma^2}{m_e \omega_e^2 C_g}[-e^{-p_1 x} - e^{-p_2 x} + 2e^{-p_3 x}] \qquad (8.11)$$

where,

$p_1 = 2k$, $p_2 = 2\gamma$, $p_3 = \gamma + k$

$$\gamma_2^2 = \frac{(4\omega^2 - \omega_e^2)}{V_e^2} - 4k^2, \quad p_4 = \frac{\omega_e}{V_e}$$

$$C_2 = \frac{a_2 k^2 (\omega_e^2 - \omega^2)(2\omega^2 + \omega_e^2)}{6\omega^2 \omega_e^2 V_e^2}$$

$$C_3 = -\frac{a_2 k^2 \gamma(\gamma - k)(3\gamma + 5k)}{V_e^2(\gamma + 3k)(p_3^2 + \gamma_2^2)}$$

$$a_2 = \frac{e\omega^2\omega_e^2}{m_e\gamma^2(\omega_e^2 - \omega^2)^2}$$

$$D_2 = \frac{(4\omega^2 - 3\omega_e^2)C_2}{\omega_e^2} + \frac{k^2(\gamma^2 - k^2)a_2}{2\omega^2}$$

$$D_3 = \frac{[\omega_e^2 - (p_3^2 - 4k^2)V_e^2]C_3}{\omega_e^2} + \frac{k^2\gamma(k - \gamma)a_2}{\omega_e^2}$$

$$C_1 = D_1 = -\frac{(p_2C_2 + p_3C_3)}{p_1} + \frac{i4\omega^2\gamma_2C_0}{p_1\omega_e^2}$$

$$C_0 = \frac{[2p_2D_2 + 2p_3(D_3 - C_2) - (p_1 + p_3)C_3]}{[p_1 - i\gamma_2(1 - 8\omega^2/\omega_e^2)]}$$

$$A_1 = \frac{2\gamma^2}{(\omega_e^2 - 4k^2V_e^2)}$$

$$A_2 = \frac{[\omega_e^2k^2 - \gamma^2(\omega_e^2 - 2\omega^2)]}{[\omega^2(\omega_e^2 - 4\gamma^2V_e^2)]}$$

$$A_3 = \frac{p_2p_3}{(p_3^2V_e^2 - \omega_e^2)} \quad \text{And} \quad A_4 = -\sum_{j=1}^{3} A_j \tag{8.12}$$

Solution for the second harmonic field quantities shows that a part of it decays in the usual way away from the interface while the other part represents wave propagating obliquely away from the plasma-vacuum interface, carrying a fraction of the surface wave energy into the bulk of the plasma.

Now, collecting the coefficients ε^3 of from both sides of the set of equations-I substituting the perturbation expansion (8.6) we get equations for the first harmonic quantities in the third order which can be put in the following matrix form:

$$\begin{bmatrix} -i\omega & N_0\dfrac{\partial}{\partial x} & ikN_0 & 0 \\ \dfrac{V_e^2}{N_0}\dfrac{\partial}{\partial x} & -i\omega & 0 & -\dfrac{e}{m_e}\dfrac{\partial}{\partial x} \\ \dfrac{ikV_e^2}{N_0} & 0 & -i\omega & \dfrac{\partial^2}{\partial x^2}-k^2 \\ -4\pi e & 0 & 0 & \dfrac{\partial^2}{\partial x^2}-k^2 \end{bmatrix}\begin{bmatrix} n_1^{(3)} \\ u_{x1}^{(3)} \\ u_{z1}^{(3)} \\ \varphi_1^{(3)} \end{bmatrix}=\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix},(x>0) \tag{8.13}$$

and

$$(\frac{\partial^2}{\partial x^2}-k^2)\varphi_{v1}^{(3)}=M_{v1}\ , \quad (x<0) \tag{8.14}$$

where,

$$M_1=-\frac{\partial n_1^{(1)}}{\partial \tau}+C_g\frac{\partial n_1^{(2)}}{\partial \xi}-N_0\frac{\partial u_1^{(2)}}{\partial \xi}-\frac{\partial}{\partial x}(n_0^{(1)}u_{x1}^{(1)}+n_1^{*(1)}u_{x2}^{(1)}+n_2^{(1)}u_{x1}^{*(1)})$$

$$-ik(n_0^{(1)}u_{z1}^{(1)}+n_1^{(1)}u_{z0}^{(1)}+n_1^{*(1)}u_{z2}^{(1)}+n_2^{(1)}u_{z1}^{*(1)})$$

$$M_2=-\frac{\partial u_{x1}^{(1)}}{\partial z}+C_g\frac{\partial u_{x1}^{(2)}}{\partial \xi}-\frac{\partial}{\partial x}(u_{x2}^{(1)}u_{x1}^{(1)*})-ik(u_{z0}^{(1)}u_{x1}^{(1)}+2u_{z1}^{(1)*}u_{x2}^{(1)}-u_{z2}^{(1)}u_{x1}^{(1)*})$$

$$+\frac{V_e^2}{N_0^2}\frac{\partial}{\partial x}(n_1^{(1)}n_0^{(1)}+n_1^{*(1)}n_2^{(1)})-\frac{V_e^2}{N_0^3}\frac{\partial}{\partial x}(n_1^{(1)^2}n_1^{*(1)})$$

$$M_3=-\frac{\partial u_{z1}^{(1)}}{\partial \tau}+C_g\frac{\partial u_{z1}^{(2)}}{\partial \xi}-\frac{V_e^2}{N_0}\frac{\partial n_1^{(2)}}{\partial \xi}+\frac{e}{m_e}\frac{\partial \varphi_1^{(2)}}{\partial \xi}-(u_{x1}^{(1)}\frac{\partial u_{z0}^{(1)}}{\partial x}+u_{x2}^{(1)}\frac{\partial u_{z1}^{(1)*}}{\partial x}+u_{x1}^{(1)*}\frac{\partial u_{z2}^{(1)}}{\partial x})$$

$$-ik(u_{z0}^{(1)}u_{z1}^{(1)}+u_{z1}^{(1)*}u_{z2}^{(1)})+\frac{ikV_e^2}{N_0^2}(n_0^{(1)}n_1^{(1)}+n_1^{*(1)}n_2^{(1)})-\frac{ikV_e^2}{N_0^3}(n_1^{(1)^2}n_1^{*(1)})$$

$$M_4=-\frac{\partial^2\varphi_1^{(1)}}{\partial \xi^2}-2ik\frac{\partial \varphi_1^{(2)}}{\partial \xi}$$

and

$$M_{v1} = -\frac{\partial^2 \varphi_{v1}^{(1)}}{\partial \xi^2} - 2ik\frac{\partial \varphi_{v1}^{(2)}}{\partial \xi} \tag{8.15}$$

The boundary conditions to be used at $x = 0$ are

$$\varphi_1^{(3)} = \varphi_{v1}^{(3)}, \qquad \frac{\partial \varphi_1^{(3)}}{\partial x} = \frac{\partial \varphi_{v1}^{(3)}}{\partial x} \quad \text{and} \quad u_{x1}^{(3)} = 0 \tag{8.16}$$

Now we -multiply Eq. (8.13) by the row matrix $[f_1, f_2, f_3, f_4]$ and integrate the resulting equation with respect to x between the limits 0 to ∞. Similarly, we multiply Eq. (8.14) by the function f_{v1} and integrate the resulting equation with respect to x between the limits $-\infty$ to 0. On adding the two equations thus obtained we get the following equation,

$$\sum_{j=1}^{4}\int_0^\infty f_j M_j dx + \int_{-\infty}^{0} f_{v1} M_{v1} dx = 0 \tag{8.17}$$

where we have chosen the functions f_1 and f_{v1} to satisfy the following equations and boundary conditions,

$$\begin{bmatrix} -i\omega & \frac{V_e^2}{N_0}\frac{\partial}{\partial x} & -\frac{ikV_e^2}{N_0} & -4\pi e \\ -N_0\frac{\partial}{\partial x} & -i\omega & 0 & 0 \\ ikN_0 & 0 & -i\omega & 0 \\ 0 & \frac{e}{m_e}\frac{\partial}{\partial x} & -iek/m_e & \frac{\partial^2}{\partial x^2} - k^2 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, (x > 0) \tag{8.18}$$

$$\left(\frac{\partial^2}{\partial x^2} - k^2\right) f_{v1} = 0, \quad (x < 0) \tag{8.19}$$

At $x = 0,\ f_4 = f_{v1},\ \frac{\partial f_4}{\partial x} = \frac{\partial f_{v1}}{\partial x},\ f_z = 0$

$$f_j \to 0 \ \text{as} \ x \to \infty \ \text{and} \ f_{v1} \to 0 \ \text{as} \ x \to -\infty \tag{8.20}$$

The solutions of the Eqs. (8.18) and (8.19) satisfying boundary conditions Eq. (8.20) are the following

$$f_1 = \frac{4\pi ie\omega A}{\gamma(\omega_e^2 - \omega^2)}[\gamma e^{-kx} - ke^{-\gamma x}]$$

$$f_2 = \frac{4\pi eN_0 kA}{(\omega_e^2 - \omega^2)}[e^{-kx} - e^{-\gamma x}]$$

$$f_3 = \frac{4\pi ieN_0 kA}{\gamma(\omega_e^2 - \omega^2)}[\gamma e^{-kx} - ke^{-\gamma x}]$$

$$f_4 = \frac{A}{\gamma(\omega_e^2 - \omega^2)}[\gamma\omega^2 e^{-kx} - k\omega_e^2 e^{-\gamma x}]$$

$$f_{v1} = Ae^{kx} \tag{8.21}$$

where A is some arbitrary constant independent of x.

It is shown that the Eq. (8.17) leads to the desired nonlinear Schrodinger equation. The linear part of M_j and M_{v1} involve first harmonic quantities in the second order which we are yet to determine. To deal with these quantities we follow the procedure adopted by Blenerhassett [26]. According to this procedure we take the help of a similar but linear problem with a slightly different wave-number $k' = k + \varepsilon$ and frequency $\omega' = \omega(k + \varepsilon)$. The problem is defined by the following equations,

$$\begin{bmatrix} -i\omega' & N_0 \dfrac{\partial}{\partial x} & ik'N_0 & 0 \\ \dfrac{V_e^2}{N_0}\dfrac{\partial}{\partial x} & -i\omega' & 0 & -\dfrac{e}{m_e}\dfrac{\partial}{\partial x} \\ ik'\dfrac{V_e^2}{N_0} & 0 & i\omega' & -\dfrac{iek'}{m_e} \\ -4\pi e & 0 & 0 & \dfrac{\partial^2}{\partial x^2} - k'^2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, (x > 0) \tag{8.22}$$

$$(\frac{\partial^2}{\partial x^2} - k'^2)F_{v1} = 0, \quad (x < 0) \tag{8.23}$$

And the boundary conditions,

$$F_4 = F_{v1}, \quad \frac{\partial F_4}{\partial x} = \frac{\partial F_{v1}}{\partial x} \quad , \; F_2 = 0 \quad \text{at} \quad x = 0$$

$$F_j \to 0 (j = 1,2,3,4) \quad \text{as}\, x \to \infty, \quad F_{v1} \to 0 \quad \text{as} \quad x \to -\infty \tag{8.24}$$

To solve this problem we make the following perturbation expansion,

$$F_j = F_j^{(1)} + \varepsilon F_j^{(2)} + \varepsilon^2 F_j^{(3)} + \cdots\cdots$$

$$F_{v1} = F_{v1}^{(1)} + \varepsilon F_{v1}^{(2)} + \varepsilon^2 F_{v1}^{(3)} + \cdots\cdots$$

$$\omega' = \omega + \varepsilon C_g + \varepsilon^2 \frac{1}{2}\frac{dC_g}{dk} + \cdots\cdots \tag{8.25}$$

Substituting these expansions in Eqs.(8.22) - (8.24) and then equating on both sides the coefficients of like powers of ε we get a sequence of equations and boundary conditions for the elements $F_j^{(l)}$ and $F_{v1}^{(l)}$ $(l = 1,2,3)$. It is found that the elements $F_j^{(1)}$ and $F_{v1}^{(1)}$ satisfy the same set of equations and boundary conditions as those satisfied by the elements $A_j^{(1)} \equiv [n_1^{(1)}, u_{x1}^{(1)}, u_{z1}^{(1)}, \varphi_1^{(1)}]$ and $\varphi_{v1}^{(1)}$. So we can make the following identification

$$F_j^{(1)} = A_j^{(1)}, \qquad F_{v1}^{(1)} = \varphi_{v1}^{(1)} \tag{8.26}$$

If we set

$$F_j^{(2)} = \alpha\psi_j^{(2)}, F_{v1}^{(2)} = \alpha\psi_{v1}^{(2)}, A_j^{(2)} = -i\frac{\partial\alpha}{\partial\xi}X_j^{(2)}, \quad \varphi_{v1}^{(2)} = -i\frac{\partial\alpha}{\partial\xi}X_{v1}^{(2)} \tag{8.27}$$

Then we find that the elements $\psi_j^{(2)}$ and $\psi_{v1}^{(2)}$ satisfy the same set of equations and boundary conditions as those satisfied by the elements $X_j^{(2)}$ and $X_{v1}^{(2)}$. So we can make the following indentification

$$\psi_j^{(2)} = X_j^{(2)}, \quad \psi_{v1}^{(2)} = X_{v1}^{(2)} \tag{8.28}$$

Setting $F_j^{(3)} = \alpha\psi_j^{(3)}$, $F_{v1}^{(2)} = \alpha\psi_{v1}^{(3)}$ we find that $\psi_j^{(3)}$ and $\psi_{v1}^{(3)}$ satisfy the Eqs. (8.13), (8.14) and boundary conditions Eq. (8.16) as satisfied by the quantities $A_j^{(3)}$ and $\varphi_{v1}^{(3)}$ with the exceptions that the quantities M_j and M_{v1} are replaced by N_j and N_{v1} which are given by

$$N_1 = i\left(C_g X_1^{(2)} - N_0 X_3^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\cdot\frac{1}{\alpha}A_1^{(1)}\right)$$

$$N_2 = i\left(C_g X_2^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\cdot\frac{1}{\alpha}A_2^{(1)} \right)$$

$$N_3 = i\left(C_g X_3^{(2)} - \frac{V_e^2}{N_0}X_1^{(2)} + \frac{ie}{m_e}X_4^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\frac{1}{\alpha}A_3^{(1)} \right)$$

$$N_4 = 2kX_4^{(2)} + \frac{1}{\alpha}A_4^{(1)}$$

and

$$N_{v1} = 2kX_{v1}^{(2)} + \frac{1}{\alpha}\varphi_{v1}^{(1)} \tag{8.29}$$

Following the same procedure as adopted in deriving Eq.(8.17) we obtain the following equation from the equations and boundary conditions satisfied by $\psi_j^{(3)}$ and $\psi_{v1}^{(3)}$:

$$\sum_{j=1}^{4}\int_0^\infty f_j N_j dx + \int_0^\infty f_{v1} N_{v1} dx = 0 \tag{8.30}$$

The operation Eq. (8.17) + $i(\frac{\partial^2\alpha}{\partial\xi^2})\cdot Eq.(8.30)$ and some straight forward integrations and algebraic manipulations give the following desired nonlinear Schrodinger equation:

$$i\frac{\partial\alpha}{\partial\tau} + P\frac{\partial^2\alpha}{\partial\xi^2} = (Q_1 - iQ_2)\alpha^2\alpha^* \tag{8.31}$$

Where, P , Q_1 , Q_2 are given by

$$P = \frac{\omega V_e^2 + 3kV_e^2 C_g - 4\omega C_g^2}{2(2\omega^2 - \omega_e^2 - k^2V_e^2)}, C_g = \frac{\omega k V_e^2}{(2\omega^2 - \omega_e^2 - k^2V_e^2)}, Q_1 = \frac{I_1}{I_2}, Q_2 = \frac{I_2}{I_3}$$

In which

$$I_1 = \frac{e^2k^3(\gamma^2-k^2)^3}{4m_e^2\gamma^2\omega(\omega_e^2-\omega^2)} +$$
$$\frac{e^2k^3\omega^3\gamma}{m_e^2(\omega_e^2-\omega^2)C_g}\left[\frac{k(\gamma^2-k^2)}{p_2} + \frac{(3\gamma^3-k^3-\gamma k p_3)}{p_3} + \frac{4k^3-3\gamma^3+\gamma k(k-2\gamma)}{p_2+p_3} + \frac{\gamma k^2+2\gamma^2 k(k-3\gamma^3)}{p_1+p_3}\right] -$$
$$\frac{e^3\omega^3k^2}{m_e^2\gamma(\omega_e^2-\omega^2)^2V_e^2}\sum_{j=1}^{4}A_j p_j^2\cdot\left[\frac{y_1+p_jy_2}{p_j+p_2} + \frac{\gamma^2 p_j}{p_j+p_1} + \frac{y_3+p_jy_4}{p_j+p_3}\right] +$$

$$\frac{e^3\omega^3k^2}{m_e^2\gamma(\omega_e^2-\omega^2)^2V_e^2}\sum_{j=1}^{4}A_j p_j^2\cdot[\frac{y_1+p_jy_2}{p_j+p_2}+\frac{\gamma^2p_j}{p_j+p_1}+\frac{y_3+p_jy_4}{p_j+p_3}]+$$

$$\frac{e\gamma}{2m_e\omega}[\sum_{j=1}^{3}D_j'(2k^2\gamma^2+\frac{x_1+p_jx_2+p_j^2x_3}{p_j+p_2}+\frac{x_4+p_jx_5+p_j^2x_6}{p_j+p_3})+\frac{4\omega^2}{\omega_e^2}(C_{or}Y_r-C_{oi}Y_i)]$$

$$I_2=\frac{e\omega\gamma(p_1^2+\gamma_2^2)(C_{or}R_i+C_{oi}R_r)}{m_e\omega_e^2}+$$

$$\frac{e\gamma}{2m_e\omega}\left[\frac{4\omega^2(C_{or}Y_i+C_{oi}Y_r)}{\omega_e^2}+D_{1i}\left\{2k^2\gamma^2+\frac{(x_1+p_1x_2+p_1^2x_3)}{p_1+p_2}+\frac{(x_4+p_1x_5+p_1^2x_6)}{p_1+p_3}\right\}\right] \tag{8.32}$$

$$I_3=2\gamma^3-k^3-\gamma^2k-\frac{p_1p_2}{p_3}$$

where,

$$y_1=2\gamma^2k-\omega_e^2(\gamma^2+k^2)\frac{k}{\omega^2},\ y_2=\frac{\gamma k(2\omega^2-\omega_e^2)}{\omega^2},\ y_3=\frac{\gamma p_3(\omega_e^2k-\omega^2\gamma)}{\omega^2},$$

$$y_4=\frac{\omega_e^2\gamma k}{\omega}-p_2k-\gamma^2,\ z_1=\frac{\omega_e^2k(\gamma^2+k^2)}{\omega^2},\ z_2=\frac{\omega_e^2k\gamma}{\omega^2},\ z_3=\gamma(3k^2+\gamma^2),$$

$$z_4=\gamma p_3,\ z_5=\gamma^2+\frac{\gamma k(\omega^2-\omega_e^2)}{\omega^2},\ x_1=4k^5,\ x_2=p_1\gamma(\gamma^2+k^2),\ x_3=k(2\gamma^2-k^2),$$

$$x_4=2k^2\gamma(\gamma^2-3k^2-2\gamma k),\ x_5=\gamma^2(k^2-\gamma^2)-3\gamma k^2p_3,\ x_6=\gamma(k^2-\gamma^2-\gamma k),$$

$$R_r=\frac{(z_1p_2+z_2\gamma^2)}{(\gamma^2+p_2^2)}-\frac{(z_3p_3+z_4\gamma^2)}{(\gamma^2+p_3^2)}+z_5$$

$$R_i=\frac{\gamma_2(z_1-p_2z_2)}{({\gamma_2}^2+p_2^2)}-\frac{\gamma_2(z_3-p_3z_4)}{({\gamma_2}^2+p_3^2)}$$

$$D_1'=D_{1r},\ D_2'=D_2,\ D_3'=D_3,\ D_1=D_{1r}+iD_{1i},\ C_0=C_{or}+iC_{oi},$$

$$Y_i=\gamma_2(x_1-x_2p_2-x_3{\gamma_2}^2)/({\gamma_2}^2+p_2^2)+\gamma_2(x_4-p_3x_5-{\gamma_2}^2x_6)/({\gamma_2}^2+p_3^2),$$

$$Y_r=\{x_1p_2+{\gamma_2}^2(x_2-x_3p_3)\}/({\gamma_2}^2+p_2^2)+\{x_4p_3+{\gamma_2}^2(x_5-x_6p_6)\}/({\gamma_2}^2+p_3^2)+2k^2\gamma_2^2 \tag{8.33}$$

8.4. MODULATION INSTABILITY

To find the criteria of modulation instability we substitute in Eq. (8.31),

$$\alpha = \alpha_0(1+\hat{\alpha})e^{-\delta\tau} exp i[-\Delta\omega\tau + \hat{\theta}] \tag{8.34}$$

Where, α_o, δ , $\Delta\omega$ are real and $\hat{\alpha}, \hat{\theta}$ are functions of ξ, τ .

For simplicity we shall confine our calculations to the initial stage of nonlinear evolution. With this in mind we substitute Eq. (8.34) in Eq. (8.31) and then equating real and imaginary parts we get,

$$\frac{\partial \hat{\alpha}}{\partial \tau} + P\frac{\partial^2 \hat{\theta}}{\partial \xi^2} + 2Q_2\alpha_0^2\hat{\alpha} = 0 \tag{8.35}$$

$$-\frac{\partial \hat{\theta}}{\partial \tau} + P\frac{\partial^2 \hat{\alpha}}{\partial \xi^2} - 2Q_1\alpha_0^2\hat{\alpha} = 0 \tag{8.36}$$

where we have chosen $\Delta\omega \approx Q_1\alpha_0^2$ and $\partial \approx Q_2\alpha_0^2$.

Assuming ξ, τ dependence of $\hat{\alpha}$ and $\hat{\theta}$ to be of the form $\exp i(l\xi - \Omega\tau)$ we get from Eq.(8.35) and Eq.(8.36) two algebraic equations in $\hat{\alpha}$ and $\hat{\theta}$, from which we get for nontrivial solution,

$$\Omega = -iQ_2\alpha_0^2 + \left[-Q_2^2\alpha_0^4 + l^2(P^2l^2 + 2PQ_1\alpha_0^2)\right]^{\frac{1}{2}} \tag{8.37}$$

The first term on the right hand side is small and is due to the oscillatory part of the second harmonic moving obliquely away from the interface. For a uniform plasma, wave train to be modulationally unstable we require PQ_1 > 0. Then we require

$$\Omega = i\left[-Q_2\alpha_0^2 + (Q_2^2\alpha_0^4 + 2|PQ_1|\alpha_0^2 l^2 - P^2l^4)^{\frac{1}{2}}\right] \tag{8.38}$$

In that a uniform plasma wave train is modulationally unstable for PQ_1 >0.

The growth rate of instability, $|\Omega|$ attains a maximum value corresponding to the wave number (l_m) given by

$$l_m^2 = \left|\frac{Q_1}{P}\right|\alpha_0^2 \tag{8.39}$$

Using this value of l_m the corresponding maximum growth rate of instability during the initial stage of nonlinear evolution is obtained as

$$g_m = \alpha_0^2[\sqrt{Q_1{}^2 + Q_2{}^2} - Q_2] \tag{8.40}$$

8.5. RESULTS AND DISCUSSION

Numerical calculation shows that $PQ_1 < 0$ for all values of *k*. So we conclude that high frequency electrostatic surface waves on a warm plasma half-space are modulationally unstable. Also it is found that $Q_2 > 0$ and $|Q_1| \ll Q_2$. So, the expression (8.40) indicates that the energy extraction by the second harmonics slightly slows down the growth rate of the instability. The growth rate of the instability as given by (8.40) has been calculated or different values of the dimensionless parameter kV_e / ω_e. The result is shown graphically in Fig. 8.1.

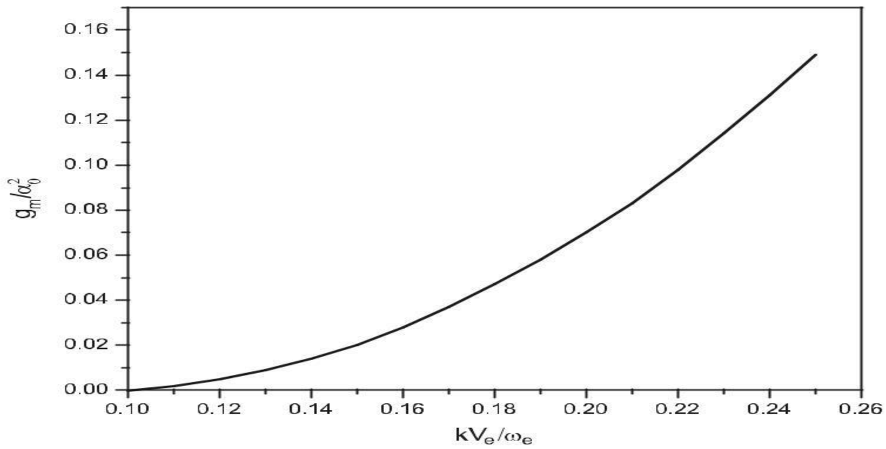

Fig. 8.1. Dependence of maximum (normalized) initial growth rate (g_m / α_0^2) on electron thermal velocity (V_e) and the wave-number (k).

Obviously, the modulation instability growth rate increases with increase in thermal velocity of electrons and also with the increase of the wave number *k*. In fact, thermal motion of the electrons introduces a number of effects. As evident from approximate dispersion relation (8.9) thermal motion broadens the frequency spectrum of surface waves. Surface waves in cold homogeneous plasma are non-dispersive. The dispersion relation (8.9) shows that inclusion of temperature makes the wave dispersive even in homogeneous plasma. Thus, it can also provide finite dispersion necessary for the formation of high frequency surface wave envelope solitons. Solutions (8.10) for the second harmonics show that the inclusion of temperature effect makes a part of the second harmonics generated through nonlinearities to carry a fraction, though small, of wave energy away from the interface into the bulk of the plasma.

As pointed out in the previous section, (8.40) gives only the initial growth rate. During the later stage of evolution the instability growth rate might be somewhat smaller than expected from (8.40) because the instability will have to grow in competition with the continuous energy extraction by the second harmonic.

It is well known that a uniform wave train may be modulationally stable or unstable depending on the sign of the product of the group dispersive and the nonlinearity coefficient, that is, PQ_1.As the coefficients depend on the plasma parameters the product of PQ_1 can have both positive and negative values over different parametric regions. The wave is modulationally unstable if $PQ_1 < 0$ and the growth rate of instability have a maximum value. There are accordingly two types of localized solitary wave solutions of the NLS equation. To obtain the soliton profile we assume

$$\rho = \alpha \exp(i\theta) \tag{8.41}$$

where ρ and θ are two real variables.

Solving the resulting equations for ρ and θ with $PQ_1 < 0$, we get the following bright envelope soliton solution:

$$\rho_B = \frac{2\sqrt{|P/Q_1|}}{L} \sec h\left(\frac{\xi - U\tau}{L}\right) \tag{8.42}$$

where U the envelope is speed and L is the spatial width of the pulse. It encloses high frequency carrier oscillations and vanishes at infinity. On the other hand, if $PQ > 0$, a stable gray or dark soliton (a potential hole or a localized region of deceased amplitude) is obtained:

$$\rho_D = \frac{2\sqrt{P/Q_1}}{Ld}\sqrt{1 - d^2 \sec h^2\left(\frac{\xi - U\tau}{L}\right)} \tag{8.43}$$

where the parameter d determines the depth of the modulation.

For $d = 1$ we get a dark soliton:

$$\rho_D = \frac{2\sqrt{P/Q_1}}{L} \tanh\left(\frac{\xi - U\tau}{L}\right) \tag{8.44}$$

Thus the sign of the product PQ determines the stability/instability profile of IAWs as well as the type of soliton structure. The soliton width is determined by the ratio $|P/Q_1|$.

8.6. SUMMARY AND CONCLUSION

In this Chapter, the modulation instability of high frequency surface waves on a plasma half-space is investigated taking into account finite temperature effects. The second harmonic generated through nonlinear self-interaction of the waves is found to carry a fraction of the surface wave energy into the bulk of the plasma. This is found to influence the modulation instability mechanism. To describe the nonlinear evolution of the wave, a nonlinear Schrödinger equation is derived in which the coefficient of the nonlinear term becomes a complex quantity. From this evolution equation we have derived the condition of instability and show that high-frequency surface waves on a warm plasma half-space are Modulational unstable.

We would like to point out here that our analysis could be extended to the whole electromagnetic spectrum of the surface waves by including electromagnetic effects. Physically, however, such extension may not result in any new result as the instability growth rate is largest at high values of wave number, where surface waves behave electrostatically. Our treatment for spatially homogeneous plasma may, however, is extended to include the effects of inhomogeneity. Such an extension may become important in many cases. For example, when a sufficiently strong laser beam interacts with plasma, the plasma undergoes strong density profile modification due to light pressure near the surface. Finally we would like to mention that the results presented in this Chapter may be of special interest in the surface physics of semiconductors and metals where ions are practically immobile and the boundary is almost rigid.

REFERENCES

[1]. M. Georgieva and A. Shivarova, Phys. Scr., **50**, p.523 (1994).
[2]. H.J. Lee and S.H. Cho, Plasma Phys. Contr. Fusion, **37,** p.989 (1995).
[3]. N.A. Azarenkov, V.O. Girka, and A.E. Sporov, Phys. Scr. , **55**, p.339 (1997)..
[4]. N.F. Cramer, L.K. Yeung, and S.V. Vladimirov, Phys.Plasmas, **5**, p.3126 (1998).
[5]. H.J. Lee and Y.K. Lim, Plasma Phys. Contr. Fusion, **40**, p.1619(1998).
[6]. D.J. Cooperberg, Phys. Plasmas**, 5**, p.862 (1998).
[7]. K.N. Ostrikov, M.Y. Yu, and N. Azarenkov, Phys. Rev. E, **58**, p.2431 (1998).
[8]. M.K. Alam, A.R. Chowdhury, and S.N. Paul, Phys. Plasmas,**6**, p.3765 (1999).
[9]. L.Stenflo and M.Y. Yu, Phys. Plasmas, **9**, p.5129 (2002).
[10]. L. Stenflo and M.Y. Yu, Phys. Plasmas, **10,** p.912 (2003).
[11]. J. Kupersztych, M. Raynaud, and C. Riconda, Phys. Plasmas,**11,** p.1669 (2004).
[12]. M. Lazar, P.K. Shukla, and A. Smolyakov, Phys. Plasmas, **14,** p**.**124501 (2007).
[13]. B. Ghosh and K.P. Das, Contrib. Plasma Phys., **28**, p.97 (1988).
[14]. H.J. Lee, Phys. Plasmas, **12**, p.1 (2005).
[15]. S.A. Dvinin, A.G. Vologirov, V.V. Mikheev, and V.S. Sviridkina, Fizika Plazmy, **34**, p.746 (2008).
[16]. B.F. Mohamed and M.A. Aziz, Int. J. Plasma Sci. Eng., **2010,** p.69304 (2010).
[17]. A. Shivarova and I. Zhelyazkov, Plasma Phys., **20,** p.1049 (1978).
[18]. O.M. Gradov and L. Stenflo, Phys. Fluids, **25**, p.983 (1982).
[19]. M.Y. Yu and I. Zhelyazkov, J. Plasma Phys., **20**, p.18 (1978).
[20]. B. Ghosh, S.R. Mazumdar, S.K. Bhattacharya, and S.N. Paul, FIZIKA, **A12,** p**.**127 (2003).
[21]. T. Lindgren, L. Stenflo, N. Kostov, and I. Zhelyazkov, J. Plasma Phys., **34**, p.427 (1985).
[22]. A. Davey and K. Stewartson, Proc. R. Soc. Lond. A**,** **388**, p.101(1974).
[23]. H.C. Barr and T.J.M. Boyd, J. Phys. A**,** **5,** p. 1108 (1972).
[24]. A.H. Nayfeh, Perturbation methods. John Wiley & Sons. 1973.
[25]. T. Taniuti and N. Yajima, J. Math. Phys., **10,** p.1369 (1969).
[26]. P.J. Blenerhasset, Philos. Trans. R. Soc. London. Ser.,**A398,** p**.**451 (1980).

PART-TWO

DOUBLE LAYERS IN DIFFERENT TYPES OF PLASMA

GENERAL INTRODUCTION

A. INTRODUCTION

Double-layers (DLs) are defined as two space charged layers of opposite electrical polarity between which an electric field exists. The DLs are essentially parallel but they may be plane or curved. In plasma, the concept of electrostatic DLs has arisen out of the fact that the charged particles are seen to undergo acceleration, particularly in the auroral region, leading to the phenomena of auroral glow. We know from classical electrodynamics that the charged particle cannot be accelerated in a magnetic field. In this regard, an electric field or a time varying magnetic field is required and in this context the concept of electrostatic DLs appears meaningful. Other names of DLs are electrostatic DLs, electric DLs, plasma DLs, electrostatic shock etc. The DLs are conceptually related to the concept of a 'sheath' . The adopted electrical symbol for a double layer is DL

B. DOUBLE LAYER CLASSIFICATION

Double layers may be classified in the following ways:

a) Weak and strong double layers.

The strength of a double layer is expressed as the ratio of the potential drop in comparison with the plasma's equivalent thermal energy, or in comparison with the rest mass energy of the electrons. A double layer is said to be strong if the potential drop across the layer is greater than the equivalent thermal energy of the plasma's components. This means that for strong double layers there are four different components to the plasma:

1. The electrons entering at the low potential side of the double layer which are accelerated;
2. The ions entering at the high potential side of the double layer which are accelerated;
3. The electrons entering at the high potential side of the double layer which are decelerated and successively reflected; and
4. The ions which enter the double layer at the low potential side of the double layer which are decelerated and reflected.

Note that in the case of a weak double layer, the electrons and ions entering from the "wrong" side are decelerated by the electric field, however most will not be reflected, as the potential drop is not strong enough.

b) Relativistic and Non-**relativistic** D**ouble Layers**.

A double layer is said to be relativistic if the potential drop over the layer is so large that the total gain in energy of the particles is larger than the rest mass energy of the electron. The charge distribution in a relativistic double layer is such that the charge density is located in two very thin layers, and inside the double layer the density is constant and very low compared to the rest of the plasma. In this respect, the double layer is similar to the charge distribution in a capacitor. As a special case of a relativistic double layer one can consider the vacuum gap at the magnetic polar cap of a pulsar.

c) Current carrying and current-free double layers.

Current carrying double layers may be generated by current-driven plasma instabilities which amplify variations of the plasma density. Current-free double layers form on the interface between two plasma regions with different characteristics, and their associated electric field maintains a balance between the penetration of electrons in either direction (so that the net current is low).

C. MATHEMATICAL METHOD AND PROFILES

In order to solve of non-linear equations for ion acoustic double layers, we have used Sagdeev potential method .In this method, the governing equations are reduced to a set a of ordinary differential equations by using the transformation η = x - V .t where 'V' is the velocity of the non-linear perturbation so that all the plasma parameters depend functionally on the variable η. Then Sagdeev potential equation is derived by the elimination of respective plasma parameter, and from which a wave equation of arbitrary amplitude is analysed.

Then using the transformation an equation of the following form in a model (as special case) is obtained:

$$\frac{d^2\varphi}{d\eta^2} = \exp(\varphi) - (1 - \frac{2\varphi}{M^2})^{\frac{1}{2}} = -\frac{d\psi}{d\varphi} \tag{C.1}$$

where, φ is the electrostatic potential, ψ is the Sagdeev potential, M is the Mach number.

Multiplying equation (1) by $d\varphi / d\eta$ and integrating with the appropriate boundary conditions we get

$$\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \tag{C.2}$$

Where,

$$\psi(\varphi)=1-\exp(\varphi)-M^2[\{1-(2\varphi/M^2)\}^{\frac{1}{2}}-1] \tag{C.3}$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi\to 0, d\varphi/d\eta\to 0$ and $d^2\varphi/d\eta^2\to 0.$

Eq. (C.2) describes the motion of a pseudo particle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $\psi(\varphi)$. The first term on the left-hand side of Eq. (C.2) can be regarded as the kinetic energy of the pseudo particle. Since the kinetic energy is always nonnegative, it follows that $\psi(\varphi)\le 0$ for the entire motion, zero being the maximum value of $\psi(\varphi)$. Eq. (C.2) shows that $d\psi/d\varphi$ is the force on the particle at the position φ . Eq. (C.1) may also be regarded as an anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the double layer solution of Eq. (C.2), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi=0$ and φ_m, where φ_m is some extremum value of the potential φ, called the amplitude of the double layers. For the double layer solutions $\psi(\varphi)$ must additionally satisfy the following boundary conditions:

(i) $\psi(\varphi)=0, d\psi/d\varphi=0$ and $d^2\psi/d\varphi^2<0$ at $\varphi=0$, (C.4a)

(ii) $\psi(\varphi)=0, d\psi/d\varphi=0$ and $d^2\psi/d\varphi^2<0$ at $\varphi=\varphi_{\max}(\neq 0)$, (C.4b)

(iii) $\psi(\varphi)<0,$ for $0<|\varphi|<|\varphi_m|$ (C.4c)

When the above conditions are satisfied, $\varphi=0$ is an unstable equilibrium position. The pseudo particle is not reflected at $\varphi=\varphi_m$ because the pseudo force and the pseudo velocity are vanished. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m.

The double-layer solution is of the form

$$\varphi_d=\frac{1}{2}\rho(g)\varphi_m(1-\tanh\theta) \tag{C.5}$$

For compressive double layers, $\rho(g)$ = + 1 for $g > 0$. When $\rho(g)$ = - 1 for $g < 0$, the rarefactive double-layers are excited.

i). Profiles of Sagdev Potential

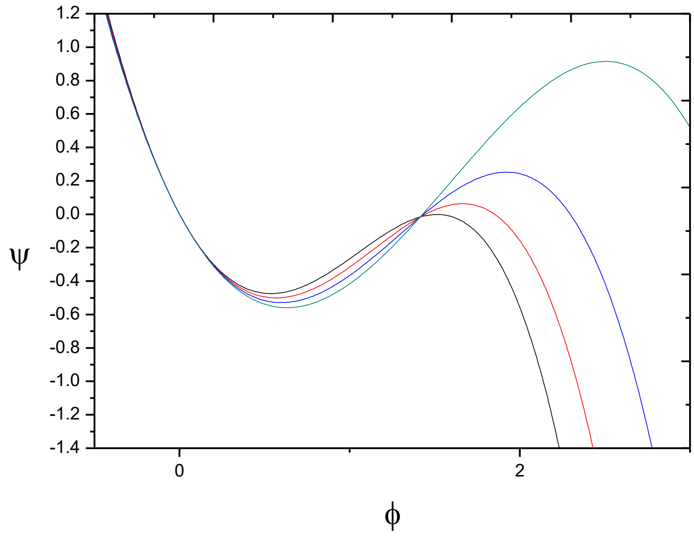

Fig.C.1.Profles Sagdeev potential in model plasma

ii). **Profiles Double Layers**

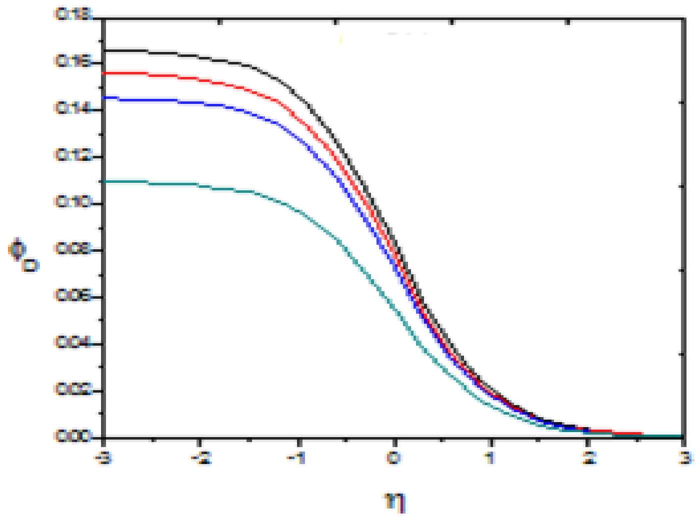

Fig.C.2. Profiles Compressive Double Layers in model plasma

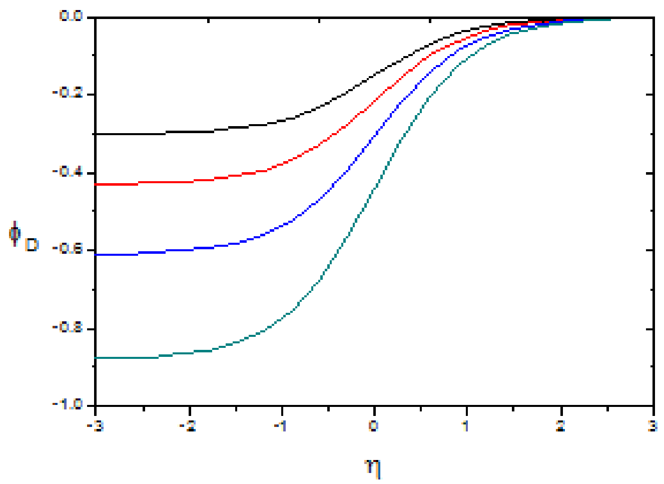

Fig.C.3.Profiles of Rarefactive Double Layers in model plasma

iv). Width of Double Layers:

The width of DLs is important from which the nature of DLs in plasma can be understood. The width of DLs can be obtained using the following expression:

$$W_d = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{C.6}$$

where φ_m is some extreme value of the potential φ, called the amplitude of the DLs and A_3 is the nonlinear term arises due to nonlinear interaction of the wave and the plasma particle. For the existence of DLs, A_3 should have the positive values.

D. SOME WORKS ON THE DOUBLE LAYERS IN PLASMA

It is very important to note that the DL cannot be excited in electron-ion plasma having single component of electrons. The works of different authors on the DLs in different types of plasma are mentioned here: (i) Electron inertia [1,2], (ii)Two-temperature electrons [3,4], (iii) Nonthermal electrons [5,6], (iv) Electron and ion beam [7,8], (v) Positrons [9,10], (vi) Negative ions [11,12] , (vii) Kappa distributed electrons [13,14], (viii) Tsallis distributed electron [15,16], (ix) Dusty plasma [17,18], (x) Quantum Plasma [19,20], (xi) Relativistically degenerate plasma [21,22] and (xii) Ultra-relativistic degenerate plasma [23,24].

REFERENCES

Double Layers in Different Types of Plasma

(i). Electron-inertia

[1]. K.K.Mondal, A. Roychowdhury and S.N.Paul, Indian J. Phys., **74 B,** p.475 (2000).

[2]. S. N. Paul, S. Chattopadhyaya, S. K. Bhattacharya and B. Bera, Pramana, **60**, p.1217 (2003)

(ii). Two-temperature electrons

[3]. T. Akhter[a], M. M. Hossain[b], A. A. Mamun, Chin. Phys. B , **22**, p.075201 (2013).

[4]. Shalini and N. S. Saini, Phys. Plasmas, **21**, p.102901 (2014).

(iii). Nonthermal electron plasma

[5]. Indrani Paul, A Chatterjee and S N Paul, Braz. J.Phys., **50**, p.272 (2020).

[6]. S. Dalui and A. Bandyopadhyay ,Indian J. Phys., **95**, p.367 (2021).

(iv).Beam Plasma

[7]. B. Das, D.K.Ghosh and P. Chatterjee, Pramana, **74,** p.973 (2010).

[8]. Indrani Paul, A.Chatterjee and S. N. Paul, Indian J. Phys.(2020) https://doi.org/10.1007/s12648-020-01899-w

(v). Positron Plasma

[9]. P. Chatterjee, T. Saha, S. V. Muniandy, C. S. Wong and R. Roychoudhury, Phys. Plasmas, **17**, p.012106 (2010).

[10]. S. K. Jain and M. K. Mishra, J. Plasma Phys. , **79** , p.6615 (2013).

(vi). Negative ion plasma

[11]. M. K. Mishra, R.S. Chhabra, Phys. Plasmas,**3**, p.4446 (1996).

[12]. K.Kumar; M. K. Mishra, IEEE Trans. Plasma Sc. , **49 ,** p.414 (2021).

(vii).Kappa- distributed electrons

[13]. S. Ali Shan and Nadia Imtiaz ,Phys. Plasmas, **24**, p.102109 (2017).

[14]. D. Dutta and K. S. Goswami , J. Plasma Phys **,85** , p.905850308 (2019).

(viii).Tsallis distributed electron

[15]. Z.Z. Li, J.F. Han ,D.N. Gao, W.S. Duan, Chinese Phys. **B**, **27,** p.105204(2018).

[16]. Indrani Paul , S.Chandra, J. Goswami , S.N.Paul and J. Sarkar , Proceedings of National Conference on Plasma Physics and Nonlinear Dynamics held at JIS University,West Bengal on 23-24 March 2017.

(ix). Dusty plasma

[17]. I. Habumugisha, S .K ,Anguma, E. Jurua and L. Nazziwa, Indian J. Phys., **92**, p.1049 (2018).

[18]. G. Khan, Z. Kumail, Y. Khan, I. Ullah and M. Adnan, Contr. Plasma Phys., **60,** e201900177(2020).

(x). Quantum Plasma

[19]. M. Shahmansouri and M.Tribeche, J. Th. Appl Phys.,**10**, p.139 (2016).

[20]. M. G. Shah, M. R. Hossen and A. A. Mamun ,IEEE Trans.Plasma Sc., **45, p.**3316 (2017).

(xi).Relativistically degenerate plasma

[21]. M.R. Hossen, S.A.Ema and A.A. Mamun, Commun. Theor. Phys., **62,** p.888 (2014)

[22]. Indrani Paul , S.Chattopadhyaya and S. N.Paul (Accepted for publication in International Journal of Research in Engineering and Science) (2021)

(xii).Ultra-relativistic degenerate plasma

[23]. A. A. Mamun and P. K. Shukla, Phys. Letts., **A 374 (41)**, p.4238 (2010)

[24]. N. Roy, S. Tasnim and A. A. Mamun, Phys. Plasmas, **19**, p.033705 (2012)

CHAPTER-9

EFFECT OF BOUNDARY OF CYLINDRICAL WAVEGUIDE AND NONTHERMAL ELECTRONS ON THE ION-ACOUSTIC DOUBLE LAYERS IN BOUNDED MULTICOMPONENT PLASMA

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9.1. INTRODUCTION

In last few years, ion acoustic solitons (IAS) and double layers (IADLs or DLs) have been theoretically studied by various authors considering different parameters in plasma e.g., ion temperature , two-temperature electrons , positrons, magnetic field, beam ions etc. But, the presence of negative ions [1-3] and nonthermal electrons [4-8] in plasma is found to be most interesting on the formation of IADLs. It is to be noted that the most of the works are done in infinite or unbounded plasma. But, the studies of IAS and IADLs in bounded plasma are important for the laboratory experiments [9-14]. Considering plasma bounded by cylindrical and spherical geometry composed of ions and different types of electrons, Das *et al* [15], Das and Singh [16], Labany *et al*. [17] have derived the Korteweg–deVries (KdV) equation for the study of IAS using perturbation method. It has been shown that interaction of isothermality on the propagation of radially ingoing IAS in various plasmas exerts a drastic modifying effect on the existence and behaviour of IAS. Subsequently, the effect of higher order nonlinearity and finite geometry has been considered by Ghosh and Das [18] and it has been shown that higher order nonlinearity is to increase the amplitude and to decrease the width of IAS as compared with that predicted values by first- order theory. Moreover, higher-order nonlinearity and finite boundary effects give rise to W-shaped IAS. Recently, Kaur [19] have numerically investigated the cylindrical and spherical IAS in plasma composed with ions, Tsallis distributed electrons and positrons. They have also studied numerically the effect of non-extensivity and no planar geometry on the amplitudes and width of potential structures of the IAS. Very recently, using the reductive perturbation method (RPM) Sahu and Roy [20] have studied the nonlinear propagation of electron acoustic waves in unmagnetized dissipative quantum plasma in unbounded planar geometry and bounded nonplanar geometry. It is found that non-dimensional parameter, equilibrium density ratio of cold to hot electron component, have crucial roles on the formation of both compressive and rarefactive solitary waves (SWs). It is also found that electron acoustic SWs in nonplanar geometry significantly differ from the planar geometry.

It is to be mentioned that the works of above authors on the SWs in unbounded plasma have been done using the RPM. But, Sayal and Sharma [21] have analysed theoretically ion Langmuir plasma oscillations in plasma filled in a cylindrical waveguide using fluid equations using Sagdeev's effective (pseudo) potential method (SPM). It is shown that propagation of such modes is possible because of the finite radius of the cylindrical waveguide. Moreover, in the nonlinear regime, small as well as large amplitude compressive SWs may be formed in the bounded plasma. Later, Sayal and Sharma [22] have theoretically investigated soliton formation in presence of an electron beam propagating in a plasma-filled cylindrical waveguide. Using the SPM, Bhattacharya *et al*. [23] have studied IAS in a multi-component bounded plasma consisting positive ions, negative ions and nonthermal electrons. It is seen that when one considers nonthermal electrons and negative ions, both compressive and rarefactive IAS will be excited. But, in absence of negative ions, IAS will not exist in bounded plasma for any value of nonthermal parameter of electrons. Later, Mondal *et al*. [24] have theoretically studied linear and nonlinear propagation of ion acoustic waves in a multi-component plasma consisting of electrons, positive ions and negative ions bounded in a cylindrical waveguide. The effect of nonlinearity on the ion acoustic wave is investigated through the derivation of the effective potential. It is shown that dimensions of the cylindrical system containing plasma have a positive influence on the stability of ion acoustic waves.

However, the IADLs in bounded plasma system have not been studied by any authors till now. Since the negative ions and the nonthermal electrons have important contributions to the excitation of IAS in plasma, we are interested here to study theoretically the IADLs in multi-component plasma consisting of positive ions, negative ions and nonthermal electrons filled in a cylindrical wave guide using the SPM. The expression of Sagdeev potential in such plasma has been derived and the conditions for the existence of IADLs are obtained. The effects of boundary of cylindrical wave guide and nonthermal electrons on the Sagdeev potential has been graphically discussed. From the nonlinear equation, the solution for small amplitude IADLs are obtained and its structure is analyzed as function of the plasma parameters. It is seen that finite geometry of bounded plasma, non-thermality of electrons, density of negative ions, stream velocity of positive and negative ions have significant contribution on the structure of IADLs. The IADLs may be compressive or rarefactive in plasma depending upon the values of the plasma parameters.

9.2. BASIC EQUATIONS

We consider a perfectly conducting cylinder of radius R filled uniformly with collisionless, unmagnified, non-relativistic plasma and having an infinite axial uniform magnetic field. This plasma consists of cold positive ions, negative ions and nonthermal electrons. It is assumed that the ions have streaming velocities along the axis of the cylinder taken to be x-axis. The plasma particles are assumed to move only along the axis of the waveguide, which we choose as *x*-axis of a (r, θ, x) cylindrical coordinate system. So, there will be no interaction with the corresponding empty waveguide modes that have only transverse electric field components (H-modes) and we shall consider only the E-modes. The set of equations governing the dynamics of such bounded plasma can be written in non-dimensional form [21, 24]:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}\left(n_s u_s\right) = 0 \tag{9.1}$$

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} = -\frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x} \tag{9.2}$$

$$\nabla_{\perp}\varphi + \frac{\partial^2 \varphi}{\partial x^2} = n_e - \sum \psi_s n_s \tag{9.3}$$

where,

$$\nabla_{\perp}^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{9.4}$$

$\nabla_{\perp}$ is the transverse Laplacian, the subscript 's' stands for positive ion and negative ion, s = i for positive ions, s = j for negative ions; n_s, u_s, m_s are respectively the density , velocity and mass of positive ions and negative ions ; n_e is the number density of electrons, $\psi_i = 1$, $\psi_j = -1$, $Q_i = 1$, $Q_j = Q = m_j / m_i$, the velocities are normalized by $(K_B T_e / m_i)^{1/2}$, the densities by equilibrium ion density, the electrostatic potential φ by $(K_B T / e)$, all length by the Debye length $(K_B T_e / 4\pi n_0 e^2)^{1/2}$, K_B is Boltzmann constant.

We assume the boundary conditions

$$n_s \to n_{so}, u_s \to u_{so}, \phi \to 0 \text{ At } |x| \to \infty \tag{9.5}$$

The charge neutrality condition of the plasma is $n_{io} = 1 + n_{jo}$, where n_{i0} and n_{j0} are the equilibrium densities of positive ions and negative ions.

Now we consider the electron distribution which includes the population of nonthermal electrons. A nonthermal plasma (, cold plasma or non-equilibrium plasma) is a plasma which is not in thermodynamic equilibrium, because the electron temperature is much more than the temperature of heavy species (ions and neutrals). As only the electrons are thermalized, their Maxwell-Boltzmann velocity distribution is very different than the ion velocity distribution. When one of the velocities of a species does not follow a Maxwell-Boltzmann distribution, the plasma is said to be non-Maxwellian. In many astrophysical and space plasma scenarios, plasma particles are not in thermodynamic equilibrium but have long-range interactions. The systems with long-range interactions believed to exhibit non-Maxwellian distributions. External forces acting on plasma or interaction of wave particle may lead to the formation of non-Maxwellian distributions. The presence of high energetic plasma particles may also destroy the equilibrium state of plasma. Nonthermal distribution of electrons with excess energetic particles can be modelled by the Cairn's distribution [25, 26]. Cairns *et al.* [25, 26] have considered plasma with nonthermal electrons and cold ions and have shown that it is possible to obtain both compressive and rarefactive SWs. This nonthermal model can explain the ion acoustic structures with density depression in the magnetosphere observed by Freja [27] and Viking [28] satellites. In fact, the study of non Maxwellian plasma is crucial to the understanding of space, astrophysical and laboratory plasma dynamics.

The nonthermal electron distribution function is chosen as [25, 26]

$$f_e(V) = \frac{n_o}{(1+3\alpha)\sqrt{2\pi V_e^2}} \left[1 + \alpha\left(\frac{V}{V_e^2} - 2\phi\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{V}{V_e^2} - 2\phi\right)\right] \tag{9.6}$$

where $V_e = \sqrt{K_B T_e / m_e}$ and m_e is the mass of electron.

From (9.6) the normalized electron density n_e is obtained as

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)\exp(\phi) \tag{9.7}$$

where $\beta = \dfrac{4\alpha}{1+3\alpha}$

β measures the deviation from thermalized state α determines the number of nonthermal electrons in the plasma. β = 0 corresponds to the Boltzmann distribution of electrons.

9.3. NONLINEAR EQUATIONS AND SOLUTION OF DOUBLE LAYERS

Now we assume that the radial behaviour of the perturbed quantities is described by the lowest power of Bessel function. Following Mondal *et al*. [24] we take the perturbed quantities in the form of $J_0(k_\perp r) f(x,t)$, where $f(x,t) = N_s(x,t), U_s(x,t), \cdots$ and $k_\perp = p_{0n}/R$. Integrating the nonlinear equations over r from 0 to R after multiplying with $rJ_0(k_\perp r)$ we get

$$\frac{\partial N_s}{\partial t} + n_{s0}\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial N_s}{\partial x} + \alpha_1 N_s \frac{\partial U_s}{\partial x} + \alpha_1 U_s \frac{\partial N_s}{\partial x} = 0 \tag{9.8}$$

$$\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial U_s}{\partial x} + \alpha_1 U_s \frac{\partial U_s}{\partial x} = -\frac{\psi_s}{Q_s}\frac{\partial \varphi}{\partial x} \tag{9.9}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \varphi = N_e - \sum \psi_s n_s \tag{9.10}$$

where

$$\alpha_1 = \frac{\left(\int_0^R J_0^3 r dr\right)}{\left(\int_0^R J_0^2 r dr\right)} \tag{9.11}$$

To obtain the expression of Sagdeev potential from the Eqs.(9.8) - (9.10) we use the transformation $\eta = x - Mt$ where M is the velocity ion acoustic wave. Then from the Eq. (9.8) to Eq.(9.10) we obtain

$$U_s = \frac{N_s(M - u_{s0})}{n_{s0} + \alpha_1 N_s} \tag{9.12}$$

$$(M-u_{s0})U_s-\frac{\alpha_1}{2}U_s^2=(\frac{\psi_s}{Q_s})\varphi \qquad (9.13)$$

From Eq.(9.10) and Eq.(9.12) we get

$$N_s=\frac{n_{s0}}{\alpha_1\left[-1+\left\{1-\frac{2\alpha_1\varphi}{Q_s(M-u_{s0})^2}\right\}^{-\frac{1}{2}}\right]} \qquad (9.14)$$

Therefore, the Poisson Eq.(9.10) becomes

$$\frac{\partial^2\varphi}{\partial\eta^2}=\varphi+(1-\beta\varphi+\beta\varphi^2)\exp(\varphi)-\sum\left[\frac{\psi_s n_{s0}}{\alpha_1\left(-1+\left\{1-\frac{2\alpha_1\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}}\right)}\right]=-\frac{\partial V}{\partial\varphi} \qquad (9.15)$$

where V is the Sagdeev potential [29].

$$V=[(1+3\beta)+\sum\frac{\psi_s n_{s0}}{\alpha_1^2}Q_s(M-u_{s0})^2\log(2)]-$$

$$\frac{\varphi^2}{2}-[(1+3\beta)-3\beta\varphi+\beta\varphi^2]\exp(\varphi) \qquad (9.16)$$

$$-\sum\frac{\psi_s n_{s0}}{\alpha^2}Q_s(M-u_{s0})^2\left[\begin{array}{l}\frac{1}{2}\left\{1-\frac{2\alpha_1\varphi}{Q_s(M-u_{s0})^2}\right\}-\left\{1-\frac{2\alpha_1\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}}\\+\log\left(1+\left\{1-\frac{2\alpha_1\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}}\right)\end{array}\right]$$

Now, we expand the right hand side of Eq. (9.15) in power series of φ. For small amplitude ion acoustic wave we take the terms up to φ^3 and obtain

$$\frac{d^2\varphi}{d\eta^2}=A_1\varphi+A_2\varphi^2+A_3\varphi^3+\cdots\cdots \qquad (9.17)$$

where,

$$A_1 = \left\{ (2-\beta) + \frac{(Qn_{i0} - n_{j0})}{Q(M-u_0)} \right\} \tag{9.18a}$$

$$A_2 = \left\{ \frac{1}{2} + \frac{3\alpha_1 (Q^2 n_{i0} - n_{j0})}{Q^2 (M-u_0)^2} \right\} \tag{9.18b}$$

$$A_3 = \left\{ \frac{1+3\beta}{6} + \frac{5\alpha_1 (Q^3 n_{i0} - n_{j0})}{Q^3 (M-u_0)^6} \right\} \tag{9.18c}$$

Now, integrating Eq.(9.15) we get

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2 + V(\varphi) = 0 \tag{9.19}$$

In deriving Eq. (9.19), we have used the boundary conditions $\varphi \to 0, d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0$.

Eq. (9.19) describes the motion of a pseudo-particle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $V(\varphi)$. The first term on the right hand side of Eq. (9.19) can be regarded as the kinetic energy of the pseudo particle. Since the kinetic energy is always nonnegative, it follows that $V(\varphi) \le 0$ for the entire motion, zero being the maximum value of $V(\varphi)$. Eq. (9.19) shows that $V'(\varphi)$ is the force on the particle at the position φ . Eq. (9.19) may also be regarded as anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the IADLs solution of Eq. (9.19), the Sagdeev potential $V(\varphi)$ should be negative between $\varphi = 0$ and φ_m, where φ_m is some extreme value of the potential φ, called the amplitude of the IADLs. The IADLs solutions must additionally satisfy the following boundary conditions:

(i) $V(\varphi) = 0, V'(\varphi) = 0$ and $V''(\varphi) < 0$ at $\varphi = 0$, (9.20)

(ii) $V(\varphi) = 0, V'(\varphi) = 0$ and $V''(\varphi) < 0$ at $\varphi = \varphi_{\max}(\neq 0)$, (9.21)

(iii) $V(\varphi) < 0$, for $0 < |\varphi| < |\varphi_m|$ (9.22)

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudo particle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity vanish. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m.

Applying boundary conditions in Eq.(9.21) and Eq.(9.22) in Eq.(9.17), we find that

$$\varphi_m = -2A_2 / 3A_3 \tag{9.23}$$

and

$$V(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{9.24}$$

The IADLs solution of Eq.(9.17) satisfying Eq.(9.22) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh[\sqrt{(A_3 / 8\varphi_m)}\eta]\right) \tag{9.25}$$

where, $\varphi_m / 2 = \varphi_{0D}$, φ_{0D} is the amplitude of IADLs.

The width of the IADLs is

$$w_d = 2\sqrt{A_3 / 8|\varphi_m|}\,. \tag{9.26}$$

It is seen that the nature, amplitude and width of IADLs depend on the plasma parameters, i.e., on the nonthermal parameter (β), density of negative ions (n_{j0}), drift velocity (u_0) of ions and finite geometry (α_1) of bounded plasma through the coefficients A_2 and A_3.

It is to be noted that Eq. (9.23) relates the amplitude φ_m of IADLs to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right hand side of Eq. (9.17). The IADLs can only exist for positive value of A_3, i.e. for plasma parameters satisfying the following conditions for β, α and n_{j0}:

$$\beta > \left\{\frac{1}{3} + \frac{10\alpha_1(Q^3 n_{i0} - n_{j0})}{Q^3(M - u_0)^6}\right\}, \tag{9.27a}$$

Or,

$$\alpha_1 > \left\{ \frac{Q^3(1+3\beta)(M-u_0)^6}{30(Q^3 n_{i0} - n_{j0})} \right\} \tag{9.27b}$$

Or,

$$n_{j0} > Q^3(M-u_{s0})^6 \left\{ n_{i0} + \frac{(1+3\beta)}{30\alpha_1} \right\} \tag{9.27c}$$

where, $M - u_{s0} > 0$.

From Eqs. (9.25) and (9.23), it follows that the since $n_{e0} = \chi$ the sign of A_3 becomes positive which is condition for the existence of IADLs, so the sign of A_2 will determine the nature of the IADLs. If A_2 is positive (negative), a rarefactive (compressive) IADLs are formed. Since A_2 is independent of β, the nature of IADLs, whether compressive or rarefactive, remains invariant under changes in β. However, the negative ion density has effective role on the nature of IADLs in the bounded plasma. There is a critical value of negative ion for compressive and rarefactive IADLs.

For compressive IADLs, A_2 is negative, i.e?

$$n_{j0} > \left[\frac{Q^2(M-u_0)^2}{3\alpha_1} \left\{ \frac{1}{2} + \frac{3n_{i0}\alpha_1}{(M-u_0)^2} \right\} \right] \tag{9.28a}$$

and for rarefactive IADLs A_2 is positive, i.e.

$$n_{j0} < \left[\frac{Q^2(M-u_0)^2}{3\alpha_1} \left\{ \frac{1}{2} + \frac{3n_{i0}\alpha_1}{(M-u_0)^2} \right\} \right] \tag{9.28b}$$

It is evident that there is a critical value of negative ion density (n_{j0C}) for the occurrence of compressive and rarefactive IADLs in the bounded plasma,

$$n_{j0C} = \left[\frac{Q^2(M-u_0)^2}{3\alpha_1} \left\{ \frac{1}{2} + \frac{3n_{i0}\alpha_1}{(M-u_0)^2} \right\} \right] \tag{9.29}$$

when $M - u_{s0} > 0$.

The value of n_{j0C} can be numerically estimated if the values of other plasma parameters $Q, \beta, \alpha_1, M, u_0$ etc. are known.

9.4. RESULTS AND DISCUSSION

We now investigate numerically the IADLs in a multi-component cold drifting plasma composed of positive ions, negative ions and nonthermal electrons filled in a cylindrical wave guide. Our analysis reveals that both compressive and rarefactive IADLs will exist in the bounded plasma under consideration for selected ranges of the plasma parameters. To see the nature of IADLs in the bounded plasma, we have considered different types of negative and positive ions e.g. (O^-, Ar^+) ions, (Cl^-, He^+) ions and (O^-, H^+) ions which exist in space and also can be produced in laboratory. Moreover, we consider plasma having physically admissible values of β, i.e. $0 \le \beta < 4/3$.

9.4.1. Profiles of Sagdeev Potential in Bounded Plasma

The nature of Sagdeev potential V is important for the existence of IADLs in bounded plasma. So, we have drawn the profiles of V for different values of α_1 and β with fixed values of other plasma parameters n_{j0}, M, u_{i0}, Q.

i). The effect of α_1 :

To start with we have plotted the Sagdeev potential V shown in Fig.9.1(a), Fig.9.1(b) and Fig.9.1(c) for different values of α_1 of the bounded plasma having (O^-, Ar^+) ions for fixed values of other plasma parameters.

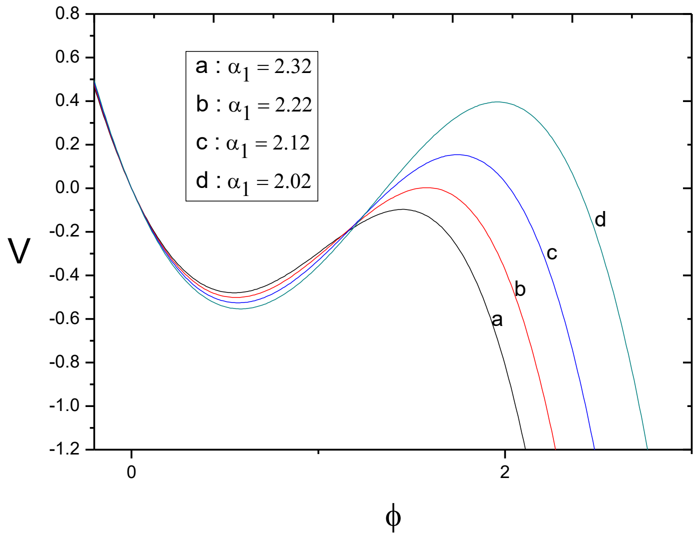

Fig.9.1 (a). The profiles of Sagdeev potential for different values of parameter α of the bounded plasma having (O^-, Ar^+) ions for fixed values of β =0.14, n_{j0} =0.12, u_{i0} =0.1, M =1.98. Graphs a, b, c and d represent α_1 = 2.32, 2.22, 2.12 and 2.02 respectively.

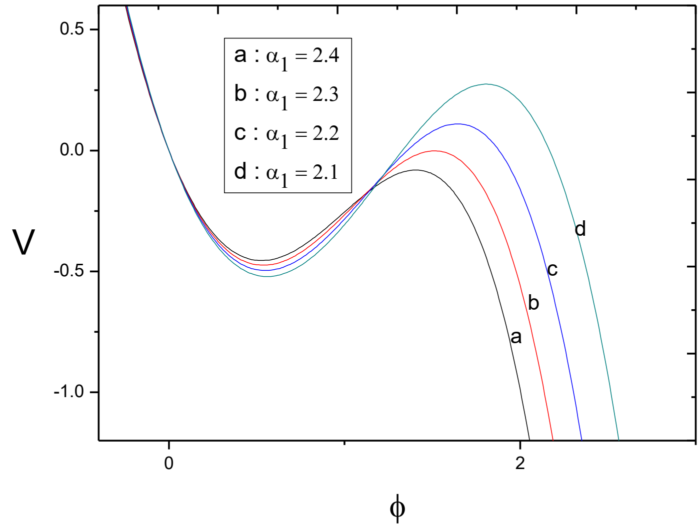

Fig.9.1 (b). The profiles of Sagdeev potential for different values of parameter α_1 of bounded the plasma having (O$^-$, Ar$^+$) ions for fixed values of β =0.1, n_{j0}=0.12, u_{i0}=0.1, M =2. Graphs a, b, c and d represent α_1 = 2.4, 2.3, 2.2 and 2.1 respectively.

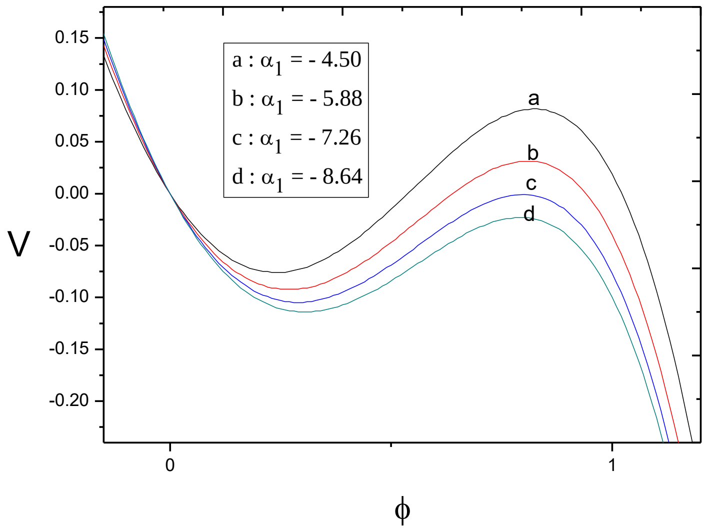

Fig.9.1(c). The Profiles of Sagdeev potential for different values of parameter α_1 of the bounded plasma having (O$^-$, Ar$^+$) ions for fixed values of n_{j0}=0.2015, β =5.065, M =4, u_{i0}=0.1. Graphs a, b, c and d represent α_1 = - 4.50, -5.88, -7.26 and -8.64 respectively.

It is seen from Fig.9.1(a) that Sagdeev potential (V) for different values of α_1 of the bounded plasma for fixed values of β =0.14, n_{j0}=0.12, u_0=0 and M =1.98 are : i) $V > 0$

for $\alpha_1 = 2.02$, ii) $V = 0$ for 2.12 and iii) $V < 0$ for $\alpha_1 = 2.22$ and 2.32. So, the IADLs are formed in bounded plasma having (O^-, Ar^+) ions when $\alpha_1 = 2.12$.

Fig.9.1(b) shows that Sagdeev potential V for different values of α_1 parameter of bounded plasma for fixed values of β =0.1, n_{j0}=0.12, u_0=0.1 and M =2 are : i) $V > 0$ for $\alpha_1 = 2.1$ ii) $V = 0$ for 2.2 and iii) $V < 0$ for $\alpha_1 = 2.3$ and 2.4. So, the IADLs are formed in bounded plasma having (O^-, Ar^+) ion when $\alpha_1 = 2.2$.

From Fig.9.1(c) it is seen that Sagdeev potential V for different values of α_1 parameter of for fixed values of n_{j0}=0.2015, β =5.065, M =4 and u_0=0.1 are: i) $V > 0$ for $\alpha_1 = -4.5$, ii) $V = 0$ for $\alpha_1 = -5.88$ and iii) $V < 0$ for $\alpha_1 = -7.26$ and -8.64 So, the IADLs are formed in bounded plasma having (O^-, Ar^+) ions when $\alpha_1 = -5.88$.

ii). The effect of β :

The effect of nonthermal parameter β of electrons on the Sagdeev potential V in the bounded plasma having (O^-, Ar^+) ions for fixed values of other plasma parameters are shown in Fig.9.2(a), Fig.9.2(b) and Fig.9.2(c).

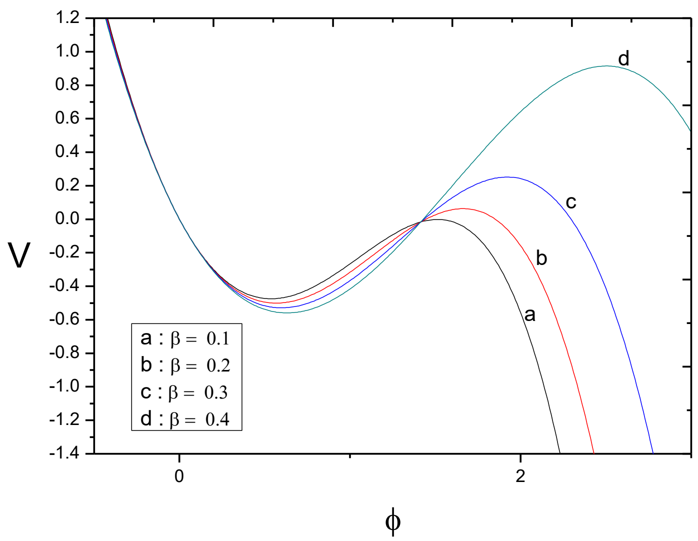

Fig.9.2 (a). The Profiles of Sagdeev potential for different values of nonthermal parameter β of electrons in the bounded plasma having (O^-, Ar^+) ions for fixed values of α_1 =2.2, n_{j0}=0.12, M =1.98 and u_{i0} =0.1. Graphs a, b, c and d represent β = 0.1, 0.2, 0.3 and 0.4 respectively.

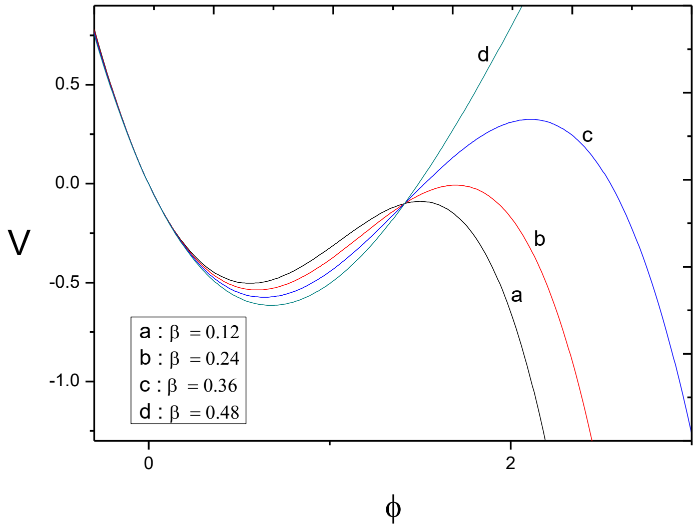

Fig.9.2 (b). The profiles of Sagdeev potential for different values of nonthermal parameter β of electrons in the bounded plasma having (O^-, Ar^+) ions for fixed values of α_1 =2.1, n_{j0} =0.12, M=1.98 and u_0 =0.1. Graphs a, b, c and d represent β = 0.12, 0.24, 0.36 and 0.48 respectively.

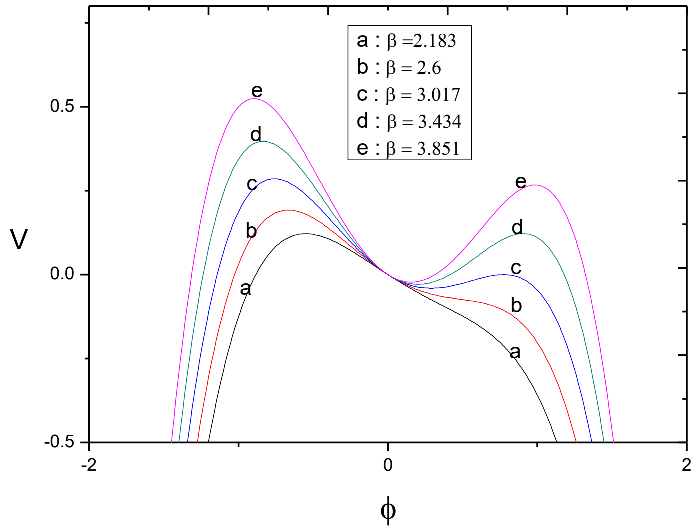

Fig. 9.2(c). The Profiles of Sagdeev potential for different values of nonthermal parameter of electrons β of electrons in the bounded plasma having (O^-, Ar^+) ions for fixed values of α_1 = - 2.8, n_{j0} = 0.12, M = 4.6 and u_{i0} = 0.1. Graphs a, b, c and d represent β = 2.183, 2.6, 3.017, 3.434, 3.851 respectively.

In Fig.9.2(a) it is observed that Sagdeev potential V for different values of β for fixed values of α_1 = 2.2, n_{j0} =0.12, M =1.98, u_{i0} =0.1 are : i) $V > 0$ for $\beta = 0.4$, ii) V = 0 for $\beta = 0.3$ and iii) $V < 0$ for $\beta = 0.2$ and $\beta = 0.1$. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ions when for $\beta = 0.3$.

Fig.9.2(b) shows that Sagdeev potential V for different values of β in bounded plasma for fixed values of α_1 =2.1, n_{j0}=0.12, M =1.98, u_{i0} =0.1 are: i) $V>0$ for $\beta=0.48$, ii) $V=0$ for $\beta=0.36$ and iii) $V<0$ for $\beta=0.24$ and $\beta=0.12$. So, the DLs is formed in the bounded plasma having (O^-, Ar^+) ions when $\beta=0.36$.

It is seen from Fig.9.2(c) that Sagdeev potential V for different values of β in bounded plasma for fixed values of α_1 = - 2.8, n_{j0}= 0.12, M = 4.6, u_{i0}= 0.1 are: i) $V>0$ for $\beta=3.434$ and $\beta=3.851$, ii) $V=0$ for $\beta=3.017$ and iii) $V<0$ for $\beta=3.183$ and $\beta=2.6$. So, the DLs is formed in bounded plasma having (O^-, Ar^+) ions when $\beta=3.017$.

9.4.2. Profiles of Ion-Acoustic Double Layers in Bounded Plasma

i). The effect of α_1 :

The effects of finite geometry of bounded medium (α_1) on the IADLs in plasma having (O^-, Ar^+) ions and (Cl^-, He^+) ions have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters.

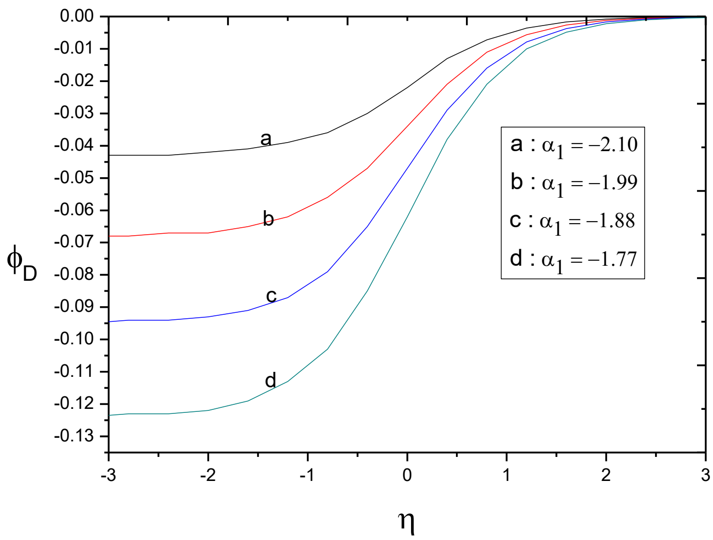

Fig.9.3 (a).The profile of IADLs for different values of α_1 in the bounded plasma having (O^-, Ar^+) ions for fixed values of n_{j0}=0.03, M =2 and u_{i0} =0.15 and β=0.5. Graphs a, b, c and d represent α_1 = -2.10, - 1.99, - 1.88, and - 1.77 respectively.

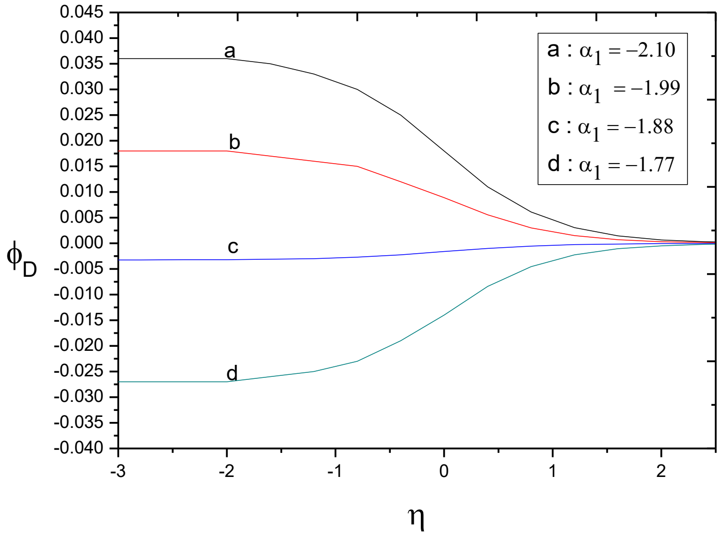

Fig.9.3 (b).The profiles of IADLs for different values of α_1 in the bounded plasma having (Cl^-, He^+) ions for fixed values of n_{j0} =0.03, M =2, u_{i0} =0.15 and β =0.5. Graphs a, b, c and d represent α_1 = -2.10, - 1.99, -1.88, and -1.77 respectively.

It is seen from Fig.9.3 (a) that the IADLs are rarefactive in plasma having (O^-, Ar^+) ions and the amplitude increases with the decrease of α_1. But, Fig.9.3 (b) shows that IADLs in plasma having (Cl^-, He^+) ions may be compressive or rarefactive depending on the values of α_1. The IADLs are compressive for α_1 = -2.10,-1.99 and rarefactive for α_1 =-1.88, -1.77. The amplitudes of compressive IADLs are decreased with the decrease of α_1 and for rarefactive DLs the amplitude increases with decrease of α_1.

ii). The effect of β:

The effect of β of nonthermal electrons on the IADLs in plasma having Q = 0.4 (O^-, Ar^+), and Q=16 (O^-, H^+) have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters i.e. dimension of finite geometry of bounded plasma, Mach number , ion stream velocity ,concentration of negative ions .

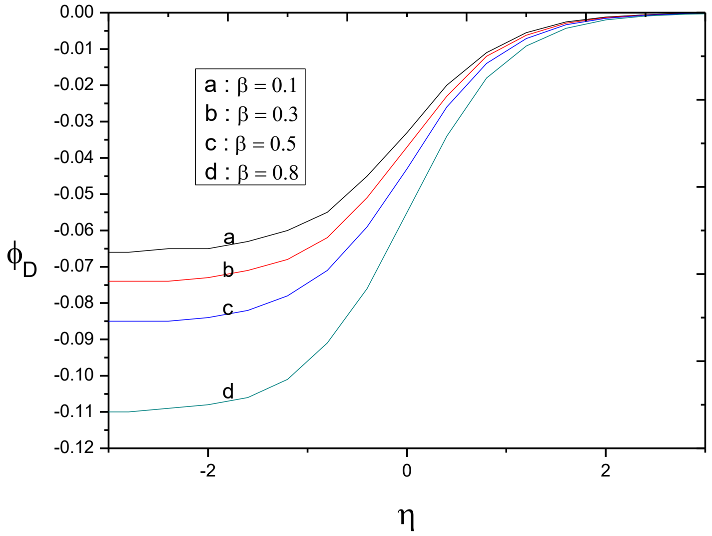

Fig. 9.4 (a). The profiles of IADLs for different values of nonthermal parameter β in the bounded plasma having (O^-, Ar^+) ions for fixed values of α_1 =-4, M =2, u_{i0} =0.2 and n_{j0} =0.1. Graphs a, b, c and d represent β = 0.1, 0.3, 0.5 and 0.8 respectively.

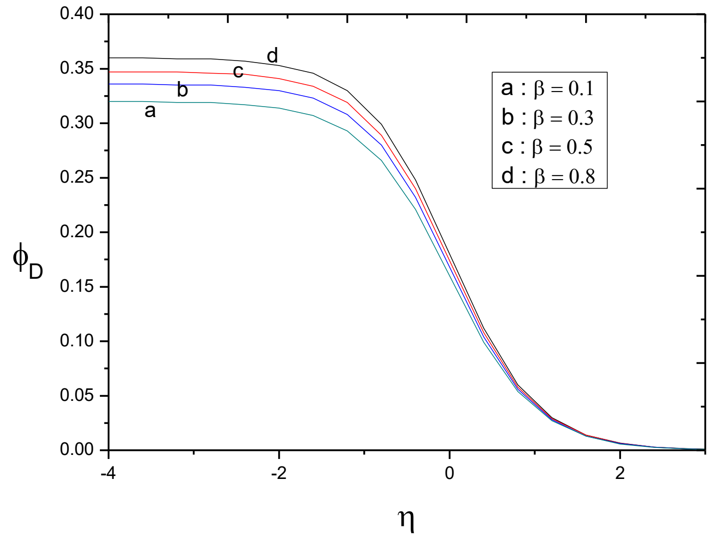

Fig.9.4(b). The profile of IADLs for different values of nonthermal parameter β in the bounded plasma having (O^-, $H^{+)}$ ions for fixed values of α_1 =-4, M =2, u_{i0} =0.2 and n_{j0} =0.1. Graphs a, b, c and d represent β = 0.1, 0.3, 0.5 and 0.8 respectively.

It is seen form Fig.9.4(a) that the IADLs are rarefactive in plasma having (O^-, Ar^+) ions and the amplitude of IADLs increases with the increase of β for fixed values of α_1 =-4, M =2, u_{i0} =0.2, n_{j0} =0. But, Fig.9.4(b) shows that the IADLs in plasma having (O^-, H^+) ions are compressive and the amplitude of IADLs increases with the increase of β.

9.4.3. Profiles of Width for Ion-Acoustic Double Layers in Bounded Plasma

It is well known that the width of IADLs in plasma is important for the experimental studies. So, we have numerically estimated the width of IADLs using Eq. (9.26) and the variations of width are graphically discussed for different parameters of bounded plasma system.

i). The effect of α_1 :

The variation of width of IADLs with negative ion density (n_{j0}) for different values of α_1 are shown in Fig. 9.5(a) and 9.5(b) for the plasma having (O^-, Ar^+) ions and (Cl^-, He^+) ions.

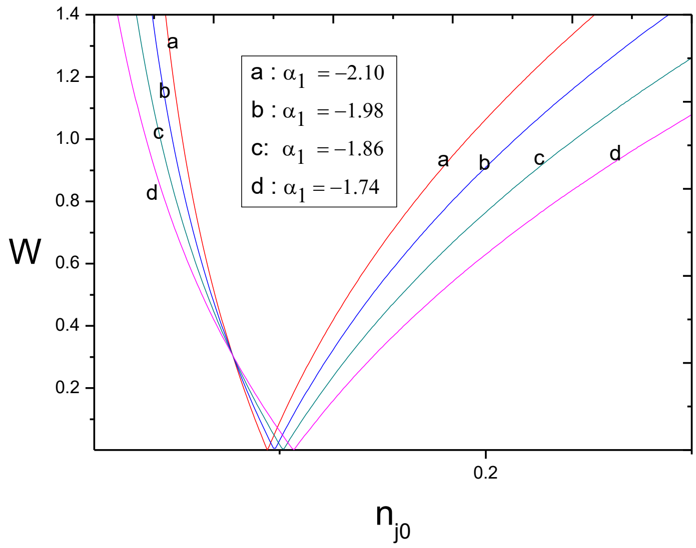

Fig.9.5(a). Variation of width of IADLs with negative ion density n_{j0} for different values of α_1 in the bounded plasma having (O^-, Ar^+) ions for fixed values of M =2, β =0.1 and u_{i0} =0.2. Graphs a, b, c and d represent α_1 = -2.10 ,- 1.98, -1.86, and - 1.74 respectively.

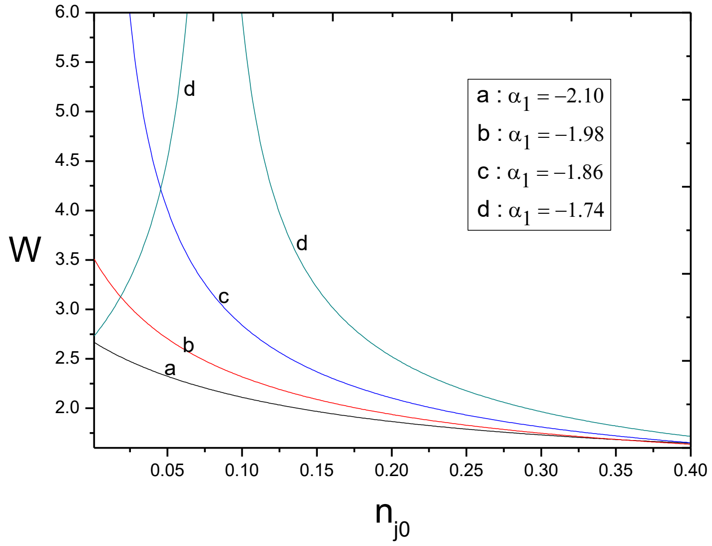

Fig.9.5(b).Variation of width of IADLs with negative ion density (n_{j0}) for different values of α_1 in the bounded plasma having (Cl^-, He^+) ions for fixed values of M =2, β =0.1 and u_{i0} =0.2. Graphs a, b, c and d represent α_1 = -2.10 ,- 1.98, -1.86, and - 1.74 respectively.

It is seen from Fig. 9.5(a) for the plasma having (O^-, Ar^+) ions that the width of IADLs decreases with increase of n_{j0} up to a critical negative ion density n_{j01} then it increases steadily with the increase of $n_{j0.}$ More over, the width of IADLs is larger for large value α_1. The variation of width of IADLs in plasma with (Cl^-, He^+) ions is shown in Fig.9.5(b). It is observed that the width decreases with n_{j0} , but for small α_1, $\alpha_1 = -1.74$, the width increases with n_{j0} and attain a maximum value at negative ion density n_{j01} then it decreases with increase of n_{j0}. Moreover, the width of IADLs is large for small value of α_1.

ii). The effect of β :

Similarly, the variation of width of IADLs with negative ion density (n_{j0}) for different values of nonthermal parameter of electrons (β) are shown in Fig. 9.6 (a) and Fig.9.6 (b) for the plasma having, (O^-, Ar^+) ions and (Cl^-, He^+) ions.

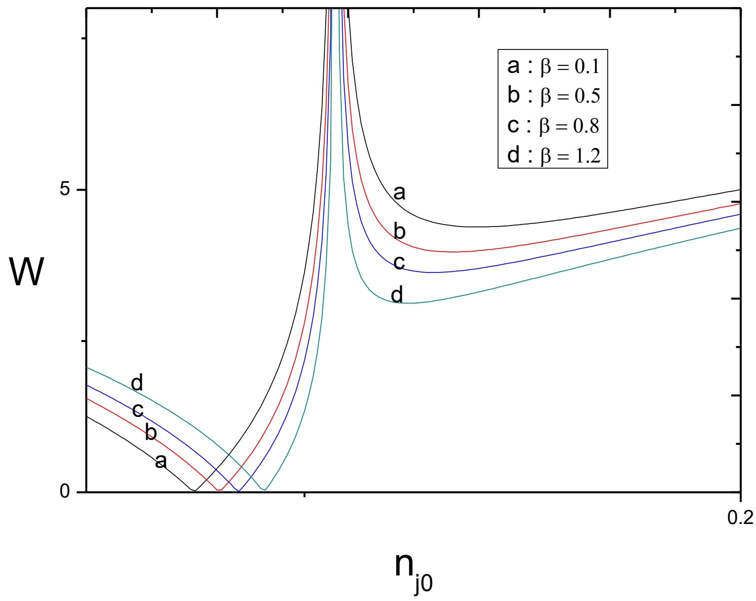

Fig.9.6(a). Variation of width of IADLs with negative ion density for different values of nonthermal parameter β in the bounded plasma having (O^-, Ar^+) ions for fixed values of M =2, α_1 = - 4 and u_{i0} =0.2. Graphs a, b, c and d represent β = 0.1, 0.5, 0.8 and 1.2 respectively.

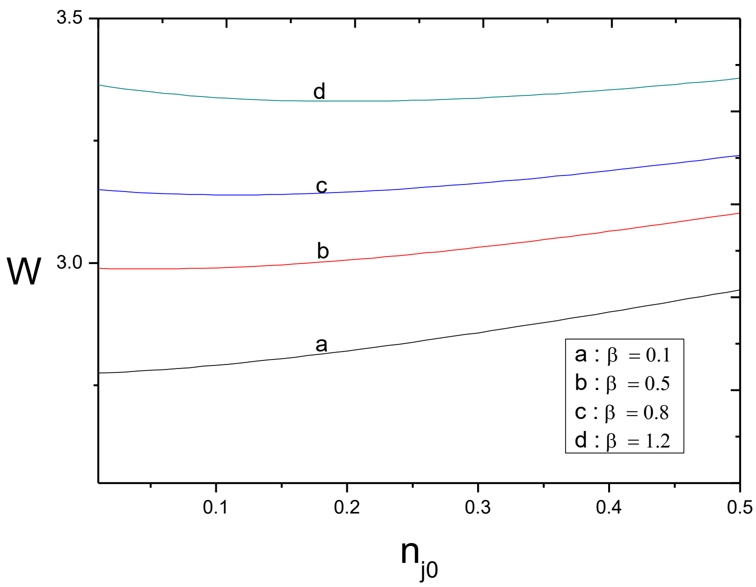

Fig.9.6 (b).Variation of width of IADLs with negative ion density for different values of values of β in the bounded plasma having (O^-, H^{+}) ions for fixed values of M =2, α_1 = - 4, u_{i0} = 0.2. Graphs a, b, c and d represent β = 0.1, 0.5, 0.8 and 1.2 respectively.

In Fig.9.6 (a) it is seen that the width of IADLs decreases with increase of n_{j0} up to negative ion density $n_{j0}(=n_{j01})$. Then it increases steadily with the increase of n_{j0} up to certain maximum value of DL width at negative ion density $n_{j0}(=n_{j02})$. After that the DL width decreases and attains certain values at negative ion density $n_{j0}(=n_{j03})$ then it again increases with the increase of n_{j0}. Moreover, at n_{j01} the width of IADLs is larger for large value of β, at n_{j03} the width of IADLs is small for large value β. But, it is observed from Fig.9.6(b) that the width increases with increase of n_{j0} and the width is large for large value β.

9.5. SUMMARY AND CONCLUSION

Considering multi-component plasma composed of cold positive ions, negative ions and warm nonthermal electrons bounded in a finite geometry cylindrical tube we have theoretically studied the excitation of IADLs in the plasma. It has been shown that formation of IADLs in bounded plasma depends on the plasma parameters i.e. finite geometry of the bounded medium, nonthermal parameters of electrons, density of negative ions, and stream velocity of ions. Numerical estimation is made taking different types positive and negative ions in the plasma and the profiles of IADLs are drawn. The IADLs in the bounded multi-component plasma may be compressive or rarefactive depending upon the values of the plasma parameters.

Finally, we would like to point out that the type of plasma considered in the present work exists in space and also can be produced in the laboratory. By our present analysis, ion acoustic IADLs can be studied in bounded plasma (e.g. dusty plasma and different type's electrons, namely, two-temperature electrons, superthermal electrons, Tsallis distributed electrons etc.). In the present study we have not considered the temperatures of positive ions and negative ions and the static magnetic field in the plasma. Considering these parameters more interesting results may be obtained which would give new information on the IADLs in bounded plasma. The DLs in plasma have been observed experimentally [11-14]. But, there is no information about experimental observation on the IADLs in bounded multi-component plasma having nonthermal electrons. So, our theoretical results may not be verified / compared with the experimental observations.

To study the IADLs in bounded multi-component plasma we have used the SPM. Using the cylindrical co-ordinates and perturbation method one can study the IADLs in bounded plasma following Das *et al*.[15] , Labany *et al*. [17] , and also Paul *et al*. [18 and Mondal *et al* [19] of cartesian co-ordinates.

REFERENCES

[1]. S. Watanabe, J. Phys. Soc. Japan, **53,**. p.950 (1984).

[2]. M. K. Mishra, R. S. Tiwari and J. K. Chawla, Phys.Plasmas, **19,** p.062303 (2012).

[3]. K. Kumar and M. K. Mishra, AIP Advances, **7,** p.115114 (2017).

[4]. R A Cairns, A A Mamun, R Bingham and P K Shukla, Physica Scripta, **T63** , p.80 (1996).

[5].B. Ghosh, S. N. Paul, C. Das, I. Paul and S. Banerjee,Braz. J. Phys., **43**, p.28 (2013).

[6].B.Ghosh, S.Banerjee and S.N.Paul, Ind. J.Pure and Appl. Phys.,**51** , p.488 (2013).

[7].Indrani Paul , S. Chandra ,S. Chattopadhyay and S. N. Paul, Indian J. Phys., **90**, p.1195 (2016)

[8].S. Chattopadhyay, Sri Lankan J. Phys., **20,** p.1 (2019).

[9].K. E. Lonngren, Plasma Phys., **25**, p.943 (1983).

[10].Y.Nakamura and H.Sugai, Chaos, Solitons and Fractals, **7**, p.1023 (1996).

[11].S. Torvén, Formation of Double Layers in Laboratory Plasmas ; Wave Instabilities in Space Plasmas , pp. 109-128 (1979).

[12].N. Hershkowitz, Review of recent laboratory double layer experiments, Space Science Reviews, **41,** p**.** 351 (1985)..

[13].M.A.Raddu, Phys. Reports, **178**, p.25 (1989.

[14]. C Charles, Plasma Sources Sci. Technol., **16,** p.R1–R25(2007).

[15].G. C. Das, Kh. I. Singh and B. Karmakar , Canadian J. Phys., **67(6),** p.609 (1989).

[16].G.C. Das and Kh .Ibohanbi. Singh, ,Aust. J. Phys., **44**(5), p.523 (1991).

[17].S. K. EL-Labany , S. A. EL-Warraki and W. M. Moslem, J. Plasma Phys.,**63(4),** p**.**343 (2000).

[18].B.Ghosh and K.Das, J.Plasma Phys., **40**, p.545 (1988).

[19].S.Kaur , Spherical and cylindrical ion-acoustic solitary waves in electron-positron-ion plasmas, Ph.DThesis-2017, Punjab Agricultural University

[20]. B.Sahu and D. Roy, Phys.Plasmas, **24**, p.112705 (2017).

[21].V. K. Sayal and S. R. Sharma , Phys. Scr,. **42,** p.475(1990).

[22]. V.K.Sayal and S.R.Sharma, Phys. Letts A, **149**, p.155 (1990).

[23].S. K. Bhattacharya and S N Paul and B Chakraborty, Indian J. Phys. **77B** (3), p.327 (2003).

[24].K.K. Mondal, S. K.. Bhattacharya and S. N. Paul, FIZIKA, **A 13** , p.77 (2004)

[25].R.A. Cairns, R. Bingham, R.O. Dendy, C.M.C. Nairn, P.K. Shukla, A.A. Mamun, J.Phys. IV France , **5,** C6–43 (1995).

[26].R.A. Cairns, A.A. Mamun, R. Bingham, R. Bostrom, R.O. Dendy,C.M.C. Nairn, P.K. Shukla, Geophys. Res. Letts., **22**, p.2709 (1995).

[27].R. Bostrom, G. Gustafsson, B. Holback, G. Holmgren, H. Koskinen, P. Kintner, Phys. Rev. Letts., **61**, p.82 (1988).

[28].P.O. Dovner, A.I. Erikson, R. Bostrom, B. Holback, Geophys.Res.Letts., **21,** p.1827 (1994)

[29].R.Z.Sagdeev, in Reviews of Plasma Physics, ed. by M.A. Leontovich (Consultants Bureau, New York, 1996), **4**, p. 23.

[30].S.N. Paul, K. K. Mondal and A. Roychowdhury, FIZIKA, **B 5**, p.73 (1996).

[31].K. K. Mondal, S. N. Paul, and A. Roychowdhury, IEEE Trans. Plasma Sci., **26,** p.987 (1998).

CHAPTER-10

INFLUENCE OF NONTHERMAL ELECTRONS AND ION- BEAMS ON THE ION-ACOUSTIC DOUBLE LAYERS IN WARM ION PLASMA

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10.1. INTRODUCTION

In recent years, there is a great deal of interest on the study of nonlinear propagation of waves in plasma consisting of beams of electron / ions because of its importance in understanding the nonlinear electrostatic disturbances in magnetosphere and solar plasma [1, 2]. The nonlinear structure of plasma is changed considerably in presence of beam of electron / ion. Krivoruchku *et al*. [3] has shown that an electron beam in plasma can change the propagation of the Trivelpiece-Gould solitons. Later, it is shown by Gell *et al*. [4] that compressive solitary waves are excited in an electron beam plasma system. However, the effects of ion-beam are also important to excite the ion- acoustic soliton (IAS) in plasma. Abrol and Tagare [5] have obtained a modified Koteweg-deVries (KdV) equation for IAS in presence of ion-beam in plasma composed of cold ions, beam ions and nonisothermal electrons using the reductive perturbation method (RPM). Later, Karmakar *et al*. [6], El-Labany[7], Das [8] and other authors [9-10] have obtained the solution of IAS in an ion-beam plasma through the derivation of a modified KdV equation. It is seen that the existence solitons and its behaviour depends effectively on the ion-beam as well as on other plasma parameters. Recently, Paul *et al*. [11] have studied nonlinear propagation of ion-acoustic waves (IAW) in unmagnetized quantum (degenerate) plasma in presence of an ion beam using the one dimensional quantum hydrodynamic model. They have derived the KdV equation using the RPM and have obtained the solution of IAS. The critical value of ion-beam density for the nonexistence of solitary wave is numerically estimated and its variation with velocity and temperature of ion-beam has been discussed graphically. It is seen that formation and structure of IAS are significantly affected by the ion-beam in quantum plasma. The IAS in the quantum plasma may be compressive or rarefactive depending upon the values of velocity, density and temperature of the ion-beam.

But, few authors have studied the ion-acoustic double layers (IADLs) in ion-beam plasma. Kaur *et al* [12] have derived the KdV, modified KdV and Gardner equations using the RPM and studied the acoustic Gardner solitons and DLs in a multi-component superthermal plasma having ion-beam. They have discussed the characteristic features of both compressive and rarefactive nonlinear excitations from the solution of these equations. On the other hand, the presence of non-Maxwellian nonthermal electrons [13,14] give rise to interesting characteristics in IADLs in the plasma [15-19].

It is seen from the works of above authors that a lot of theoretical works have been done on the IAS in the plasma having beams of nonthermal electrons and ions. But, IADLs in ion-beam plasma has not been studied by any authors. So, we are interested here to study the IADLs in a multi-component plasma composed of warm positive ions and beam ions together with nonthermal electrons. To solve the problem we have used the Sagdeev potential method (SPM) for deriving the expression of Sagdeev potential and nonlinear equation for the propagating IAW. The solution for IADLs is obtained and graphically discussed for different values of density, velocity and temperature of the ion-beam, and non-thermality of electrons. The widths of the IADLs are also discussed graphically.

10.2. BASIC EQUATIONS

We assume**,** a collisionless unmagnetized plasma consisting of warm positive, beam ions and nonthermal electrons. Moreover, we consider the constant ion-beam velocity in unperturbed state and ion-beams are in the direction of propagation of waves. The normalized basic equations in unidirectional propagation for such plasma in dimensionless form are:

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}\left(n_s u_s\right)=0 \tag{10.1}$$

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}+\frac{\sigma_s}{Q_s n_i}\frac{\partial p_i}{\partial x}=\psi_s\frac{Z_s}{Q_s}\frac{\partial \varphi}{\partial x} \tag{10.2}$$

$$\frac{\partial p_s}{\partial t}+u_s\frac{\partial p_s}{\partial x}+3p_s\frac{\partial u_s}{\partial x}=0 \tag{10.3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+\sum_s \psi_s Z_s n_s \tag{10.4}$$

where, the subscript s = i and b denote positive ions and beam ions; n_s, u_s, p_s, σ_s are respectively the density , velocity, pressure and temperature of positive ion and beam ions; The concentration of nonthermal electrons is represented by n_e, $Q_b = m_b / m_i$, $Q_i = 1$, m_i and

m_b are the mass of positive ions and beam ions, $\psi_i = \psi_b = -1$, Z_i and Z_b are the charges of positive ions and ion-beam and negavtive ions. For single charged ions, $Z_i = 1$, multiple charged ion-beams $Z_b = Z$, Z is the number of positive charges of ion-beams. The velocities are normalized by the ion-acoustic speed $(K_B T_e / m_i)^{1/2}$, the densities by the equilibrium ion density n_0, all lengths by the Debye length $(K_B T_e / 4\pi n_0 e^2)^{1/2}$, the time variable by the ion plasma period $\omega_p^{-1} = (m_i / 4\pi n_0 e^2)^{1/2}$ and the potential φ by $(K_B T_e / e)$ where , T_e is the temperature of electron, K_B is the Boltzmann constant and other symbols have their usual meanings.

The nonthermal electron distribution function is chosen as [15, 16]

$$f_e(V) = \frac{n_o}{(1+3\alpha)\sqrt{2\pi V_e^2}}\left[1+\alpha\left(\frac{v}{V_e^2}-2\phi\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{v}{V_e^2}-2\phi\right)\right] \tag{10.5}$$

where α is a parameter that determines the proportion of the fast energetic electrons, v is the velocity of electrons in phase space and is normalized by the average thermal speed $V_e = \sqrt{K_B T_e / m_e}$ and m_e is the mass of electron.

The normalized density of nonthermal electron n_e is

$$n_e = \left(1 - \beta\varphi + \beta\varphi^2\right)\exp(\varphi) \tag{10.6}$$

where $$\beta = \frac{4\alpha}{1+3\alpha} \tag{10.6a}$$

β measures the deviation from thermalized state.

For obtaining the solution of IADLs from Eq.(10.1) to (10.4) we use the transformation $\eta = x - Vt$, where V is the velocity of the IAW and we assume the boundary conditions:

$$n_i \to n_{io}, n_b \to n_{bo}, u_i \to 0, u_b \to u_{bo}, p_i \to p_{io}, p_b \to p_{bo}, \varphi \to 0 \text{ at } |x| \to \infty \tag{10.7}$$

The charge neutrality condition in the plasma is $n_{i0} = 1 - Z n_{b0}$.

By using Eq. (10.4) and the boundary conditions (10.7) we get from Eqs. (10.1) to (10.5)

$$\frac{d^2\phi}{d\eta^2}=\left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi)$$

$$-\frac{n_{io}^{\frac{3}{2}}}{\sqrt{12\sigma_i p_{io}}}\left[\sqrt{\left\{\left(V+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}-\sqrt{\left\{\left(V-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}\right]$$

$$-\frac{Zn_{bo}^{\frac{3}{2}}Q^{\frac{1}{2}}}{\sqrt{12\sigma_b p_{bo}}}\left[\sqrt{\left\{\left(V-u_{bo}+\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^2-\frac{2Z}{Q}\varphi\right\}}-\sqrt{\left\{\left(V-u_{bo}-\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^2-\frac{2Z}{Q}\varphi\right\}}\right] \tag{10.8}$$

$$=-\frac{d\psi}{d\varphi} \tag{10.9}$$

where $\psi(\phi)$ is known as the Sagdeev Potential and it is given by

$$\psi(\phi)=(1+3\beta)-[(1+3\beta)-3\beta\varphi+\beta\varphi^2]\exp(\varphi)$$

$$-\frac{n_{io}^{\frac{3}{2}}}{6\sqrt{3\sigma_i p_{io}}}\cdot\left[\begin{array}{l}\left\{\left(V+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}-\left\{\left(V-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}^{\frac{3}{2}}\\ -\left(V+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^3+\left(V-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^3\end{array}\right]$$

$$-\frac{Q^{\frac{3}{2}}n_{bo}^{\frac{3}{2}}}{6\sqrt{3\sigma_b p_{bo}}}\left[\begin{array}{l}\left\{\left(V-u_{bo}+\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^2-\frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}}-\left\{\left(V-u_{bo}-\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)-\frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}}\\ +\left\{\left(V-u_{bo}+\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^3\right\}-\left\{\left(V-u_{bo}-\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^3\right\}\end{array}\right] \tag{10.10}$$

Integrating (10.9) we obtain

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2+\psi(\varphi)=0 \tag{10.11}$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi \to 0, d\varphi / d\eta \to 0$ and $d^2\varphi / d\eta^2 \to 0.$

For the solutions of IADLs, $\psi(\varphi)$ must additionally satisfy the boundary conditions mentioned in Chapter-9. When the conditions are satisfied, $\varphi = 0$ at the unstable equilibrium position. The pseudo particle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity are vanished. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m .

10.3. SOLUTION FOR ION-ACOUSTIC DOUBLE LAYERS

Now we expand the right hand side of Eq. (10.8) and Eq. (10.10) in power series of ϕ and we obtain

$$\frac{d^2\varphi}{d\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + \cdots \tag{10.12}$$

where

$$A_1 = \left[\begin{array}{l} (1-\beta) - \dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-1} - \left(V - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-1}\right\} \\ -\dfrac{Z^2 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{1}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V - u_{bo} + \sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-1} - \left(V - u_{bo} - \sqrt{\dfrac{3\sigma_b p_{jo}}{Qn_{bo}}}\right)^{-1}\right\} \end{array}\right] \tag{10.13a}$$

$$A_2 = \frac{1}{2}\left[\begin{array}{l} 1 - \dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3} - \left(V - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}\right\} \\ -\dfrac{Z^3 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V - u_{bo} + \sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-3} - \left(V - u_{bo} - \sqrt{\dfrac{3\sigma_b p_{jo}}{Qn_{bo}}}\right)^{-3}\right\} \end{array}\right] \tag{10.13b}$$

$$A_3 = \frac{1}{6}\left[\begin{array}{l} (1+3\beta) + \dfrac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5} - \left(V - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5}\right\} \\ + \dfrac{3Z^4 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V - u_{bo} + \sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-5} - \left(V - u_{bo} - \sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-5}\right\} \end{array}\right] \tag{10.13c}$$

And

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2 - \frac{1}{3}A_2\varphi^3 - \frac{1}{4}A_3\varphi^4 \cdots\cdots \tag{10.14}$$

For small amplitude ion acoustic waves ($\varphi < 1$), we take the terms up to φ^3 of Eq. (12) to get the solution of IADLs in plasma. Applying the boundary conditions mentioned in Chapter-9 [Eqs. (9.20) to (9.22)] for the existence of IADLs we obtain

$$\varphi_m = -\frac{2A_2}{3A_3} \tag{10.15}$$

And

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{10.16}$$

The small amplitude IADLs solution of Eq. (10.12) satisfying Eq. (10.15) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh[\sqrt{(A_3/8\varphi_m)}\eta]\right) \tag{10.17}$$

The width of IADLs is given by

$$w_d = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{10.18}$$

It is seen that width of IADLs depends on the plasma parameters e.g. ion-beam density, ion-beam temperature, velocity of ion-beams and nonthermal parameter of electrons. Eq. (10.15) relates the amplitude φ_m of the IADL to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (10.12). It is important to be noted that the IADLs in plasma can only exist for positive value of A_3 i.e. $A_3 > 0$. To satisfy the condition, the plasma parameters should have some critical values. The nonthermal parameter β must satisfy the condition $\beta < \beta_C$, β_C being the critical value of β. for the existence of IADLs, the parameter β_C obtained from (10.13c) is

$$\beta_C = \left(\frac{1}{3}\right)\left[\frac{3n_{i0}}{2\sigma_1}\left\{(V-\sigma_1)^{-5}-(V+\sigma_1)^{-5}\right\}+\frac{3n_{b0}}{2Q^{\frac{5}{2}}\sigma_2}\left\{\left(V-u_{bo}-\sigma_2\right)^{-5}-\left(V-u_{bo}+\sigma_2\right)^{-5}\right\}-1\right] \quad (10.19)$$

where, $\sigma_1 = \sqrt{3\sigma_i p_{i0}/n_{i0}}$, $\sigma_2 = \sqrt{3\sigma_b p_{b0}/n_{b0}}$ and $n_{i0} = 1 - Zn_{b0}$.

The parameter β_c for the existence of small amplitude IADLs in plasma can be numerically estimated from Eq. (10.19), if the values of other parameters of plasma are known. Similarly, for the existence of IADLs when $A_3 > 0$, the critical values of ion-beam density (n_{b0C}), ion-beam velocity (u_{b0C}) and ion-beam temperature (σ_{bC}) can be obtained from Eq. (10.13c).

If IADLs are excited in plasma, it is very important to see the nature of IADLs (compressive and rarefactive). From Eq. (10.15) and Eq. (10.17), it follows that the sign of A_2 determines the nature of IADLs. If A_2 is positive (negative), a rarefactive (compressive) IADL is excited. Since A_2 is independent of β , the nature of IADLs, whether compressive or rarefactive, remains invariant under changes of β. But, it is seen that A_2 depends on the parameters n_{b0}, u_{b0} and σ_b . For compressive IADLs, $A_2 < 0$, and for rarefactive IADLs, $A_2 > 0$. Expressions for the critical values of $n_{b0}(n_{b0C})$, $u_{b0}(u_{b0C})$ and $\sigma_b(\sigma_{bC})$ for compressive and rarefactive IADLs in warm plasma can be obtained from Eq. (10.13b). In warm plasma, it is difficult to get the expressions of n_{b0C}, u_{b0C} and σ_{bC}. For cold plasma, $\sigma_i = 0$ and $\sigma_b = 0$, the critical values of n_{b0C} and u_{b0C} for compressive and rarefactive IADLs are obtained from Eq. (10.13b):

$$n_{b0C} = \frac{Q^2(V^4-3)(V-u_{bo})^4}{3[V^4 - Q^2(V-u_{b0})^4]} \quad (10.20)$$

and

$$u_{b0C} = V\left[1-\left\{\frac{3n_{b0}}{V^4-3+3n_{b0}}\right\}^{\frac{1}{4}}\right] \tag{10.21}$$

When,

i) $n_{b0} < n_{b0C}$ or $u_{b0} < u_{b0C}$, positive potential (compressive) IADLs will be formed,

ii) $n_{b0} > n_{b0C}$ or $u_{b0} > u_{b0C}$, negative potential (rarefactive) IADLs will be excited.

The critical values of ion-beam density (n_{b0C}) and ion-beam velocity (u_{b0C}) for the excitation of IADLs in cold plasma can be numerically estimated from Eq. (10.20) and Eq. (10.21), if the values of other plasma parameters are known.

10.4. RESULTS AND DISCUSSION

From Eq. (10.17) we see that the plasma parameter e.g. ion-beam density, ion-beam temperature, ion-temperature, ion-beam velocity and nonthermality of electrons have significant influence on the excitation of IADLs in ion-beam plasma. To ascertain the actual nature of the IADLs we have numerically evaluated for various parameters and discussed graphically. The Figures in the article illustrate the results of evaluations of the characteristics of the IADLs that occur in ion-beam plasma, and are obtained by using Eq. (10.17). The evaluated curves are the dimensionless dependences of the values of the evaluated characteristics of IADL excitations, along the axes of the ordinates and abscissa as of which the values of the dimensionless quantities in arbitrary units are used.

10.4.1. Profile of Ion-Acoustic Double Layers

To start with we have plotted the IADLs potential with the plasma parameters:

i). Nonthermal parameter of electron shown in Figs. 10.1(a) and 10.1(b)

ii). Ion-beam density shown in Figs. 10.2(a) and 10.2(b)

iii). Ion-beam temperature shown in Figs. 10.3 (a) and 10.3(b)

iv). Ion-beam velocity shown in Figs. 10.4 (a) and 10.4(b)

i). Effect of nonthermal parameter of electron:

The effects of nonthermal electrons on the IADLs in the beam plasma having H^+ ion- beam and Ar^+ ion-beam has been numerically estimated and the normalized potential of IADLs are shown Fig.10.1(a) and 10.1(b) for the fixed values of other normalized plasma parameters. We consider the plasma having the values of nonthermal parameter of electron which is less than 1.3 (β<1.3).

To see the effects of nonthermal parameter electrons (β) on the IADLs in plasma having H^+ beams and Ar^+ beams, we have followed the conditions for the existence of IADLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of IADLs given by Eq. (10.15) is first numerically estimated using the values of parameters of our model plasma. Numerical estimation shows that the amplitude of IADLs having H^+ beam is positive and Ar^+ beam is negative, which gives compressive and rarefactive IADLs in plasma respectively. The positive and negative values of amplitude of IADLs depend on the critical values of β which is obtained from the coefficient A_2 of Eq. (10.13b). Using Eq. (10.17), the profiles of IADLs for H^+ beams and Ar^+ beams are drawn as shown in Fig.10.1 (a) and Fig. 10.1(b).

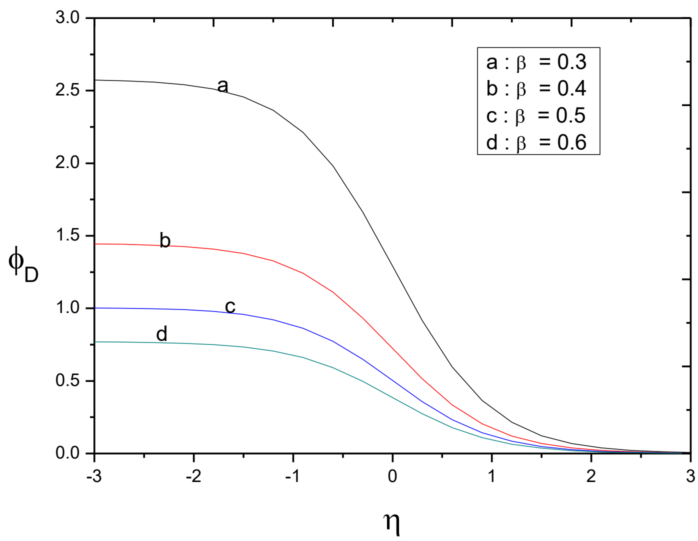

Fig.10.1(a). Profiles of the IADLs for different values of nonthermal parameter (β) in H^+ ion-beam plasma having σ_i =0.001, σ_b =0.003, V =1.6, n_{b0} =0.1 and u_{b0} =0.2; Graphs a, b, c, and d represent β = 0.3, 0.4, 0.5 and 0.6 respectively.

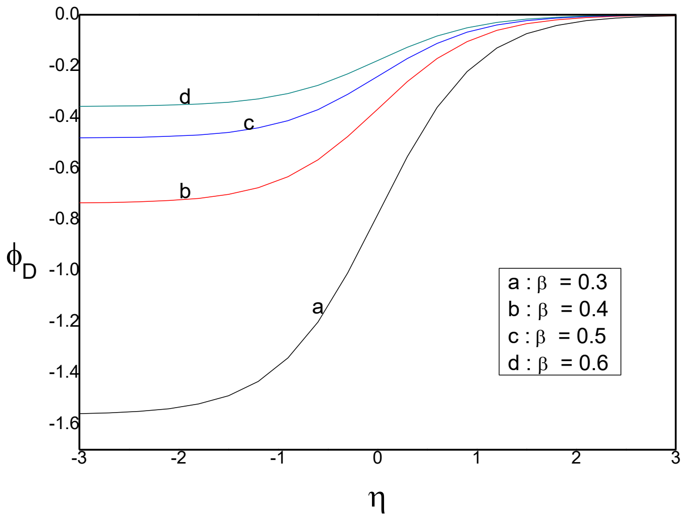

Fig.10.1(b). Profiles of the IADLs for different values of nonthermal parameter (β) in Ar^+ ion-beam plasma having σ_i =0.001, σ_b =0.003, V =1.6, n_{b0} =0.3 and u_{b0} =0.9; Graphs a, b, c, and d represent β = 0.3, 0.4, 0.5 and 0.6 respectively.

It is seen from Fig.10.1(a) that the coefficient A_2 is negative so compressive IADLs are excited in plasma having H^+ ion-beam for β = 0.3, 0.4 , 0.5 and 0.6 with fixed values of σ_i = 0.001, σ_b = 0.003, V =1.6, n_{b0} = 0.1 and u_{b0} = 0.2. It is also observed that the amplitudes of compressive IADLs decreases with the increase of β.

In Fig10.1(b), the Ar^+ ion-beam in plasma shows that A_2 is positive when β = 0.3, 0.4, 0.5 and 0.6 with fixed values of σ_i =0.001, σ_b =0.003, V =1.6, n_{b0} =0.3 and u_{b0} =0.9. Hence, the amplitude of IADLs becomes negative giving rise to rarefactive IADLs. The amplitudes of rarefactive IADLs are decreased with the increase of the nonthermal parameter β .

ii). Effect of ion-beam density: The effects of ion-beam density (n_{b0}) on the IADLs in plasma having H^+ ion-beams and Ar^+ ion-beams are understood from the numerical estimation of amplitude and electrostatic potential given by Eq.(10.15) and Eq.(10.17). From these numerical estimation using the values of parameters of our model plasma, it is observed that the amplitude and electrostatic potential IADLs are negative for both H^+ ion-beams and Ar^+ ion-beams giving rise to rarefactive IADLs shown in Fig.10.2(a) and Fig.10.2(b).

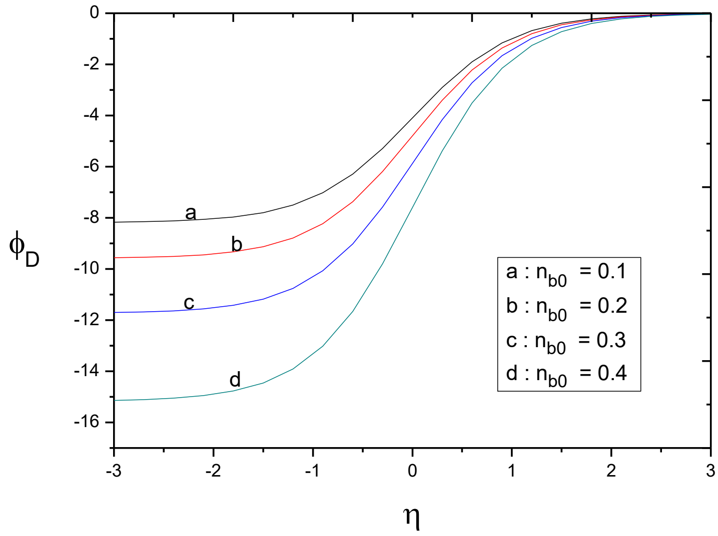

Fig.10.2(a). Profiles of the IADLs for different values of ion- beam density (n_{b0}) of H^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6, β =1.1 and u_{b0} =0.4; Graphs a, b, c, and d represent n_{b0} = 0.1, 0.2, 0.3 and 0.4 respectively.

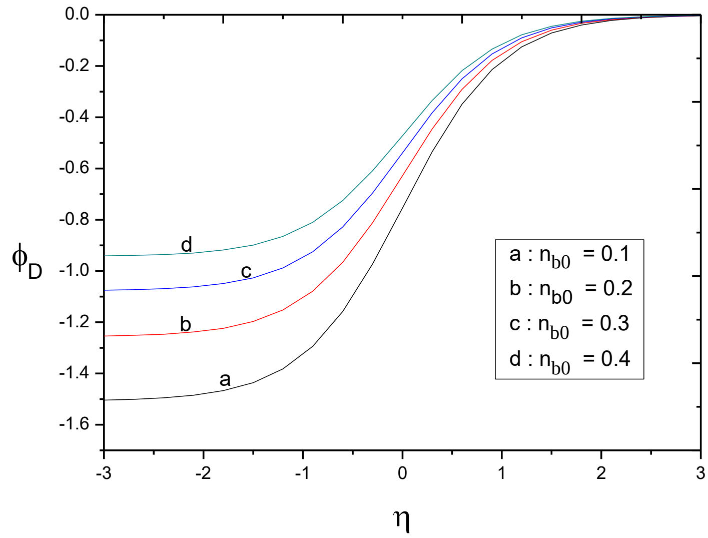

Fig.10.2(b). Profiles of the IADLs for different ion- beam density (n_{b0}) of Ar^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6, β = 0.3 and u_{b0} =0.4; Graphs a, b, c, and d represent n_{b0} = 0.1, 0.2, 0.3 and 0.4 respectively.

The Fig.10.2(a) shows that potential of IADLs are negative in the plasma having H^+ ion-beams for n_{b0} = 0.1, 0.2, 0.3 and 0.4 with fixed values of σ_i = 0.001, σ_b = 0.003, V =1.6, β =1.1 and u_{b0} = 0.4. The amplitudes of rarefactive IADLs of H^+ ion-beams increase with the increase of ion-beam density.

The Fig.10.2(b) illustrates the rarefactive IADLs in the plasma having Ar^+ ion-beams for n_{b0} = 0.1, 0.2, 0.3 and 0.4 with fixed values of σ_i =0.001, σ_b =0.003, V =1.6, β =0.3, and

u_{b0} =0.4. The amplitudes of rarefactive IADLs of Ar^{+} beams decrease with the increase of the ion-beam density.

iii). Effect of ion-beam temperature: The effects of ion-beam temperature on the normalized potential of IADLs are shown in Fig. 10.3(a) and 10.3(b) with fixed values of other normalized plasma parameters. The effect of ion-beam temperature on the IADLs in plasma is understood from numerical estimation for the amplitude given by Eq. (10.15) and electrostatic potential from Eq. (10.17).

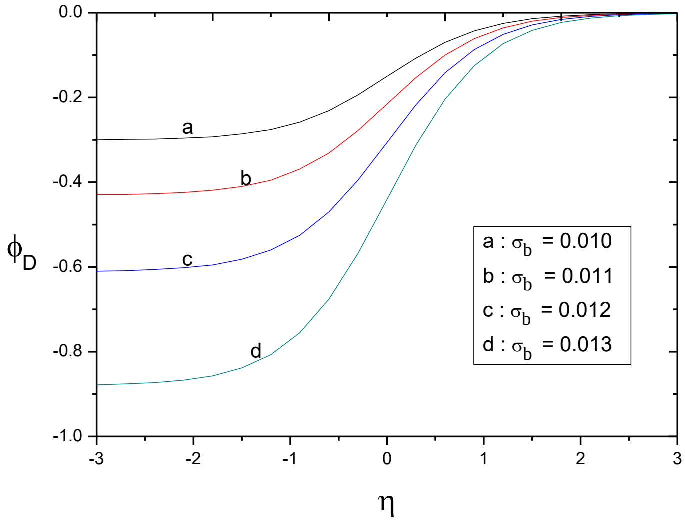

Fig.10.3(a). Profiles of the IADLs for different values of ion-beam temperature (σ_b) of H^{+} beam in plasma having σ_i =0.01, β =1.18, V =1.6, n_{b0} =0.3 and u_{b0} =0.4; Graphs a, b, c, and d represent σ_b = 0.01, 0.011, 0.012 and 0.013 respectively.

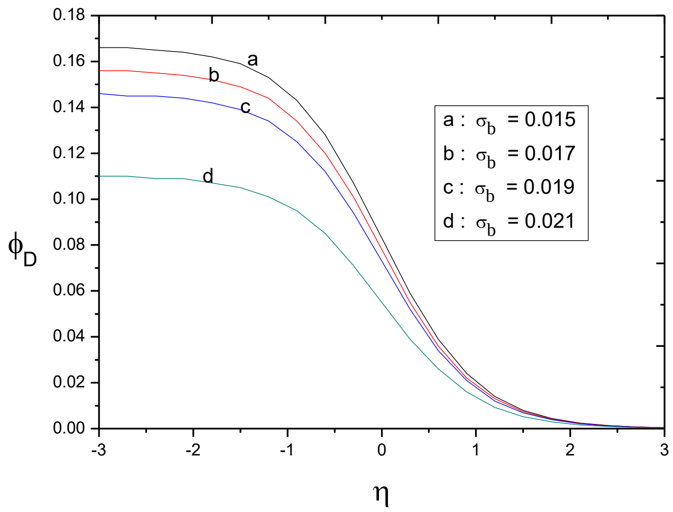

Fig.10.3(b). Profiles of the IADLs for different ion-beam temperature (σ_b) of Ar^{+} beam in plasma having σ_i =0.001, β =1.29, V =1.6, n_{b0} =0.3 and u_{b0} =0.75; Graphs a, b, c, and d represent σ_b = 0.015, 0.017, 0.019 and 0.021 respectively.

It is seen from numerical estimation that the amplitude for H^+ beam is negative (since A_2 is positive) in our plasma having σ_b = 0.01 0.011, 0.012 and 0.013 with fixed values of σ_i =0.01, β =1.18 V =1.6, n_{b0} =0.3 and u_{b0} =0.4. So, the profile of IADLs for H^+ beam plasma is rarefactive. Fig.10.3 (a) shows that the amplitude of rarefactive IADLs increases with the increase of ion-beam temperature.

In a similar way, numerical estimation are made for IADLs for Ar^+ beam in the plasma and the profile of IADLs for Ar^+ beam in the plasma is shown in Fig.10.3(b) which is compressive. The amplitude of IADLs is positive (since A_2 is negative) in our plasma having σ_b = 0.015, 0.017, 0.019 and 0.021 with fixed values of σ_i =0.001, β =1.29 V =1.6, n_{b0} =0.3 and u_{b0} =0.75. For this reason, the IADLs in Fig.10.3 (b) are compressive. It is seen that the amplitude of compressive IADLs in Fig.10.3(b) increases with the decrease of ion-beam temperature.

iv) Effect of ion-beam velocity: The effects of ion-beam velocity on the normalized potential of IADLs in plasma having H^+ beam and Ar^+ beam are shown in Fig.10. 4(a) and 10.4(b) with fixed values of other normalized plasma parameters.

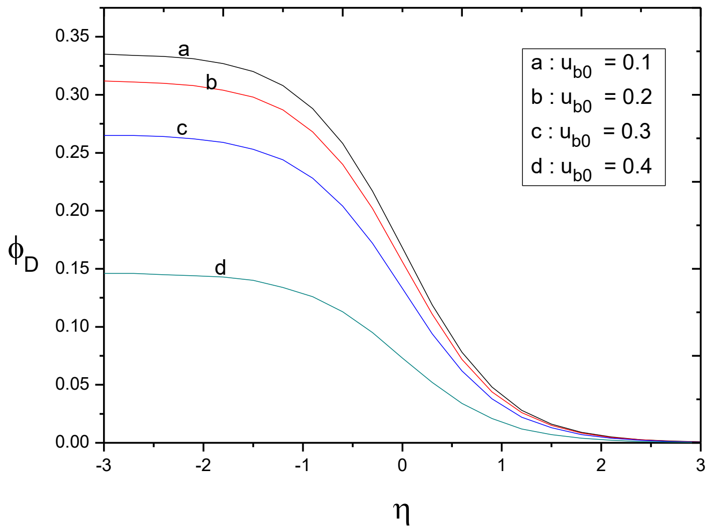

Fig.10.4(a). Profiles of the IADLs for different values of ion-beam velocity (u_{b0}) of H^+ beam in plasma having σ_i =0.001, σ_b =0.003, β =1.2, V =1.6 and n_{b0} =0.3; Graphs a, b, c, and d represent u_{b0} = 0.1, 0.2, 0.3 and 0.4 respectively.

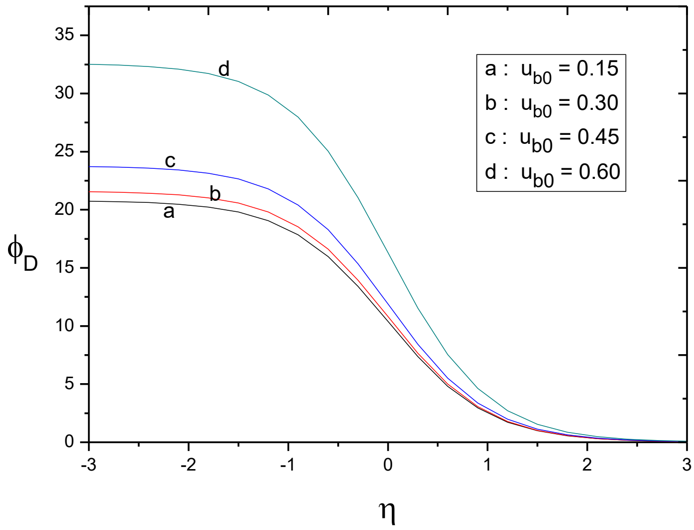

Fig.10.4(b).Profiles of the IADLs for different ion-beam velocity (u_{b0}) of Ar^+ beam in plasma having σ_i =0.001, σ_b =0.003, β=0.1, V =1.6 and n_{b0}=0.1; Graphs a, b, c, and d represent u_{b0} = 0.15, 0.30, 0.45 and 0.60 respectively.

It is seen in Fig.10.4 (a), the amplitudes of IADLs for H^+ beam in plasma are positive (since A_2 is negative) for u_{b0} = 0.1, 0.2, 0.3 and 0.4 having fixed values of σ_i =0.001, σ_b =0.003, β=1.2, V =1.6 and n_{b0}=0.3. Hence, the IADLs for H^+ beam are compressive. The amplitude of compressive IADLs increases with the decrease of ion-beam velocity.

Similarly, numerical estimation is made for amplitude of IADLs of Ar^+ beams in plasma, from which it is seen that amplitude is positive (as A_2 is negative) for u_{b0} = 0.15, 0.30, 0.45 and 0.60 having fixed values of σ_i =0.001, σ_b =0.003, β=0.1, V =1.6 and n_{b0}=0.1. As a result, the IADLs in Fig.10.4 (b) for Ar^+ beams in plasma become compressive in nature. The amplitude of compressive IADLs for Ar^+ beams increases with increase of ion-beam velocity in plasma.

10.4.2. Profiles of Width for Ion-Acoustic Double Layers

It is known that width is important for the study of IADLs in plasma for experimental studies in laboratory and space. We have numerically estimated the width of IADLs using Eq. (10.18). The variation for different parameters of ion-beam plasma is graphically discussed in below figures. It is to be remembered that for the numerical estimation we have followed the conditions for the existence of IADLs (i.e. A_3 must be positive).

Now, we have plotted the width of IADLs with the ion-beam density (n_{b0}) in plasma:

i). For different values of nonthermal parameter (β) of electron are shown in Fig.10.5(a) for H^+ beam and Fig.10.5(b) for Ar^+ beam

ii). For different values of ion-beam temperature (σ_b) are shown in Fig.10.6(a) for H^+ beam and Fig.10.6(b) for Ar^+ beam

iii). for different values of ion-beam velocity (u_{b0}) are shown in Fig.10.7 (a) for H^+ beam, and Fig. 10.7(b) for Ar^+ beam

i). Effect of nonthermal parameter of electrons:

The variation of width of IADLs with ion-beam density (n_{b0}) for different values of nonthermal parameter (β) of electrons in plasma having H^+ beam and Ar^+ beam in plasma are shown in Fig.10.5(a) and Fig. 10.5(b).

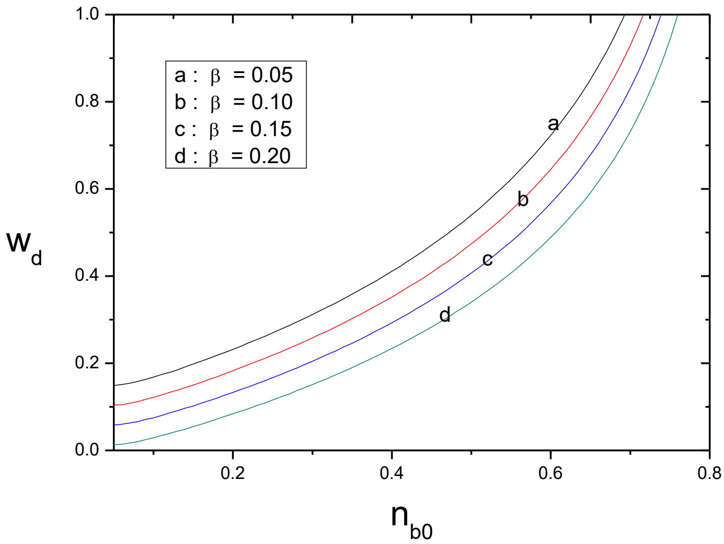

Fig. 10.5(a). Variation of width of IADLs with ion-beam density for different values of nonthermal parameter (β) in H^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6 and u_{b0} =0.3; Graphs a, b, c, and d represent β = 0.05, 0.1, 0.15 and 0.2 respectively.

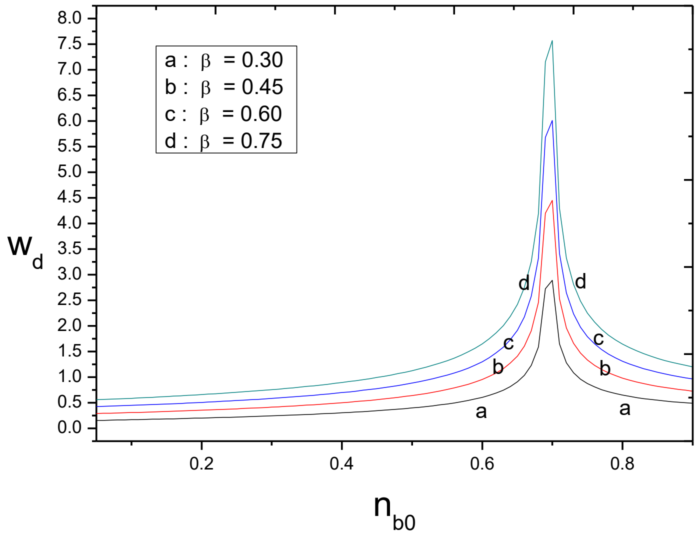

Fig. 10.5(b). Variation of width of IADLs with ion-beam density for different values of nonthermal parameter (β) in Ar^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6 and u_{b0} =0.2; Graphs a, b, c, and d represent β = 0.3, 0.45, 0.6 and 0.75 respectively.

From Fig.10.5 (a) for H^+ beam, it is seen that the widths of IADLs increase with the increase of n_{b0} when β = 0.05, 0.1, 0.15 and 0.2 in a plasma having fixed values of σ_i =0.001, σ_b =0.003, V =1.6, and u_{b0} =0.3. The widths of IADLs are large for small values of β.

In Fig. 10.5(b), Ar^+ beam shows that widths of IADLs increase up to certain maximum value when β = 0.3, 0.45, 0.6 and 0.75 having fixed values of σ_i =0.001, σ_b =0.003, V =1.6 and u_{b0} =0.2; Then the widths decrease to a minimum value , after that they again increase steadily with increase of ion-beam density. The widths of IADLs are large for small values of nonthermal parameter of electrons.

ii). Effect of ion-beam temperature: Similarly, the variation of width of IADLs with ion-beam density (n_{b0}) for different values of ion-beam temperature (σ_b) in plasma having H^+ beam and Ar^+ beam are shown in Fig.10.6(a) and Fig.10.6(b).

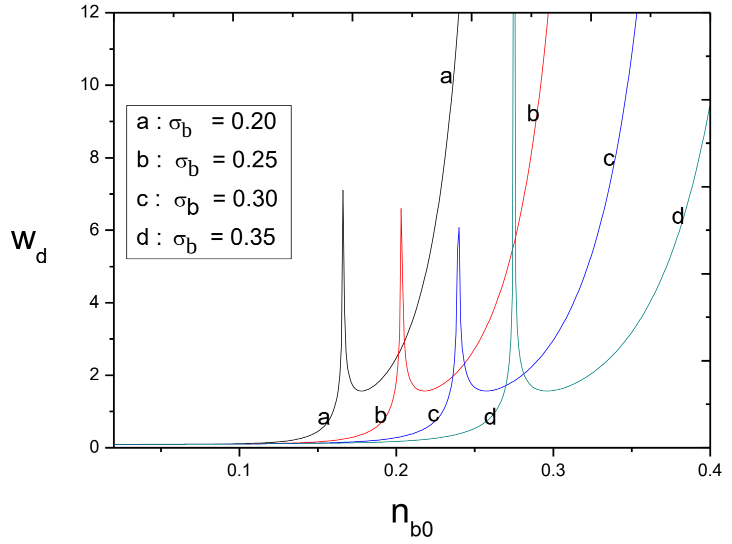

Fig. 10.6(a). Variation of width of IADL with ion-beam density (n_{b0}) for different values of ion-beam temperature (σ_b) in H^+ beam in plasma having σ_i =0.01, V =1.6, β =0.3 and u_{b0} =0.2; Graphs a, b, c, and d represent σ_b = 0.20, 0.25, 0.3 and 0.35 respectively.

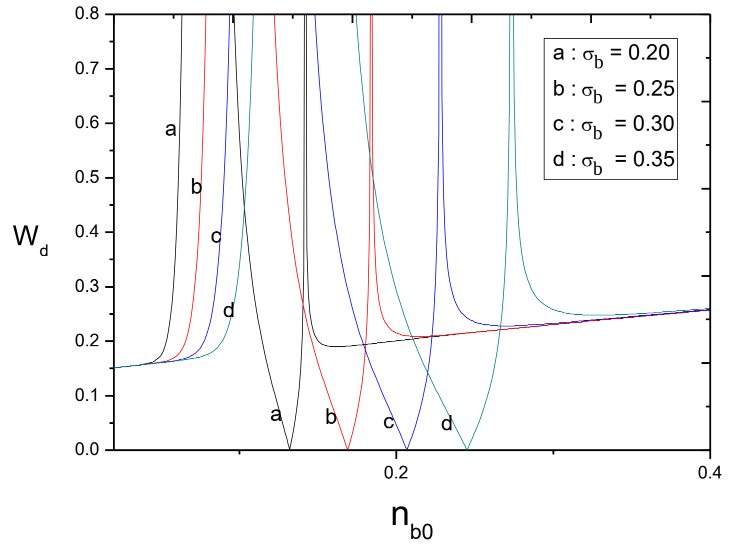

Fig. 10.6(b). Variation of width of IADL with ion-beam density (n_{b0}) for different values of ion-beam temperature (σ_b) in Ar^+ beam in plasma having σ_i =0.01, V =1.6, β =0.3 and u_{b0} =0.2; Graphs a, b, c, and d represent σ_b = 0.20, 0.25, 0.3 and 0.35 respectively.

It is seen from Fig.10.6 (a) that the widths of IADL of H^+ beam in plasma increase up to a certain maximum value with the increase of n_{b0} when σ_b = 0.20, 0.25, 0.3 , 0.35 with fixed values of σ_i =0.01, V =1.6, β=0.3 and u_{b0}=0.2. Then widths decrease to a minimum value with the increase of n_{b0}, after that widths increase again with the increase of n_{b0}.

On the other hand, the variations of widths of IADLs in plasma having Ar^+ beams are shown in Fig.10.6 (b). It is seen that the widths of Ar^+ beams increases to a maximum value with the increase of n_{b0} when values of σ_b, σ_i, V, β and u_{b0} are same as in H^+ beam plasma. After that the widths decrease to a minimum value; again they increase to a maximum value and finally get decreased to a minimum value and remain constant with increase of ion-beam density.

iii). Effect of ion-beam velocity

The variation of width of IADLs with ion-beam density (n_{b0}) for different values of ion-beam velocity (u_{b0}) in plasma having H^+ beams and Ar^+ beams are shown in Fig.10.7(a) and Fig.10.7(b).

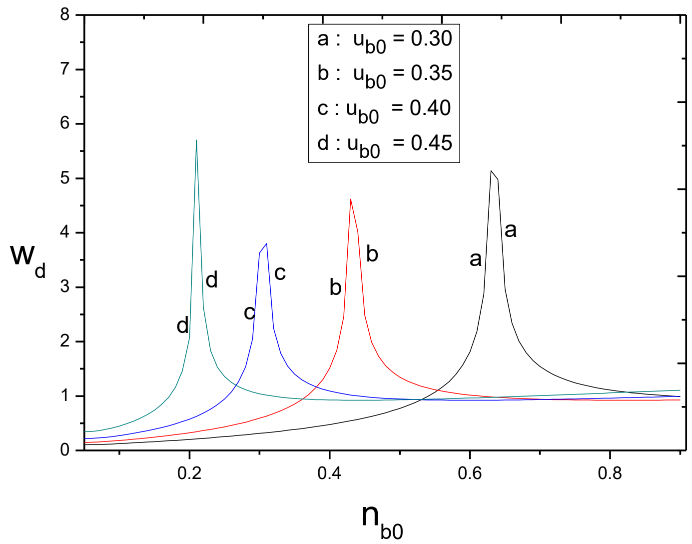

Fig. 10.7(a). Variation of width of IADL with ion-beam density (n_{b0}) for different values of ion-beam velocity (u_{b0}) of H^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6 and β =0.1; Graphs a, b, c, and d represent u_{b0} = 0.25, 0.30, 0.35 and 0.40 respectively.

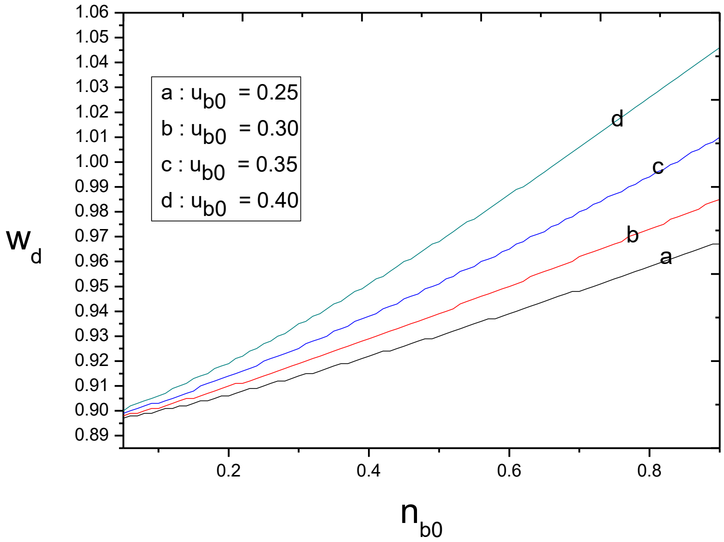

Fig. 10.7(b). Variation of width of IADL with ion-beam density (n_{b0}) for different values of ion-beam velocity (u_{b0}) of Ar^+ beam in plasma having σ_i =0.001, σ_b =0.003, V =1.6 and β =1.2; Graphs a, b, c, and d represent u_{b0} = 0.25, 0.30, 0.35 and 0.40 respectively.

It is seen in Fig.10.7 (a) that the widths increase with the increase of n_{b0} up to a certain maximum value for H^+ beam. Then they decrease with the increase of n_{b0}. this figure is drawn with the value of u_{b0} = 0.25, 0.30, 0.35 and 0.40 in a plasma having fixed values of σ_i =0.001, σ_b =0.003, V =1.6 and β =0.1.

It is observed in Fig.10.7 (b) that the widths increase steadily with increase of n_{b0} for Ar^+ beam, when u_{b0} = 0.25, 0.30, 0.35 and 0.40 in a plasma having fixed values of σ_i =0.001, σ_b =0.003, V =1.6 and β = 1.2.

10.5. SUMMARY AND CONCLUSION

We have theoretically studied the IADLs for small amplitude waves in the plasma composed of warm positive ions, warm positive beam ions and warm nonthermal electrons using pseudo potential method. It has been shown that the IADLs in such plasma depends on the plasma parameters; i.e. ion-beam density, ion-beam temperature, ion-beam velocity and nonthermal parameters of electrons. Numerical estimation is made considering a model plasma having beam of ions and the profiles of IADLs including widths are drawn.

A).Profiles of Ion-Acoustic Double Layers

i). The IADLs are compressive and rarefactive in plasma having H^+ beams and Ar^+ beams depending on the values of nonthermality of electrons and other plasma parameters. The IADLs are compressive in plasma having H^+ beam for the nonthermal parameter β = 0.3 to 0.6. The amplitudes of compressive IADLs are decreased with increase of β. But , in presence of Ar^+ ion beams in plasma the IADLs are rarefactive for β = 0.3 to 0.6 . The amplitudes rarefactive IADLs are decreased with the increase of nonthermal parameter of electrons.

ii).The rarefactive IADLs are excited in plasma having H^+ beams for n_{b0} = 0.1 to 0.4 when other plasma parameters have fixed values. The amplitudes of rarefactive IADLs for H^+ beams are increased with the increase of ion beam density. In plasma having Ar^+ beams, the rarefactive IADLs are also excited for n_{b0} = 0.1 to with some fixed values of other plasma parameters. The amplitudes of rarefactive IADLs for Ar^+ beams are decreased with the increase of the ion beam density.

iii).The IADLs are rarefactive in plasma having H^+ beams for σ_b = 0.01 to 0.013 when other plasma parameters have some fixed values. The amplitude of rarefactive DLs increases with the increase of ion beam temperature. But, the IADLs are compressive in plasma having Ar^+ beams for σ_b = 0.011 to 0.014 when other plasma parameters have some fixed values. The amplitude of compressive IADLs for Ar^+ beams increases with decrease of ion beam temperature.

iv).The IADLs are compressive in plasma having H^+ beams for u_{b0} = 0.1 to 0.4 when other plasma parameters have some fixed values. The amplitude of compressive IADLs for H^+ beams increases with the decrease of ion beam velocity. In plasma having Ar^+ beams, the IADLs are also compressive for u_{b0} = 0.15 to 0.60 for same fixed values of plasma parameters. The amplitude of compressive IADLs for Ar^+ beams increases with increase of ion beam velocity.

B). Profiles of Width for Ion-Acoustic Double Layers

i). In H^+ beam plasma the width of IADLs increases with increase of n_{b0} when β = 0.05 to 0.2 having some fixed values of other plasma parameters. The width is large for small values of nonthermal parameter of electrons. In Ar^+ beam plasma the width of IADLs increases up to certain maximum value with increase of n_{b0} when β = 0.3 to 0.45 having some fixed values of other plasma parameters . The width decreases to a minimum value with the increase of n_{b0} .The width is large for large value of β.

ii).In plasma having H^+ beams the width of IADLs increases up to a certain maximum value with increase of n_{b0} when σ_b = 0.20 to 0.35 having some fixed values of other plasma parameters. After that width decreases to a minimum value with increase of n_{b0}, then the width increases again with increase of n_{b0}. In plasma having Ar^+ beams the width of IADLs increases to a maximum value with increase of ion beam density when values of other plasma parameters are same as in H^+ beam plasma. Then the width decreases to a minimum value; again it increases to a maximum value and it is decreased to a minimum value and remains constant with increase of ion beam density.

iii). In plasma having H^+ beams the width of IADLs increases with increase of u_{b0} up to a certain maximum value; and then it decreases steadily with increase of n_{b0} when u_{b0} = 0.25 to 0.40 having fixed values of other plasma parameters. In Ar^+ beam plasma the width of IADLs increases steadily with increase of n_{b0} when u_{b0} = 0.25 to 0.40 having some fixed values of other plasma parameters.

It is to be stated that the plasma considered in present work exists in space and also can be produced in the laboratory. This analysis can be useful to study of the IADLs in auroral and magnetospheric plasmas which can explain the particle acceleration in astrophysical plasmas. Our study is important because of the fact high energy charged particles (electrons and ions) are found to exist in solar flares and upstream solar wind etc. During solar bursts, evolution of stars and pulsar radiation when relativistic and non-relativistic ions and electrons are ejected and entered the space and the ionosphere. Following our present analysis, IADLs can also be studied in plasma having electron beams and negative ions. Moreover, EADLs and IADLs can be studied in plasma having beams of electrons / ions considering different types non-Maxwellian electrons, namely, nonisothermal electrons, superthermal electrons , Tsallis distributed electrons etc. In our present study, static magnetic field in the ion-beam plasma has not been considered. By considering the magnetic field and different types of non-Maxwellian electrons in presence of beams (electron / ion), more interesting results on the

IADLs in plasma would be obtained which may give new information about acceleration process of ionized particles in space and laboratory plasma. The IADLs in degenerate plasma in presence of electron/ ion-beams can also be studied following our present work. It is to be mentioned that for the study of large amplitude IADLs in ion-beam plasma the terms φ^4, φ^5 etc. of r. h.s. of Eq. (10.12) should be taken into account and the solution of IADLs can be obtained using tanh-method [20-23]. The IADLs in ion-beam plasma can also be studied from the modified KdV equation using the reductive perturbation method [24, 25]

It is to be mentioned the IADLs in plasma have been experimentally observed by various authors [26-30]. But, there is no information in experimental observations on the IADLs in ion-beam plasma consisting of nonthermal electrons.

REFERENCES

[1]. R. A. Hoffmann and D. S. Evans, J. Geophys. Res., **73**, p.6201 (1968).
[2]. M. V. Goldman, Sol. Phys.**, 89**, p.403 (1983).
[3]. S. M. Krivoruchku, Ya. B. Fainberg, V. D. Shapiro, and V. I . Shevchenko, Sov. Phys. JETP, **40**, p.1039 (1974).
[4].Y. Gell and I. Roth, Plasma Physics, **19**, p.915 (1977).
[5]. P. S. Abrol and S. G .Tagare, Phys. Letts**., A 75**, p. 74 (1979).
[6]. B. Karmakar, G. C. Das and K.I. Singh, Plasma Phys. Contri. Fusion, **30**, p.1167 (1988).
[7]. S. K. El-Labany, J. Plasma Phys., **54(2),** p.85 (2009).
[8]. R. Das, Astrophys. Sp. Science., **341,** p.543 (2012).
[9]. R. Das and P. Deka, Inter. J. Sc. & Eng. Research, **6,** p.864 (2015).
[10]. N. Kaur, K. Singh , Y. Ghai and N. S. Saini, Plasma Sci. Technol., **20**, p.074009 (2018).
[11].Indrani Paul, A. Chatterjee and S. N. Paul,Laser and Particle Beams, November 2019 DOI: 10.1017/S0263034619000697
[12]. N. Kaur, K. Singh and N. S. Saini, Phys. Plasmas, **24**, p.092108 (2017).
[13]. R. A.Cairns, R.Bingham, R.O. Dendy, C. M. C Nairn, P. K.Shukla, A. A.Mamun, J. Phys., IV **France 5**, C6–43 (1995).
[14]. R A Cairns, A A Mamun, R Bingham, R Bostrom, R O Dendy, C M C Nairn, P K Shukla, Geophys. Res. Letts., **22,** p.2709 (1995).
[15]. T. S. Gill, P. Bala, H. Kaur, N. S. Saini, S. Bansal and J. Kaur, Euro. Phys. J., **D 31**, 91-100 (2004).
[16]. S. K .Bhattacharaya, S. N. Paul and B. Chakraborty, Indian J. Phys., **77B**, p.327 (2003) [26]. B.
[17]. B. Ghosh , S. N. Paul , C. Das, I. Paul and S. Banerjee, Braz. J. Phys., **43,** p.28 (2013)
[18]. D. N . Gao, J. Zhang,Y. Yang,W. S. Duan, Plasma Physics Reports , **43 (8)**, p.833 (2017).
[19]. I. Paul, A. Chatterjee and S. N. Paul, Braz. J.Physics **(2020) DOI : 10.1007/s13538-020-00741-2**
[20]. G. C. Das and J. Sarma ,Phys. Plasmas, **5,** p.63 (1998).
[21]. A . Bandyopadhyay and K. P. Das , Phys. Scr., **61**, p.92 (2000).
[22]. B. Sahu, R. Roychowdhury, Phys.Plasmas, **11,** p.1947(2004).
[23]. S. N. Paul, A. Chatterjee , I. Paul and B. Ghosh , International J. Chemical and Physical Sciences, IJCPS, 4 Special Issue-6 , APST–2015, p.38 (2015).
[24]. S N Paul, K K Mondal, A Roychowdhury, FIZIKA, **B 5**, 73–84 (1996).
[25]. K. K. Mondal, S. N. Paul, A. Roychowdhury, IEEE Trans. Plasma Sci., **26,** p.987 (1998)
[26]. Y. Nakamura and H. Sugai, Chaos, Solitons and Fractals, **7**, p.1023 (1996).
[27].S. Torvén, Formation of Double Layers in Laboratory Plasmas ; Wave Instabilities in Space Plasmas , pp. 109-128 (1979).
[28]. N. Hershkowitz, Space Science Rev., **41,** p.351 (1985).
[29]. H. Schamel, Phys. Reports (Review Section of Physics Letters), **140,** p.161(1986)
[30].C. Charles, Plasma Sources Sci. Technol., **16,** R1–R25(2007).

CHAPTER-11

ION-ACOUSTIC DOUBLE LAYERS IN DRIFTING MULTICOMPONENT PLASMA HAVING POSITRONS AND NONTHERMAL ELECTRONS

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11.1. INTRODUCTION

In last few years, there has been considerable interest in the study of ion-acoustic double layers (IADLs) in multispecies plasmas including the effects of negative ions. The negative ions, which result from electron attachment to neutral particles when an electronegative gas is introduced in an electrical gas discharge, can be found in the D-region of the ionosphere [1], plasma processing reactors [2], and neutral beam sources [3]. Even a small amount of negative ions can have significant effect on the formation of nonlinear ion-acoustic wave structures [4]. Merlino and Loomis [5] have first experimentally investigated the formation of strong double layers in plasma consisting of positive ions, negative ions, and electrons in a triple plasma device. When a strong double layer is present in positive ion/electron plasma, the introduction of negative ions results in the formation of a second double layer which is separated from the first double layer by a region of quasineutral plasma. Subsequently, Jain *et al* [6] have theoretically investigated the effect of second-ion species on the characteristics of a large-amplitude ion-acoustic double layers (IADLs) in a collisionless, unmagnetized plasma consisting of hot and cold Maxwellian populations of electrons and two cold-ion species with different masses, concentrations, and charge states. After deriving the criteria for the existence of large-amplitude IADLs, they have found that the presence of a positive-ion impurity does not considerably modify the characteristics of large-amplitude double layers. However, the presence of a negative-ion impurity significantly changes the characteristics of the IADLs. They have also presented an analytic discussion of small-amplitude IADLs using a reductive perturbation method (RPM). Later, Mishra *et al* [7] have studied IADLs in plasma consisting of warm positive- and negative-ion species with different masses, concentrations, and charge states along with electrons with two-electron temperature distributions. It is found that there exist two critical concentrations of negative ions. One of them generally decides the existence of IADLs , whereas the other one decides the nature of the IADLs. It is found that the system supports IADLs only when the negative-ion concentration is greater than the critical concentration It is also found that below the critical concentration compressive IADLs exist and the IADLs become rarefactive above the critical concentration. Paul *et al*. [8] have considered the effect of electron inertia for theoretical investigation on ion acoustic solitons (IAS) and IADLs in a multi-component plasma having warm positive ions, negative ions and isothermal electrons using pseudo potential method. It has been found that drift velocity and electron-inertia have significant contribution on the formation of IADLs in plasma having ($H+$, Cl^-), ($H+$, O^-), ($He+$, H^-) and ($He+$, O^-) ions. Baboolal *et al* [9] have also investigated IADLs in negative-ion plasmas and have resolved the discrepancy of the results of previous authors on the existence of IADLs in plasma.

On the other hand, the non-Maxwellian nonthermal electrons give rise to many interesting characteristics in the nonlinear propagation of waves, including the excitation of IAS and IADLs in plasma. Gill *et al* [10] have studied the properties of IAS and IADLs in plasma consisting of nonthermal electrons, warm positive and negative ions with different concentration of masses and charged states using small amplitude approximation. They have used the RPM to derive KdV and modified-KdV (mKdV) equations. The existence of IAS and IADLs is explored over a wide range of parameter space. However, when nonthermal parameter of electrons is increased beyond a critical value, no IADLs are excited. Das *et al*. [11] have considered the non-thermal electrons to have vortex-like velocity distribution and have derived a combined Schamel's modified Korteweg–de Vries–Zakharov– Kuznetsov (S–KdV–ZK) equation . Then have studied the existence and stability of IADLs in magnetized plasma consisting of warm adiabatic ions and nonthermal electrons. Using the SPM, Das and Chatterjee [12] have studied the formation of large amplitude DLs in dusty plasma with non-thermal electrons and two temperature isothermal ions. It is found that the temperature of the dust particles, nonthermal electrons and ion-temperature all play significant role in determining the region of the existence of DLs . Later, Ghosh *et al* [13] have applied the SPM to study the IADLs in multi-component plasma consisting of nonthermal electrons and warm positive and negative ions with drift motion. They have obtained the conditions for the existence of such DLs which control the formation and the character of DLs. The effects of nonthermal electrons, negative-ion concentration, and negative-ion temperature on the formation and structure of IADLs are also investigated. **R**ecently, Kaur *et al* [14] have investigated IADLs in inhomogeneous plasma consisting of Maxwellian and nonthermal distributions of electrons using the RPM. They have derived an mKdV equation for IADLs propagating in collisionless inhomogeneous plasma. It is observed that the nonthermal parameter of electrons significantly affect the amplitude and width of IADLs which further depend on the density of the plasma.

However, most fascinating results on the IADLs are observed in a plasma containing positrons in plasma. The IADLs in electron-ion-positron plasma have different characteristic from that obtained in electron-ion (e.g. nonthermal electrons and ions) plasma. To understand the nature of the IADLs in e-i-p plasma, Bharuthram [15] have investigated the existence and the properties of arbitrary amplitude IADLs in four-component plasma, consisting of two species of hot electrons, a hot and a cold positron species. The results are discussed with applications to the polar-cusp region of pulsars. Later, Mishra *et al*. [16] have studied the IADLs in a multi-component plasma with positrons. Using the RPM, they have derived the mKdV equation for the system and discussed about the solution of IADLs of mKdV equation. It is found that there exist two critical concentrations of positrons which decide the existence and nature of the IADLs. It is also found that the system supports IADLs only when the positron concentration is less than the critical concentration. It is also investigated that for the given sets of parameter values the system supports rarefactive or compressive IADLs for different values of positron density. It is predicted that for the given sets of parameter values, the amplitude of rarefactive (compressive) IADLs decreases (increases) with the increase of positron concentration but the width of the rarefactive (compressive) IADLs increases (decreases). Subsequently, Jain and Mishra [17] have studied the large-amplitude IADLs in collisionless plasma consisting of isothermal positrons, warm adiabatic ions and two-temperature distribution of electrons using the SPM. It is found that for a selected set of physical parameters, the rarefactive IADLs exist in the e-i-p plasma. The existence regime of IADLs is very sensitive to the plasma parameters, e.g. cold electron concentration and temperature ratio of two electron species. The increase in the finite ion temperature ratio increases the amplitude of rarefactive IADLs. For the study of small-amplitude IADLs , they have expanded the Sagdeev potential and found that the amplitude of IADLs increases with increase in ion temperature ratio and cold electron concentration. However, due to the increase of positron concentration and temperature ratio of positrons to electrons, the amplitude of IADLs decreases.

In many physical situations, it seems therefore more realistic to consider the simultaneous presence of nonthermal electrons and positrons in multi-component plasma with negative ions for the study of IADLs in drifting plasmas. To the best of our knowledge, this problem has not been studied till now. We are hence motivated to study the excitation and characteristics of the IADLs in a multicomponent drifting plasma composed of nonthermal electrons, isothermal positrons and warm positive ions as well as negative ions.

11.2. BASIC EQUATIONS

We assume**,** collisionless unmagnetized multicomponent plasma consisting of warm positive and negative ions, nonthermal electrons and isothermal positrons. Moreover, we consider the stream velocity of positive and negative ions in unperturbed state. The normalized basic equations in unidirectional propagation for such plasma in dimensionless form are:

For positive ions:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}\left(n_i u_i\right)=0 \tag{11.1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{11.2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial u_i}{\partial x}=0 \tag{11.3}$$

For negative ions:

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j u_j\right)=0 \tag{11.4}$$

$$\frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}+\frac{\sigma_j}{Qn_j}\frac{\partial p_j}{\partial x}=\frac{Z}{Q}\frac{\partial \phi}{\partial x} \tag{11.5}$$

$$\frac{\partial p_j}{\partial t}+u_j\frac{\partial p_j}{\partial x}+3p_j\frac{\partial u_j}{\partial x}=0 \tag{11.6}$$

For Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_p-n_i+Zn_j \tag{11.7}$$

Where, the subscript e, i, j and p denote electron, positive ions, negative ions and positrons. n_i, u_i, p_i, σ_i and n_j, u_j, p_j, σ_j are respectively the concentration , velocity, pressure and temperature of positive and negative ions, $Q = m_j / m_i$, m_i and m_j are the mass of positive ions and negative ions, Z is number of electrons attached to negative ion, $Z = 1$.

The normalized electron density n_e is given by [18,19]

$$n_e = \left(1-\beta\phi+\beta\phi^2\right)e^{\phi} \tag{11.8}$$

where $$\beta = \frac{4\alpha}{1+3\alpha} \tag{11.8a}$$

β measures the deviation from thermalized state and α determines the presence of fast particles in the model.

The density of the positrons is [20]

$$n_p = \chi_p \exp(-\sigma_p\varphi) \tag{11.9}$$

where, $\chi_p = n_{p0} / n_{e0} = 1 - n_{i0}$, $\sigma_p = T_e / T_p$.

For obtaining the IADL solution from Eqs.(11.1) to (11.7) we use the Gallelian transformation $\eta = x - Vt$, where V is the velocity of the ion-acoustic wave and we assume the boundary conditions

$$n_i \to n_{io}, n_j \to n_{jo}, u_i \to u_{io}, u_j \to u_{jo}, p_i \to p_{io}, p_j \to p_{jo}, \phi \to 0 \quad \text{at } |x| \to \infty \tag{11.10}$$

By using the boundary conditions (11.10) we get from Eqs.(11.1) to (11.6)

$$\frac{d^2\phi}{d\eta^2} = \left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi) - \chi_p \exp(-\sigma_p\varphi) +$$

$$\frac{n_{io}^{\frac{3}{2}}}{\sqrt{12\sigma_i p_{io}}}\left[\sqrt{\left\{\left(V-u_{io}+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}-\sqrt{\left\{\left(V-u_{io}-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}\right]+$$

$$\frac{Zn_{jo}^{\frac{3}{2}}Q^{\frac{1}{2}}}{\sqrt{12\sigma_j p_{jo}}}\left[\sqrt{\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2+\frac{2Z}{Q}\varphi\right\}}-\sqrt{\left\{\left(V-u_{jo}-\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2+\frac{2Z}{Q}\varphi\right\}}\right]$$

$$= -\frac{d\psi}{d\varphi} \tag{11.11}$$

where $\psi(\phi)$ is known as the Sagdeev Potential.

$$\psi(\phi)=(1+3\beta)-[(1+3\beta)-3\beta\varphi+\beta\varphi^2]\exp(\varphi)-\frac{\chi}{\sigma_p}[1-\exp(-\sigma_p\varphi)]$$

$$-\frac{n_{io}^{\frac{3}{2}}}{6\sqrt{3\sigma_i p_{io}}}\cdot\left[\begin{array}{l}\left\{\left(V-u_{io}+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}-\left\{\left(V-u_{io}-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}^{\frac{3}{2}}\\ -\left(V-u_{io}+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^3+\left(V-u_{io}-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^3\end{array}\right]$$

$$-\frac{Q^{\frac{3}{2}}n_{jo}^{\frac{3}{2}}}{6\sqrt{3\sigma_j p_{jo}}}\left[\begin{array}{l}\left\{\left(V-u_{jo}+\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2+\dfrac{2Z}{Q}\varphi\right\}^{\frac{3}{2}}-\left\{\left(V-u_{jo}-\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)+\dfrac{2Z}{Q}\varphi\right\}^{\frac{3}{2}}+\\ \left\{\left(V-u_{jo}+\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^3\right\}-\left\{\left(V-u_{jo}-\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^3\right\}\end{array}\right] \tag{11.12}$$

Integrating Eq.(11.11) we get

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2+\psi(\varphi)=0 \tag{11.13}$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi\to 0, d\varphi/d\eta\to 0$ and $d^2\varphi/d\eta^2\to 0.$

Eq. (11.13) describes the motion of a pseudo particle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $\psi(\varphi)$. For the double layer solution of Eq. (11.13), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi=0$ and φ_m, where φ_m is some extremum value of the potential φ, called the amplitude of IADL. The double layer solutions $\psi(\varphi)$ must additionally satisfy the boundary conditions mentioned in Chapter-9 [Eqs. (9.20) to (9.21)].

11.3. SOLUTION OF ION-ACOUSTIC DOUBLE LAYERS

Now we expand the r.h.s of Eq.(11.11) and Eq.(11.12) in power series of φ and taking the terms up to φ^5 we obtain

$$\frac{d^2\phi}{d\eta^2} = A_1\phi + A_2\phi^2 + A_3\phi^3 + \cdots\cdots \tag{11.14}$$

$$A_1 = \left[(1-\beta) + \chi\sigma_p - \frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i p_{io}}{n_{io}}} - \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j p_{jo}}{n_{jo}}} \right] \tag{11.15a}$$

$$A_2 = \frac{1}{2}\left[\begin{array}{l} 1 - \chi\sigma_p^2 + \dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}} \right)^{-3} - \left(V - u_{io} - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}} \right)^{-3} \right\} \\ - \dfrac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-3} - \left(V - u_{jo} - \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-3} \right\} \end{array} \right] \tag{11.15b}$$

$$A_3 = \frac{1}{6}\left[\begin{array}{l} (1+3\beta) + \chi\sigma_p^3 + \dfrac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}} \right)^{-5} - \left(V - u_{io} - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}} \right)^{-5} \right\} \\ + \dfrac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} - \left(V - u_{jo} - \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} \right\} \end{array} \right] \tag{11.15c}$$

And

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2 - \frac{1}{3}A_2\varphi^3 - \frac{1}{4}A_3\varphi^4 - \cdots\cdots \tag{11.16}$$

Now, for small amplitude ion acoustic wave we take the terms up to φ^3 of Eq. (11.16). Applying boundary conditions mentioned in Chapter-9 [Eqs. (9.20) to (9.22)], we find that

$$\varphi_m = 2A_2 / 3A_3 \tag{11.17}$$

and

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{11.18}$$

The small amplitude double solution of Eq.(11.14) satisfying Eq.(11.17) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh[\sqrt{(A_3/8\varphi_m)}\eta]\right) \tag{11.19}$$

where,

$\varphi_m / 2 = \varphi_{0D}$, φ_{0D} is the amplitude of IADLs.

The width of double layers is given by

$$W_d = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{11.20}$$

Eq. (11.17) relates the amplitude φ_m of the DLs to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (11.14). It is important to note that double layers in the plasma can only exist for positive value of A_3, i.e., for plasma parameters satisfying the following condition:

i) The nonthermal parameter β must be,

$$\beta > \frac{1}{3}\left[\begin{array}{l} 1 + \chi\sigma_p^3 + \dfrac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V - u_{io} + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5} - \left(V - u_{io} - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5}\right\} \\ + \dfrac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{\left(V - u_{jo} + \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-5} - \left(V - u_{jo} - \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-5}\right\} \end{array}\right] \tag{11.21}$$

The critical values of nonthermal parameter β_c for the existence of small amplitude DL in the muticomponent plasma can be numerically estimated from Eq.(11.21), if the values of other parameters are known.

Again, from Eqs. (11.17) and (11.19), it follows that the sign of A_2 determines the nature of the IADL. If A_2 is positive (negative), a rarefactive (compressive) IADL is formed. Since A_2 is independent of β , the nature of IADL, whether compressive or rarefactive, remains invariant under changes in β. But A_2 is dependent of positron density χ and positron temperature σ_p , so the nature of IADL, whether compressive or rarefactive, remains variable under changes in χ and σ_p .For a cold ion plasma ($\sigma_i = \sigma_j = 0$), A_2 is negative or positive in presence of positrons and so both compressive and rarefactive IADL will be formed; but in absence of positrons only compressive IADLs can be formed in cold ion plasma.

For compressive IADL, A_2 must be positive and the positron density χ should be less than χ_c i.e. $\chi < \chi_c$, χ_c is the critical value of positron density. For rarefactive IADL, A_2 must be negative and the positron density χ should be greater than χ_c i.e. $\chi > \chi_c$.

The critical values of positron density is given by

$$\chi_c = \frac{1}{\sigma_p^2}\left[\begin{aligned} &1+\frac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V-u_{io}+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}-\left(V-u_{io}-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}\right\} \\ &-\frac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-3}-\left(V-u_{jo}-\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-3}\right\}\end{aligned}\right] \tag{11.22}$$

Knowing the values of $n_{i0}, n_{j0}, \sigma_i, \sigma_j, \sigma_p, V, u_{io}, u_{jo}, Q, Z$ of the plasma the value χ_c can be numerically estimated from Eq.(11.22).

11.4. RESULTS AND DISCUSSION

To see the nature of ion-acoustic double layers in multicomponent plasma composed of warm positive and negative ions, nonthermal electrons and isothermal positron, we consider model plasma. The solution of Eq.(11.19) shows that IADL is dependent on

i). the nonthermal parameter of electrons

ii). the density of negative ions

iii). the density of positrons

iv). the temperature of positrons

v). the temperature of ions

vi). the stream velocity of ions.

The role of these parameters on the structures of IADLs is shown in Fig. 11.2 - Fig.11.6. The variation of amplitude and widths with plasma parameters are shown in Fig.11.7- Fig. 11.8. Let us first see the profiles of IADL for different values of the plasma parameters

11.4.1.Profiles of Ion-Acoustic Double Layers

(i). Effect of nonthermal parameter of electrons

The effect of nonthermal electrons (β) on the double layers in the plasma having Q= 8.875 (He^+. Cl^-) has been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He^+, Cl^-) plasma for different values of nonthermal parameter of electrons are shown below in Fig.11.1.

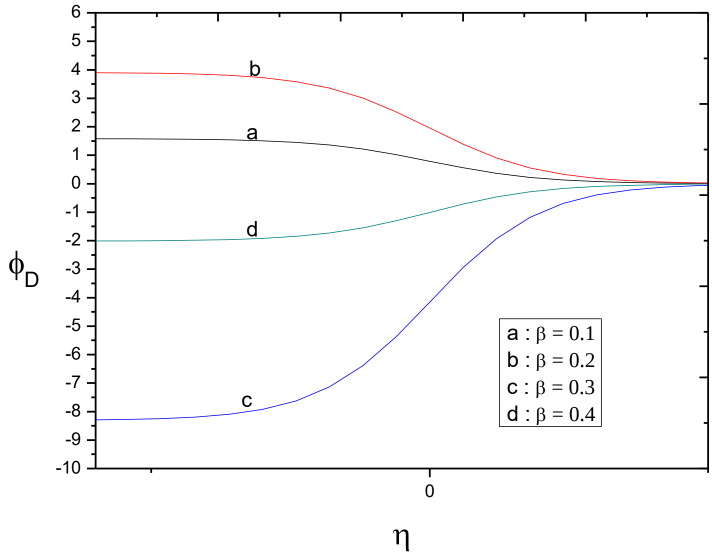

Fig.11.1. Profiles of the IADLs for different nonthermal parameter (β) in (He^+,Cl^-) with fixed values of σ_i =0.001, σ_j =0.003, χ =0.1, σ_p =0.1, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1. Graphs a, b, c and d represent β = 0.1,0.2,0.3 and 0.4 respectively.

It is seen from Fig.11.1 that IADLs may be compressive or rarefactive depending on the values of nonthermal parameter of electrons (β). For β =0.1 and 0.2 the IADLs are compressive but for β =0.3 and 0.4 the IADLs are rarefactive in nature. The amplitude of compressive IADLs increases with the increase of β , but the amplitude of rarefactive IADLs decreases with the increase of β.

(ii). Effect of negative ion density

The effects of concentration of negative ions on the double layers in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He+,Cl-) plasma for different negative ion density are shown below in Fig.11.2(a) and Fig.11.2(b) respectively.

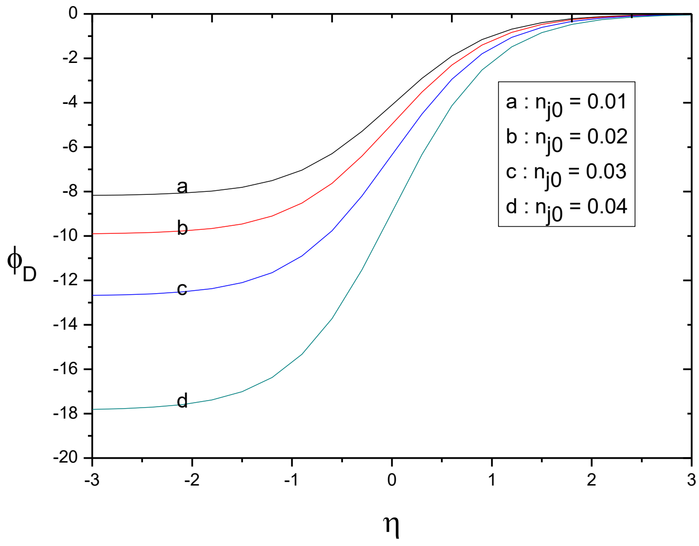

Fig.11.2(a). Profiles of the IADLs for different negative ion density in (He^+, Cl^-) plasma with fixed values of β = 0.3, σ_i =0.001, σ_j =0.003, σ_p =0.2, χ =0.1, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent n_{j0} = 0.01, 0.02, 0.03 and 0.04 respectively.

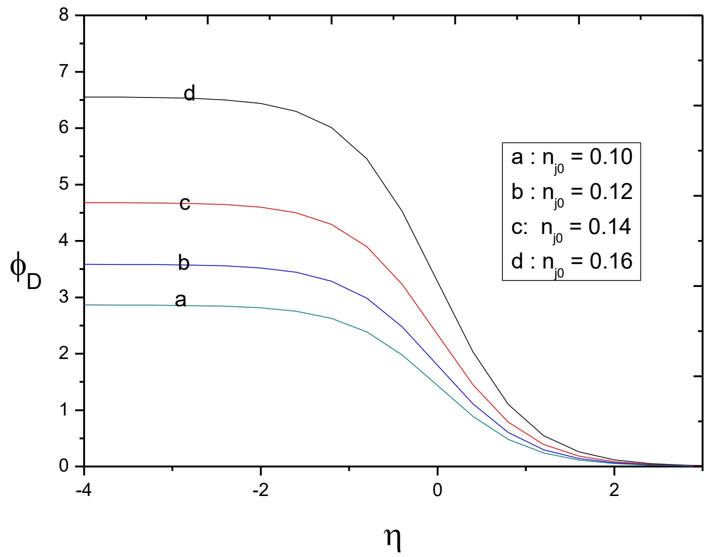

Fig.11.2(b). Profiles of the IADLs for different negative ion density in (He^+, Cl^-) plasma with fixed values of β = 0.3, σ_i =0.001, σ_j =0.003, σ_p =0.2, χ =0.1, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent n_{j0} = 0.10,0.12, 0.14 and 0.16 respectively.

It is observed from Fig.11.2(a) and Fig.11.2(b) that the IADLs may be compressive or rarefactive depending on the values of negative ion density. For n_{j0} =0.01 to 0.04 , the IADLs are rarefactive but for n_{j0} =0.10 to 0.16 the IADLs are compressive in nature. The amplitude of rarefactive IADLs increases with the increase of n_{j0} , but the amplitude of compressive IADLs decreases with the increase of n_{j0}.

(iii). Effect of positron density

The effects of positron density (χ) on the double layers in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He^+, Cl^-) plasma for different values of positron density are shown below in Fig.11.3.

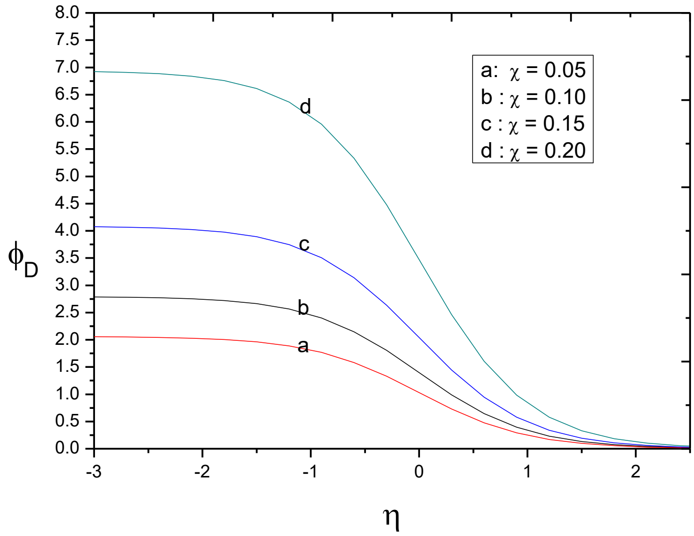

Fig.11.3. Profiles of the IADLs for different values of positron density (χ) in (He^+, Cl^-) plasma with fixed values of n_{j0} =0.1, β = 0.3, σ_i =0.001, σ_j =0.003, σ_p =0.2, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent χ = 0.05,0.10, 0.15 and 0.20 respectively.

(iv). Effect of positron temperature

The effects of positron temperature (σ_p) on the IADLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADL in (He^+, Cl^-) plasma for different values of positron density are shown below in Fig.11.4.

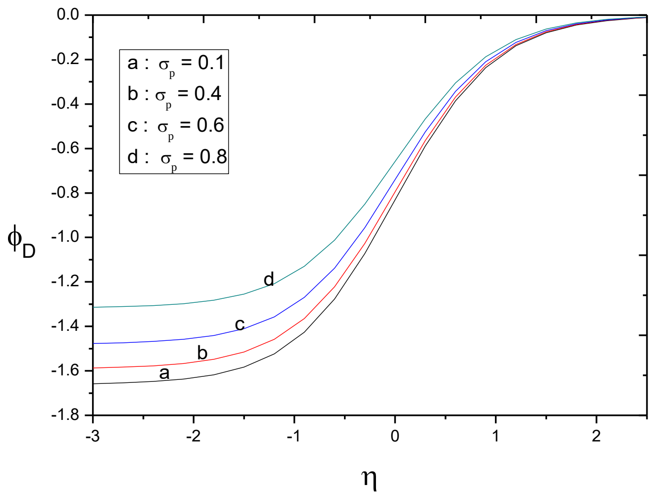

Fig.11.4. Profiles of the IADLs for different positron temperature (σ_p) in (He^+,Cl^-) plasma with fixed values of n_{j0} =0.1, β = 0.3, σ_i =0.001, σ_j =0.003, χ =0.1, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent σ_p = 0.1,0.4, 0.6 and 0.8 respectively.

From Fig.11.4 it is seen that the IADLs are rarefactive for σ_p = 0.1 to 0.8 and the amplitudes of IADLs decreases with the increase of positron temperature in the plasma.

(v). Effect of ion temperature

The effects of ion temperature (σ_i) on the IADLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He^+, Cl^-) plasma for different values of ion temperature are shown below in Fig. 11.5.

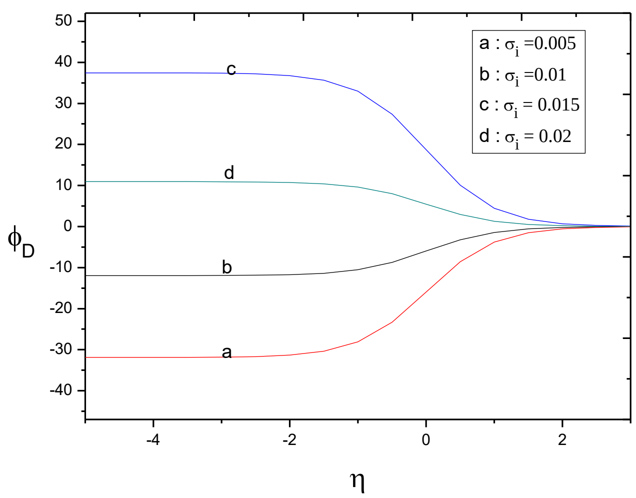

Fig.11.5. Profiles of the IADLs for different ion temperature (σ_i) in (He^+,Cl^-) plasma with fixed values of n_{j0} =0.1, β = 0.3, σ_j =0.003, χ =0.1, σ_p =0.2, V=1.6, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent σ_i = 0.005,0.01, 0.015 and 0.02 respectively.

It is seen in Fig.11.5. that the IADLs may be rarefactive or compressive depending on the values of ion temperature. For σ_i = 0.005 to 0.01, the IADLs are rarefactive, but for σ_i = 0.015 to 0.02 the IADLs are compressive in nature. The amplitudes of rarefactive IADLs decrease with the increase of ion temperature. Similarly, the amplitude of compressive IADLs decreases with the increase of ion temperature in the plasma.

(vi). Effect of ion stream velocity

The effects of ion stream velocity (u_{i0}) on the IADLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He^+, Cl^-) plasma for different values of ion stream velocity are shown below in Fig.11.6.

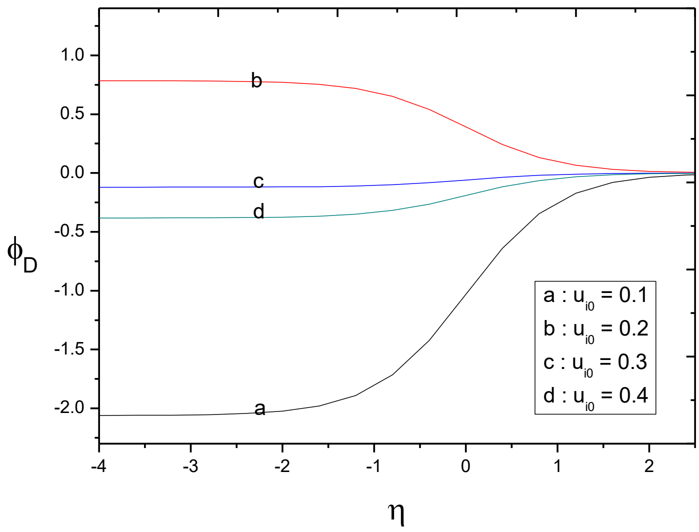

Fig.11.6. Profiles of the IADLs for different ion stream velocity(u_{i0}) in (He^+, Cl^-) plasma with fixed values of n_{j0} =0.1, β = 0.4, σ_i =0.001, σ_j =0.003, χ =0.1, σ_p =0.2, V=1.6, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent u_{i0} = 0.1,0.2, 0.3 and 0.4 respectively.

It is seen from Fig.11.6. that the IADLs may be rarefactive or compressive depending on the values of ion stream velocity. For u_{i0} = 0.1 and 0.4 the IADLs are rarefactive, but for u_{i0} = 0.2 and 0.3 the IADLs are compressive in nature. The amplitude of rarefactive IADLs for u_{i0} = 0.1 is larger than the values at u_{i0} =0.4; the amplitude of compressive IADLs for u_{i0} = 0.2 is larger than the values at u_{i0} =0.3.

11.4.2. Profiles of Amplitude and Width for Ion-Acoustic Double Layers

In experimental observation, the amplitude and width of IADLs in plasma are generally studied. So, we have numerically estimated the amplitude and width of IADLs in (He^{+},Cl^{-}) plasma for different values of the plasma parameters. The variation of amplitude and width of IADLs with negative ion concentration for the different values of nonthermal parameter (β) are shown in Fig.11.7 (a) and Fig.11.7 (b) respectively.

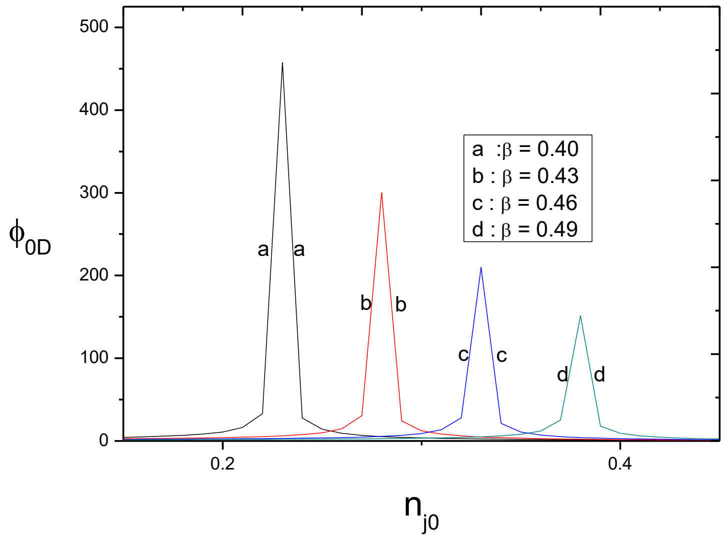

Fig. 11.7(a). Variation of amplitude of IADLs with negative ion concentration (n_{j0}) for different values of nonthermal parameter (β) in (He^{+},Cl^{-}) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, χ = 0.1, V=1.6, u_{i0} =0.1 and u_{j0} =0.2; Graphs a, b, c and d represent β = 0.40, 0.43, 0.46 and 0.49 respectively.

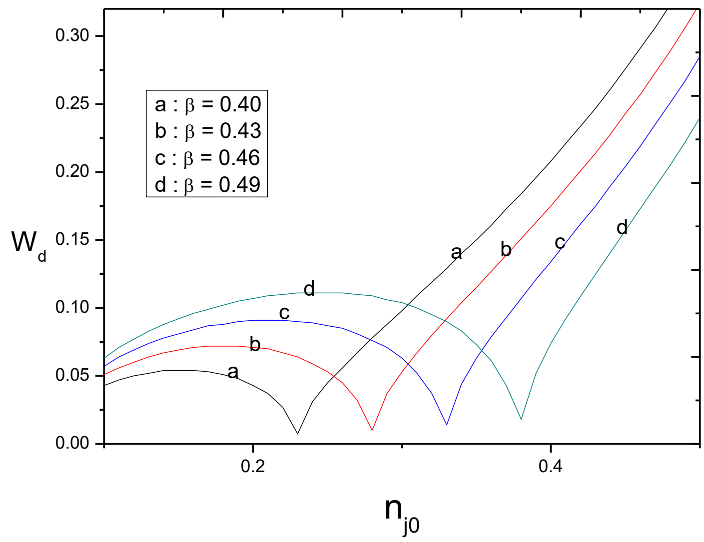

Fig. 11.7(b). Variation of width of IADLs with negative ion concentration (n_{j0}) for different values of nonthermal parameter (β) in (He^{+},Cl^{-}) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, χ = 0.1, V=1.6, u_{i0} =0.1 and u_{j0} =0.2; Graphs a, b, c and d represent β = 0.40, 0.43, 0.46 and 0.49 respectively.

It is seen from Fig.11.7(a) that the amplitude of IADLs is large for large values of β. Fig. 11.7(b) shows that the width of IADL is large for large values of β. T. The charactistic of the variation of width would be understood from the figure.

Similarly, Fig.11.8(a) and Fig.11.8(b) show the variation of amplitude and width of IADLs with negative ion concentration (n_{j0}) in (He^{+},Cl^{-}) plasma for different values of positron density (χ).

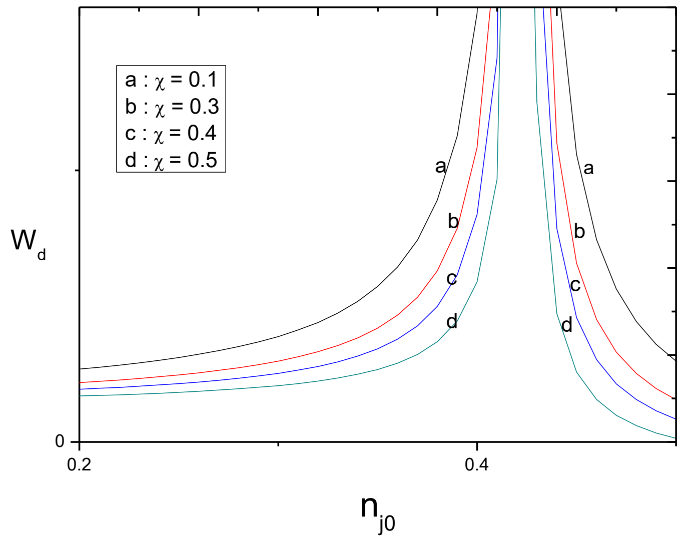

Fig.11.8 (a). Variation of amplitude of IADLs with negative ion concentration (n_{j0}) for different values of positron density (χ) in (He^{+},Cl^{-}) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, V=1.6, β = 0.5, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent χ = 0.1, 0.3, 0.4 and 0.5 respectively.

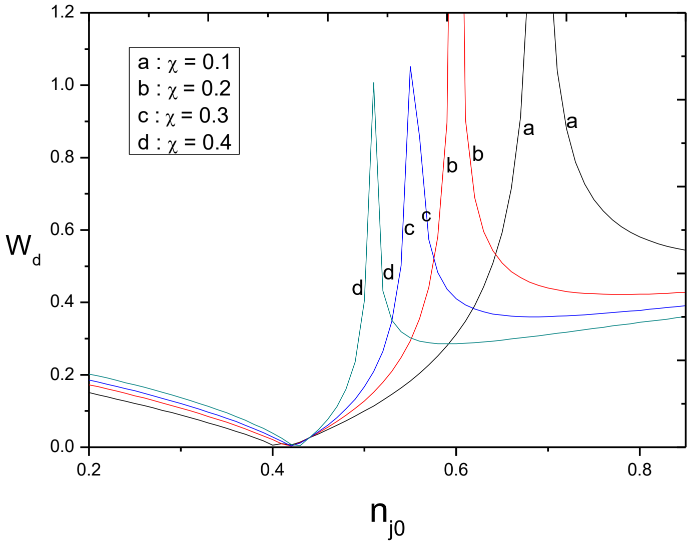

Fig. 11.8(b). Variation of width of IADLs with negative ion concentration (n_{j0}) for different values of positron density (χ) in (He^{+},Cl^{-}) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, V=1.6, β = 0.5, u_{i0} =0.1, u_{j0} =0.2 and p=1; Graphs a, b, c and d represent χ = 0.1, 0.2, 0.3 and 0.4 respectively

It is seen from Fig.11.8 (a) and Fig.11.8 (b) that the width of IADLs is large for small values of χ and the widths of IADLs are small for small value of χ . It decreases with the increase of positron density (χ).

11.5. SUMMARY AND CONCLUSION

Considering a multi-component plasma composed of warm positive ions and negative ions, positrons and nonthermal electrons, we have theoretically studied the excitation of small amplitude IADLs in the plasma. It has been shown that the IADLs in such plasma depend on nonthermal electrons, density of negative ions, density and temperature of positrons, ion temperature and ion stream velocity. Numerical estimation is made taking model plasma and the profiles of IADLs are drawn. The IADLs may be compressive or rarefactive depending on the values of plasma parameters.

However, for the present study of small amplitude ion acoustic IADLs we have considered an unmagnetized plasma. Inclusion of static magnetic field in the plasma may yield more interesting results. For the study of large amplitude IADLs the terms φ^4, φ^5 etc. of right hand side of Eq. (11.14) should be taken into account and the solution of IADLs may be obtained using tanh-method [21-24].

REFERENCES

[1]. H. Massey, Negative ions, 3rd edn. (Cambridge University Press,Cambridge, 1976), p. 663.
[2].R.A. Gottscho, C.E. Gaebe, IEEE Trans. Plasma Sci. **PS- 14**, p.92 (1986).
[3]. M. Bacal, G.W. Hamilton, Phys.Rev.Letts., **42,** p.1538 (1979).
[4].S.R. Majumdar, A. Roychowdhury, S.N. Paul, Phys. Scripta, **69**, p.355 (2004).
[5].Robert L. Merlino and Jackson J. Loomis. Phys. Fluids B: Plasma Physics, **2,** p.2865 (1990).
[6].S. L. Jain, , R. S. Tiwari, and , S. R. Sharma , Canadian J. Phys., **68(6**), p.474 (1990).
[7]. M.K.Mishra, A.K. Arora, R.S.Chhabra , Phys. Rev. E,. p.046402 (2002).
[8].S N Paul, S Chattopadhyaya, S K Bhattacharya and B Bera ,PRAMANA , **60,** p.1217 (2003).
[9].S. Baboolal , R. Bharuthram and M. A. Hellberg, J. Plasma Phys., **46**, p.247 (2009).
[10].T. S. Gill, P. Bala,H. Kaur, N. S. Saini, S. Bansal and J. Kaur , Eur. Phys. J., D **31**, p. 91 (2004).
[11].J. Das, A.Bandyopadhyay and K. P. Das, J. Plasma Phys., **74**, p.163 (2008).
[12].B. Das and P.Chatterjee, Phys. Letts. A, **373**, p.1144 (2009).
[13].B. Ghosh, S. N. Paul, C. Das, I. Paul, S. Banerjee , Brazilian J. Phys., **43** (1-2) DOI 0.1007/s13538-012-0115-February (2013).
[14].H.Kaur, T.S. Gill and Parveen Bala, Pramana, **89,** p.28 (2017).
[15].R. Bharuthram, Astrophys.Sp. Science, **189**, p. 213 (1992).
[16].M. K. Mishra, R. S. Tiwari, and S. K. Jain, Phys. Rev., **E 76,** p.036401 (2007).
[17].S. K. Jain and M K Mishra J. Plasma Phys., **79,** p. 661(2013).
[18].R.A. Cairns, R. Bingham, R.O. Dendy, C.M.C. Nairn, P.K. Shukla, A.A. Mamun, J. Phys. IV France **5**, C6–43 (1995).
[19].R.A. Cairns, A.A. Mamun, R. Bingham, R. Bostrom, R.O. Dendy, C.M.C. Nairn, P.K. Shukla, Geophys. Res. Letts., **22,** p.2709 (1995).
[20]. S. I. Popel, S. V. Vladimirov and P. K. Shukla,Phys. Plasmas, **2**, p.716 (1995).
[21].G. C. Das and J. Sarma, Phys. Plasmas, **5**, p.63 (1998).
[22].A. Bandyopadhyay and K. P. Das , Phys. Scripta, **61,** p.92 (2000).
[23].B.Sahu, R.Roychowdhury, Phys.Plasmas, **11,** p.1947(2004).
[24].S.N.Paul, A.Chatterjee , I.Paul and B.Ghosh , International J. Chemical and Physical Sciences, ISSN:2319-6602 ,IJCPS **Vol. 4** Special Issue-6 , APST–2015, p.38 (2015).

CHAPTER--12

EFFECT OF RELATIVISTICALLY DEGENERATE ELECTRONS AND POSITRONS ON ION-ACOUSTIC DOUBLE LAYERS IN WARM ION PLASMA

[This Chapter is based on the paper: International Journal of Research in Engineering and Science (IJRES),Vol. 10(2), pp. 01-11 (2022)]

12. 1. INTRODUCTION

Nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently in the plasma consisting of electrons and ions. The relativistic effect on solitons e-i-p plasma is found to be interesting. Mushtaq and Shah [1], Gill *et al.* [2], Saeed [3] and other authors [4-9] have shown that both positrons and relativistic factor cause to increase the amplitude of soliton and to shrink the width of solitons. But in the plasmas, where the density is quite high and temperature is very low, the thermal deBroglie wavelength may become comparable to the interparticle distances. In such situations, quantum effects become important due to the overlapping of wave functions of the neighbouring particles. The quantum effects can significantly modify the linear and nonlinear characteristics of waves in degenerate plasma [10-16]. Considering an unmagnetized two-species relativistic quantum plasma system composed of electrons and ions, Sahu [17] have derived the Korteweg–deVries (KdV) equation by reductive perturbation method (RPM) using one-dimensional quantum hydrodynamic model (QHD).

But, in relativistic degenerate (RD) dense plasma, propagation of waves is generally studied following the works of Chandrasekhar [18-19] instead of using D-P model [20]. The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the nonrelativistic limit, and $P_e \propto n_e^{4/3}$ for the ultra relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Shah *et al.* [21] have undertaken investigation on the effect of trapping on the formation of solitons in RD plasmas. They have used the Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. The Sagdeev potential approach has been employed to obtain the arbitrary amplitude solitons when the plasma has been considered cold and when small temperature effects have been taken into account. The theoretical results obtained have been analyzed numerically for different parameter values, and the results have been presented graphically. Masood and Eliasson [22] have studied electrostatic solitons in quantum plasma with RD electrons and cold ions. The inertia is given by the ion mass while the restoring force is provided by the relativistic electron degeneracy pressure, and the dispersion is due to the deviation from charge neutrality. A nonlinear KdV equation is derived for small but finite amplitude waves and is used to study the properties of localized ion acoustic solitons (IAS) for parameters relevant for dense astrophysical objects such as white dwarf stars. Later, Chandra *et al.* [23] have studied electron-acoustic solitons in a RD quantum plasma with two-temperature electrons using the QHD model and degenerate electron pressure $P_e \propto n_e^{4/3}$ .They have shown that degeneracy parameter significantly influences the conditions of formation and properties of soliton structures. Recently, shock waves and the formation of solitons in electron-acoustic wave in inner magnetosphere plasma with RD particles are studied by Goswami *et al*. [24]. They have considered collisions which amount to dissipation actions and shocks. By using the RPM, they have derived a KdV-Burger equation and numerically analyzed by employing the tanh-method. The results are important in explaining the observed geomagnetic storm in recent years.

Since, sufficient works have not been done on the electro-static double layers (DLs) in RD plasma, in this Chapter, we have studied the ion- acoustic DLs (IADLs) in unmagnetized fully nonlinear plasma with RD electrons and positrons and non-degenerate warm ions using Sagdeev potential method (SPM) [25]. From the nonlinear equation, the solution of IADLs of IAW has obtained in such plasma. The profiles of IADLs are drawn and discussed for different values of the plasma parameters. The effects of degenerate electrons and positrons on the IADLs in RD plasma are shown graphically.

12.2. BASIC EQUATIONS

We consider unmagnetized plasma whose constituents are RD electrons and positrons, and nondegenerate warm ions. The dynamics of such plasma are governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{12.1}$$

$$\frac{\partial u_i}{\partial t} + \frac{1}{2}\frac{\partial u_i^2}{\partial x} + \frac{\partial \varphi}{\partial x} + \frac{\sigma_i}{2}\frac{\partial n_i^2}{\partial x} = 0 \tag{12.2}$$

$$en_e \frac{\partial \varphi}{\partial x} - \frac{\partial P_e}{\partial x} = 0 \quad (12.3)$$

$$en_p \frac{\partial \varphi}{\partial x} + \frac{\partial P_e}{\partial x} = 0 \quad (12.4)$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \beta_2 n_e + n_i + \alpha_2 n_p = 0 \quad (12.5)$$

Where,

$\alpha_2 = n_{p0} / |Z_i| n_{i0}, \beta_2 = n_{e0} / |Z_i| n_{i0}, \sigma_i = 3T_i / T_{Fe}$. All other physical quantities are dimensionless. The number density of electrons, ions and positrons are represented by n_e, n_i and n_p respectively and rescaled by their equilibrium values. u_i is the ion velocity, T_i and T_{Fe} are the ion temperature and electron Fermi temperature respectively. Other parameters have their usual meanings. The equilibrium condition of charge neutrality is $\beta_2 = 1 + \alpha_2$.

Following Chandrasekhar [18, 19], the degeneracy pressure of electron and positron in fully degenerate and relativistic configuration can be expressed in the form:

$$P_j = \left(\pi m_e^4 c^5 / 3h^3\right)\left[R_j (2R_j^2 - 3)\sqrt{1 + R_j^2} + 3\sinh^{-1}(R_j)\right] \quad (12.6)$$

In which $R_j = p_{Fj} / m_e c = \left[3h^3 n_j / 8\pi m_e^3 c^3\right]^{\frac{1}{3}} = R_{j0} n_j^{\frac{1}{3}}$ (12.7)

Where $R_{j0} = \left(n_{j0} / n_0\right)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3 / 3h^3 \approx 5.9 \times 10^{29} cm^{-3}$, 'c' being the speed of light in vacuum. p_{Fj} is the Fermi relativistic momentum; the index j will denote either electrons (e) or positrons (p) in the plasma. It is to be noted that the parameter R_{j0} is a measure of the relativistic effects and may be called relativistic degeneracy parameter. For ultra- relativistic case $R_{j0} >> 1$ and for weakly relativistic case $R_{j0} << 1$. The parameter R_{j0} can also be related to mass density as $\rho(gr / cm^3) = 1.97 \times 10^6 \cdot R_{j0}^3$. The density of white dwarfs can be in the range $10^5 < \rho < 10^9$. So in this case, the relativity parameter R_{j0} can be in the range $0.37 < R_{j0} < 8$.

In the limits of very small and very large values of relativity parameter R_j, we obtain:

$$P_j = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e} n_j^{\frac{5}{3}} \text{ (For } R_j \to 0\text{)}, \tag{12.8}$$

$$P_j = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} hc n_j^{\frac{4}{3}} \text{ (For } R_j \to \infty\text{)} \tag{12.9}$$

Note that the degenerate electron (positron) pressure depends only on the electron (positron) number density but not on the temperature. Now, from Eqs. (12.1) to (12.9) the densities of degenerate electrons and positrons are obtained as

$$n_e = R_{e0}^{-3}[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2]^{3/2} \tag{12.10}$$

$$n_p = p^{-1}R_{e0}^{-3}[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)]^{3/2} \tag{12.11}$$

where, R_{e0} is the relativistic degeneracy parameter of electrons, $p = n_{p0}/n_{e0} = \alpha_2/\beta_2$.

To obtain localized solutions of Eqs. (12.1) – (12.5) using the SPM we have considered a new moving frame of a single variable η = *x*−*Mt*, which is moving with velocity (Mach number) *M*. Using this transformation and equilibrium quasi-neutrality condition as well as the boundary conditions i) $n_i \to n_{i0}$, ii) $u_i \to 0$, iii) $\varphi \to 0$ at $|x| \to \infty$ we obtain from Eqs.(12.1) and (12.2) the nondegenerate ion density as

$$n_i = \frac{1}{4\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\} \pm \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}]^{1/2} \tag{12.12}$$

Now, using Eqs. (12.10), (12.11) and (12.12) in Eq. (12.5) we get

$$\begin{aligned}\frac{d^2\varphi}{d\eta^2} &= \beta_2 R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2)\right]^{3/2} \\ &-\alpha_2 p^{-1}R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)\right]^{3/2} \\ &-\frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2} - \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2}] = -\frac{d\psi}{d\varphi}\end{aligned} \tag{12.13}$$

(Taking negative branch of ion density).

Where, ψ is the Sagdeev potential and it is given by

$$\psi = -\int \beta_2 R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi\sqrt{1+R_{e0}^2} + \varphi^2 \right]^{3/2} d\varphi$$
$$+\int \alpha_2 p^{-1} R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi\sqrt{1+p^{2/3}R_{e0}^2} + \varphi^2) \right]^{3/2} d\varphi \qquad (12.14)$$
$$+\int\left[\frac{[(\sigma_i + M^2)/2 - \varphi)] - \sqrt{[(\sigma_i + M^2)/2 - \varphi]^2 - \sigma_i M^2}}{\sigma_i} \right]^{1/2} d\varphi$$

After simplification ψ becomes

$$\psi = \frac{\beta_3}{8}\left[-2(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{3/2} + 3(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2} - 3\ln\{\frac{(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2}}{(R_{eo}+\gamma_1)}\} \right]$$
$$+\frac{\alpha_3}{8}\left[2(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{3/2} - 3(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2} + 3\ln\{\frac{(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2}}{(p^{1/3}R_{eo}-\gamma_2)}\} \right]$$
$$+\beta_3\gamma_1(2R_{eo}^2+3) + \alpha_3\gamma_2(2p^{2/3}R_{eo}^2-3) + (M^2+\frac{\sigma_i}{3}) - \frac{2^{1/2}\sigma_i}{3}(M_1-M_2)^{1/2}(2M_1+M_2)$$

(12.15)

where

$$\gamma_1 = (1+R_{e0}^2)^{1/2}, \gamma_2 = (1+p^{2/3}R_{e0}^2)^{1/2}, \beta_3 = \beta_2 / \mathrm{R}_{eo}^3, \alpha_3 = \alpha_2 / pR_{eo}^3,$$
$$M_1 = 1 + M^2/\sigma_i - 2\varphi/\sigma_i, M_2 = (M_1^2 - 4M^2/\sigma_i)^{1/2}, \beta_2 = 1+\alpha_2, p = \alpha_2/\beta_2.$$

Now, integrating (12.14) we obtain an equation (the Energy law) in terms of a single independent variable

$$\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \qquad (12.16)$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi \to 0, d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0.$

For the double layer solution of Eq. (12.16), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi = 0$ and φ_m, where φ_m is some extreme value of the potential φ, called the amplitude of the IADL. Moreover, $\psi(\varphi)$ must satisfy the boundary conditions mentioned in Chapter-9 [Eqs. (9.20) to (9.22)]. .

12.3. SOLUTION FOR ION-ACOUSTIC DOUBLE LAYERS

Now we expand the right hand side of Eq.(12.13) and Eq.(12.14) in power series of ϕ and we obtain

$$\frac{d^2\varphi}{d\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + A_4\varphi^4 + \cdots\cdots \tag{12.17}$$

where,

$$\begin{aligned} A_1 &= \beta_2 R_{e0}^{-3}[\{3(1+R_{e0}^2) - \frac{3}{8}(1+R_{e0}^2)^{-1} - \frac{3}{16}(1+R_{e0}^2)^{-2} - \frac{15}{128}(1+R_{e0}^2)^{-3}\}] \\ &-\alpha_2 p^{-1} R_{e0}^{-3}[\{-3(1+p^{2/3}R_{e0}^2) + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-1} + \frac{3}{16}(1+p^{2/3}R_{e0}^2)^{-2} \\ &+\frac{15}{128}(1+p^{2/3}R_{e0}^2)^{-3}\}] - \frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^{-1}\} - \{(M-\sqrt{\sigma_i})^{-1}\}] \end{aligned} \tag{12.18a}$$

$$\begin{aligned} A_2 &= \beta_2 R_{e0}^{-3}[\{3(1+R_{e0}^2)^{1/2} + \frac{3}{8}(1+R_{e0}^2)^{-3/2} + \frac{3}{8}(1+R_{e0}^2)^{-5/2} + \frac{45}{128}(1+R_{e0}^2)^{-7/2}\}] \\ &-\alpha_2 p^{-1} R_{e0}^{-3}[\{-3(1+p^{2/3}R_{e0}^2)^{1/2} + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-3/2} \\ &+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-5/2} + \frac{45}{128}(1+p^{2/3}R_{e0}^2)^{-7/2}\}] \\ &-\frac{1}{4\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^{-3}\} - \{(M-\sqrt{\sigma_i})^{-3}\}] \end{aligned} \tag{12.18b}$$

$$\begin{aligned} A_3 &= \beta_2 R_{e0}^{-3}[\{1 - \frac{3}{8}(1+R_{e0}^2)^{-2} - \frac{5}{8}(1+R_{e0}^2)^{-3} - \frac{105}{128}(1+R_{e0}^2)^{-4}\}] \\ &-\alpha_2 p^{-1} R_{e0}^{-3}[\{1 + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2} + \frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3} + \frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}] \\ &-\frac{1}{8\sqrt{g_i}}[\{(M+\sqrt{g_i})^{-5}\} - \{(M-\sqrt{g_i})^{-5}\}] \end{aligned} \tag{12.18c}$$

and

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2 - \frac{1}{3}A_2\varphi^3 - \frac{1}{4}A_3\varphi^4 - \cdots\cdots \tag{12.19}$$

Assuming small amplitude waves $\varphi < 1$ and taking the terms up to φ^3 of Eq.(12.18) we get the solution of IADLs in plasma. Applying boundary conditions for the existence of IADLs, we obtain

$$\varphi_m = -\frac{2A_2}{3A_3} \tag{12.20}$$

and

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{12.21}$$

The solution of Eq.(12.17) for small amplitude IADLs satisfying Eq.(12.19) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh[\sqrt{(A_3 / 8\varphi_m)}\eta]\right) \tag{12.22}$$

Where,

$\varphi_m / 2 = \varphi_{0D}$, φ_{0D} is the amplitude of IADLs.

The width of IADLs is given by

$$W_d = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{12.23}$$

It is seen that width of IADLs depends on the plasma parameters e.g. RD parameter, density of RD electrons, density of RD positrons, ion temperature etc. Eq. (12.20) relates the amplitude φ_m of the IADLs to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (12.17). It is important to be noted that the IADLs in plasma can only exist for positive value of A_3 i.e. $A_3 > 0$. To satisfy the condition, the plasma parameters should have some critical values. The degenerate electron density β_2 must satisfy the condition $\beta_2 < \beta_{2c}$, β_{2c} is the critical value of β_2. For the existence of IADL, the parameter β_{2c} obtained from Eq. (12.18c) is

$$\beta_{2c} = (p^{-1}C_2 + D) / C_1 \tag{12.24}$$

Where,

$$C_1 = \mathrm{R}_{e0}^{-3}[\{1 - \frac{3}{8}(1+R_{e0}^2)^{-2} - \frac{5}{8}(1+R_{e0}^2)^{-3} - \frac{105}{128}(1+R_{e0}^2)^{-4}\}$$
$$-p^{-1}\{\{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2} + \frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3} + \frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$

$$C_2 = \{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2} + \frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3} + \frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$

$$D = \frac{1}{8\sqrt{\sigma_i}}[\{(M - \sqrt{\sigma_i})^{-5}\} - \{(M + \sqrt{\sigma_i})^{-5}\}]$$

Now, if IADLs are excited in plasma for $A_3 > 0$, we see that the sign of A_2 will determine the nature of IADLs. If $A_2 > 0$, the IADLs will be rarefactive and for $A_2 < 0$ the IADLs will be compressive. It is seen that A_2 depends on the parameters R_{e0} , α_2 , β_2 , p , σ_i and M of RD e-i-p plasma.

12.4. RESULTS AND DISCUSSION

From the solution of Eq.(12.22) for IADLs in a relativistically degenerate e-i-p plasma it seen that excitation of IADLs depends on

i). relativistic degeneracy parameter (R_{e0})

ii). degenerate positron density (p)

ii). degenerate electron density (α_2)

iv). ion temperature (σ_i)

The role of these parameters on the IADLs is shown in Fig. 12.1 to Fig.12.4. To draw the Figs, we have followed the conditions for the existence of IADLs, i.e. the coefficient A_3>0 for the values of the plasma parameters. To understand the nature of IADLs, the amplitude of DLs given by (12.20) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs.

12.4.1. Profiles of Ion-Acoustic Double Layers

i). Effect of relativistic degeneracy parameter

To see the effects of relativistic degeneracy parameter (R_{e0}) on the IADLs in degenerate plasma the Fig.12.1 is drawn.

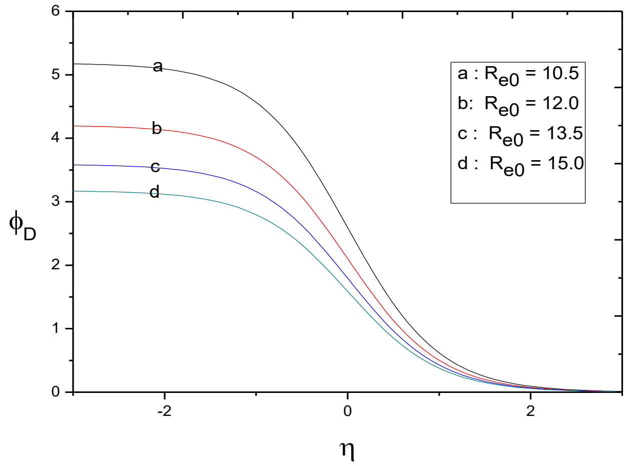

Fig.12.1. Profiles of IADLs in RD plasma with fixed values of p =0.1, σ_i =0.01, α_2 =0.4 and M =2; Graphs a, b, c and d represent R_{e0} = 10.5, 12.0, 13.5 and 15.0 respectively.

It is seen from Fig.12.1 that the IADLs are compressive and the amplitude decreases with increase of R_{e0}. Similar information is obtained from the Eq.(12.20). The coefficient A_2 is negative for R_{e0} = 10.5, 12.0, 13.5 and 15.0 indicating compressive IADLs in relativistically degenerate plasma.

ii). Effect of degenerate positron density

The Fig.12.2 is drawn to find the effects of RD positron density (p) on the IADLs in relativistically degenerate plasma.

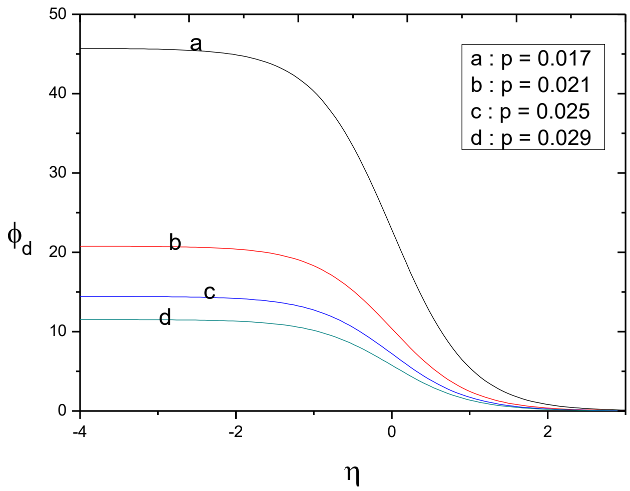

Fig. 12.2. Profiles of IADLs for different values of p in RD plasma with fixed values of α_2 = 0.3, σ_i =0.01, M =2 and R_{e0} =10; Graphs a, b, c and d represent p = 0.017, 0.021, 0.025, 0.029 respectively.

It is seen from Fig, 12.2 the IADLs are compressive and the amplitude of IADLs decreases with the increase of relativistically degenerate positron density (p). Similar information is obtained from the Eq. (12.20) as the coefficient A_2 is negative for same values of p which indicate the compressive IADLs in RD plasma.

iii). Effect of degenerate electrons

In similar way, Fig.12.3 is drawn using the Eq.(12.22) to see the effects of RD electron density (α_2) on the profiles of IADLs.

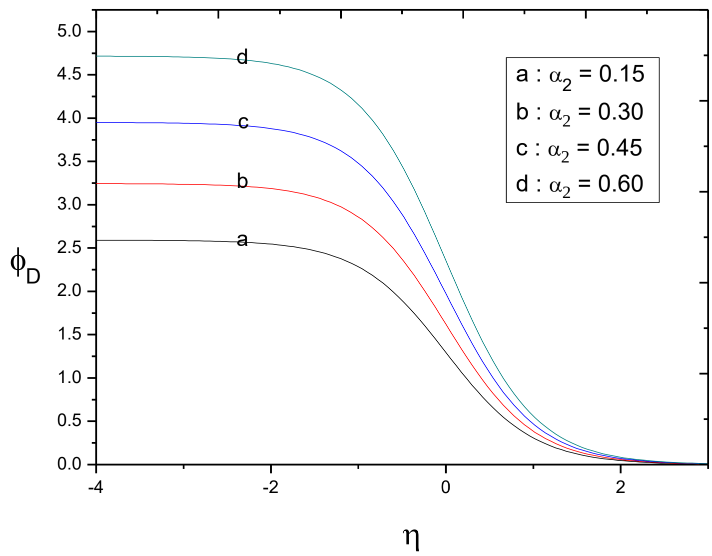

Fig.12.3. Profiles of IADLs for different values of α_2 in RD plasma with fixed values of σ_i =0.01, R_{e0} =11, M =1.9 and p =0.1; Graphs a, b, c and d represent α_2 = 0.15, 0.3, 0.45 and 0.6 respectively.

It is observed from Fig.12.3 that the compressive IADLs are excited in plasma and the amplitude of compressive IADLs increases with increase of α_2. The results of this figure is supported from the expression Eq.(12.20) as the coefficient A_2 is negative indicating the excitation of compressive IADLs since A_3 is positive

iv). Effect of ion temperature

Similarly, Fig.12.4 is drawn to observe the effects of ion temperature (σ_i) on the profiles of IADLs in RD plasma.

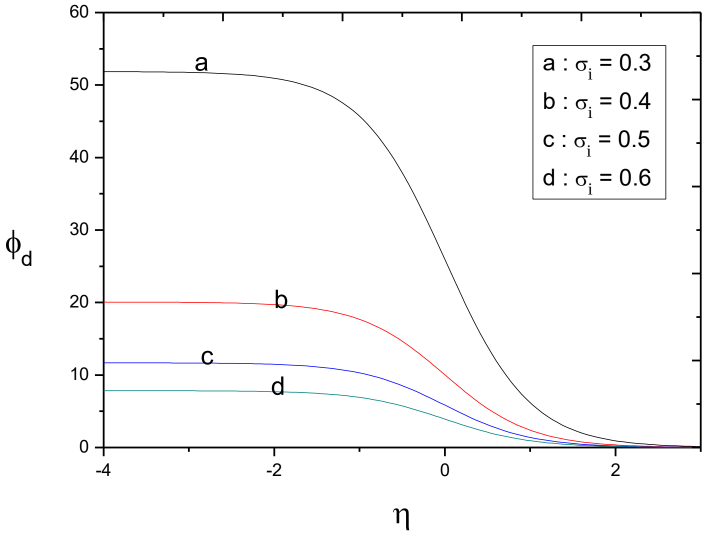

Fig.12.4. Profiles of IADLs for different values of ion temperature (σ_i) in RD plasma with fixed values of α_2 = 0.3, p =0.01, M =2 and R_{e0} =10. Graphs a, b, c and d represent σ_i = 0.3, 0.4, 0.5 and 0.6 respectively.

It is observed from Fig.12.4 that the compressive IADLs are excited in RD plasma for the different values of ion temperature (σ_i) from 0.3 to 0.6. The IADLs are compressive because the amplitude of IADLs is positive since $A_2 < 0$ which is numerically obtained from Eq. (12.18b). Moreover, the amplitude of compressive IADLs decreases with increase of σ_i .

12.4.2. Profiles of Width for Ion-Acoustic Double Layers

It is known that the width of IADLs in plasma is important for experimental studies of the IADLs in laboratory and in space plasma. We have numerically estimated the width of IADLs using Eq.(12.23) and its variation for different parameters of RD plasma is graphically discussed. It is to be remembered that for the numerical estimation we have followed the conditions for the existence of IADLs (i.e. A_3 must be positive).

Now, Fig.12.5 is drawn for the variation of the width of IADLs with the RD parameter (R_{e0}) in plasma for different values of RD positron density (p).

Similarly, the Fig.12.6 is drawn to observe the effects of RD electrons (α_2) and R_{e0} on the width of IADLs.

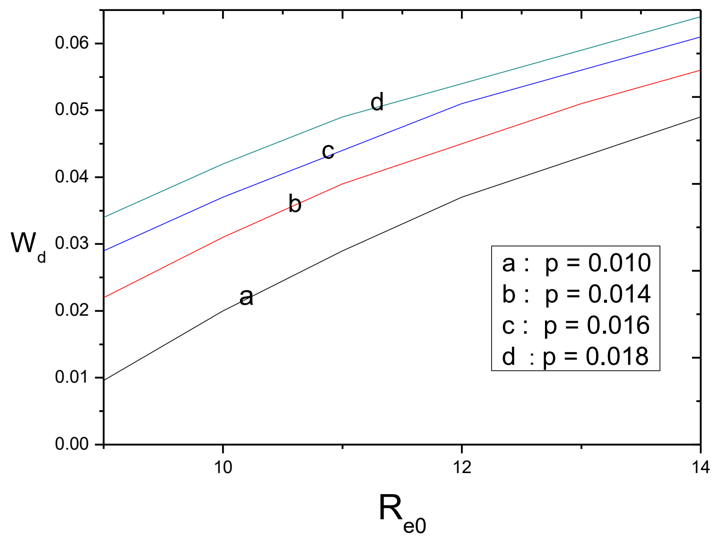

Fig.12.5. Variation of width of IADLs with R_{e0} for different values of p in RD plasma with fixed values of σ_i = 0.01, α_2 =0.12 and M =2; Graphs a, b, c and d represent p = 0.01, 0.014, 0.016 and 0.018 respectively.

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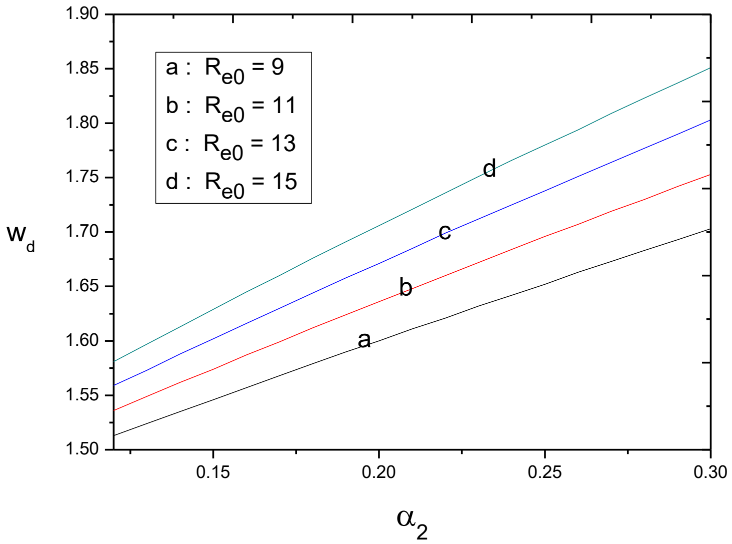

Fig.12.6. Variation of width of IADLs with α_2 for different values of R_{e0} in RD plasma with fixed values of σ_i =0.01, p =0.1 and M =2; Graphs a, b, c and d represent R_{e0} = 9, 11, 13 and 15 respectively.

It is observed from Fig.12.5 that the width of IADLs increases with increase of R_{e0} for p from 0.01 to 0.018.

On the other hand, it is seen from Fig.12.6 that the width of IADLs increases with increase of α_2 for R_{e0} from 9 to 15.

12.5. SUMMARY AND CONCLUSION

Considering a multi-component plasma composed of RD electrons and RD positrons with nondegenerate warm ions, we have theoretically studied the excitation of IADLs in the plasma using the pseudo-potential method. It has been shown that the IADLs in such plasma depend on the plasma parameters. From numerical estimation the profiles of IADLs are drawn. The IADLs in RD plasma are compressive and the amplitudes depend on the values of the plasma parameters. It is seen that:

i). The IADLs in RD e-i-p plasma are compressive in nature for different values of plasma parameters i.e. RD positron, RD electrons and ion temperature.

ii). The amplitudes of IADLs are small for large values of RD positron and ion temperature. But, the amplitude of IADLs is small for small values of degenerate electron.

iii).The width of IADLs increases with increase of R_{e0} for different values of p when other plasma parameters have fixed values. Moreover, the width of IADLs increases with increase of p .

iv). The width of IADLs increases with increase of α for different values of R_{e0} when other plasma parameters have fixed values. Moreover, the width of IADLs increases with increase of R_{e0} .

However, the effect of ion drift has not been considered in the present investigation. It is known that ion- acoustic wave will propagate through the plasma with two modes, fast-mode and slow- mode, in presence of drifting (streaming) ions. Considering the ion drift some new results on nonlinear propagation of waves in degenerate plasma may be obtained from which the causes of some phenomena in astrophysical objects may be understood.

REFERENCES

[1]. A.Mushtaq and H. A. Shah, Phys. Plasmas, **12**, p.072306 (2005).

[2]. T. S. Gill, A. Singh, H.Kaur, N. S.Saini, P.Bala, Phys. Letts. A, **361**, p. 364 (2007).

[3]. R. Saeed, A. Shah, and M. N.Haq, Phys. Letts. A, **373 (45)**, p.4164 (2009).

[4]. A. Shah and R.Saeed, Phys. Plasmas, **17**, p.102301 (2010) .

[5]. A. Shah, S. Mahmood, and Q. Haque , Phys. Plasmas, **18**, p.114501 (2011).

[6]. K. Javidan and D. Saadatmand, Astrophys. Sp. Science ,**333**, p.471 (2011).

[7]. A. Shah, Q. Haque and S. Mahmood, **,**Astrophys.Sp. Science., **335 (2)**, p.529 (2011).

[8]. E. Saberian, A. Esfandyari-Kalejahi and M. Akbari-Moghanjoughi, Canad. J. Phys. ,**89 (3**), p.299 (2011).

[9]. R. Das and R. Sarma, International Journal of Scientific & Engineering Research, **6 (7)** , p.1115 (2015).

[10]. F. Hass, Phys. Plasmas,**12**, p.062117 (2005).

[11]. P. K. Shukla and B. Eliasson, Phys. Rev. Letts., **96**, p.245001 (2006).

[12]. A. P. Misra and A. Roychowdhury, Phys. Plasmas, **13**, p.072305 (2006).

[13]. A .P. Misra and C. Bhowmik, Phys. Plasmas, **14**, p.012309 (2007).

[14]. B. Ghosh, S. Chandra and S. N .Paul, Phys. Plasmas, **18**, p.012106 (2011).

[15]. B. Ghosh , S. Chandra and S. N. Paul, Pramana, **78 (5), p.**779 **(** 2012).

[16]. M. McKerr,F. Haas and I. Kourakis , Phys Plasmas, **23,** p.052120 **(** 2016).

[17]. B. Sahu, _Pramana , **76 (6),** p. 933 **(** 2011).

[18]. S. Chandrasekhar , Philosophical Magazine, **11 (7),** p**.**592 **(** 1931).

[19]. S. Chandrasekhar , Mon. Not. R. Astron. Soc. , **170,** p.405 **(** 1935).

[20]. G. C. Das, and S. N. Paul , Phys. Fluids , **28,** p.823 (1985).

[21]. H. A. Shah, W.Masood, M. N. S. Qureshi and N. L. Tsintsadze, Phys. Plasmas, **18,** p.102306 **(** 2011).

[22]. W Masood and B.Eliasson , Phys Plasmas, **18,** p. 034503 **(**2011).

[23]. S. Chandra , B.Ghosh and S.N.Paul, Astrophys. Sp Science**, 343 ,** p**.**21**3 (**2013).

[24]. J. Goswami, S. Chandra and B. Ghosh, Astrophys. Sp Science., **364**, p.65 (2019).

[25]. R.Z.Sagdeev, in Reviews of Plasma Physics, ed. by M.A. Leontovich (Consultants Bureau, New York, 1996), **4**, p. 23.

SUMMARY AND FUTURE SCOPE OF THE THESIS

Summary of the Thesis

In the Thesis, we have theoretically and numerically studied the "Ion Acoustic Solitons and Double Layers in Different Types of Plasma" using standard mathematical methods, i.e. Reductive perturbation method, Sagdeev potential method, KBM method, tanh-method etc. For numerical analysis, we have used software like MathCAD, Origin etc. For the study, we have considered both non-relativistic and relativistic plasma consisting of positive and negative ions, isothermal electrons, non-isothermal two-temperature electrons, nonthermal electrons, positrons, superthermal (kappa distributed) electrons, q-nonextensive electrons, charged dust particles, beam ions, degenerate electrons, degenerate positrons, ultra-relativistic degenerate electrons and positrons, relativistically degenerate electrons and positrons etc.

The Thesis has two parts:

Part One: We have studied "Solitons in Different Types of Plasma", e.g. Small amplitude KdV solitons, Large amplitude higher-order solitons, Dressed solitons etc. (in Chapter- 2 to Chapter-7); and Envelope solitons (in Chapter- 8).

Part Two: We have theoretically investigated "Double Layers in Different Types of Plasma), e.g. unbounded plasma, bounded plasma (in Chapter- 9 to Chapter-12)

In the last Chapter, "Summary and Future Scope of the Thesis" are stated.

Chapter-2: The exact analytical solutions of the nonlinear equations governing the propagation of ion-acoustic solitons (IAS) in a plasma consisting of warm streaming positive ions and nonthermal electrons has been obtained with necessary and sufficient conditions for the existence of IAS. From these conditions, the critical values of the nonthermal parameter, ion temperature and velocity of IAS have been numerically evaluated and their dependence on various plasma parameters is discussed. The structure of the IAS as a function of the non-thermal electron parameter are analyzed which gives the deviation from the thermalized state. Moreover, the expression of Sagdeev potential has been derived and the conditions of soliton solution are obtained. Slow and fast IAS would be excited in presence of stream of ions and nonthermal electrons in plasma. The profiles of small amplitude IAS of fast-and slow-mode of the wave are graphically shown and discussed for different values of the parameters of the plasma. It is seen that small amplitude IAS for fast-mode is rarefactive but for slow-mode it is compressive.

Chapter-3: For the study of W-type IAS in a plasma consisting of cold positive ions and nonthermal electrons, the nonlinear equations are analysed with the help of the Krylov-Bogoliubov-Mitropolsky (KBM) method. The IAS are evaluated in first-, second-and third-order and the profiles are drawn for different values of the nonthermal parameter of electrons. It is seen that nonthermal electrons have a considerable impact on the shape of IAS in each order. The plasma consisting of cold positive ions without negative ions can support the formation of compressive as well as W-type solitons in second- and third- order for a certain value of the nonthermal parameter of electrons. The results are new and interesting because W-type IAS have been found by earlier authors in plasma having the negative ions. The IAS near-critical value of the nonthermal parameter and arbitrary amplitude solitons has also been studied in this Chapter. The results would be useful to understand the nonlinear wave processes in ionospheric and magnetospheric multi-component plasma having nonthermal electrons.

Chapter-4: Using the SP Method, we theoretically investigated arbitrary amplitude IAS in an electron-ion-positron (e-i-p) plasma. The expressions of phase velocities of slow- and fast-modes of the wave have been derived and graphically analyzed. The critical values of densities of positrons and ions, Mach number and positron temperature for the excitation of IAS have also been obtained. The solutions of the first-order and next higher-order IAS have been obtained and the profiles of the solitons have been drawn taking different values of the parameters of e-i-p plasma. It is observed that both compressive and rarefactive IAS would be excited for the positrons in e-i-p plasma. The variation of amplitudes of the IAS has been shown graphically for different values of density and temperature of positrons, drift velocity and temperature of the ions. Moreover, small and large amplitude IAS of fast-and slow-mode of the wave are graphically shown and discussed for different values of the parameters of e-i-p plasma. It is seen that small amplitude and large amplitude IAS for fast-mode are rarefactive but the IAS for slow-mode may be compressive and rarefactive depending on the values of parameters of e-i-p plasma.

Chapter-5: Using the reductive perturbation method (RPM), we have studied the IAS in self-gravitating dusty plasma consisting of warm positive ions, weak non-isothermal two-temperature electrons and negatively charged dust particles having variable charges. We have derived the uncoupled third-order partial differential equations instead of coupled nonlinear equations in self-gravitating plasma for the normal non-isothermal electrons and also for weak non-isothermal electrons. From the uncoupled nonlinear equation, the solution of IAS in weak non-isothermal electrons has been obtained and the effects of two-temperature electrons, gravity, and dust charge fluctuations on the IAS have been discussed.

Chapter-6: We have derived the integral form of governing equation in terms of pseudo-potential for the ion-acoustic waves (IAW) in an ultra-relativistic degenerate (URD) plasma consisting of cold inertial ions, degenerate two-temperature electrons and negatively charged dust particles. Considering the effect of higher-order nonlinear and dispersive effects, we have studied the dressed soliton of IAW using the KBM method. The profiles of dressed solitons are drawn and discussed for different values of density and temperature of URD two-temperature electrons. Moreover, the solution for the dressed solitons at a critical state is obtained and discussed graphically. It is seen that the degenerate two-temperature electrons have a considerable impact on the structure of dressed solitons in URD plasma.

Chapter-7: The KdV equation has been derived using the RPM in unmagnetized quantum plasma in presence of an ion beam and the solution of IAS is obtained from the KdV equation. The theoretical results have been analyzed numerically and graphically for different values of ion beam density, ion beam velocity and ion beam temperature. It is seen that the formation and structure of IAS are significantly affected by the ion beam in quantum plasma. The results would be useful for understanding the beam-plasma interactions and formation of nonlinear wave structures in dense quantum plasma.

Chapter-8: Using the multiple scale method, we have studied the modulation instability and envelope soliton of high-frequency surface waves on warm plasma half-space. The second harmonic generated through nonlinear self-interaction of the wave is found to carry a fraction of the surface wave energy into the bulk of the plasma. This is found to influence the modulation instability mechanism. To describe the nonlinear evolution of the wave a nonlinear Schrodinger (NLS) equation is derived in which the coefficient of the nonlinear term becomes a complex quantity. From this evolution equation, we have derived the condition of instability and show that high-frequency surface waves on a temperate plasma half-space are modulationally unstable. The dependence of the growth rate of instability on temperature and wave number is computed and presented graphically. The solution for the envelope solitons (bright and dark) is obtained and discussed.

Chapter-9: Using the SP method, ion-acoustic double layers (IADLs) in multi-component plasma consisting of positive ions, negative ions and nonthermal electrons filled in a cylindrical waveguide has been theoretically investigated. The effects of the boundary of a cylindrical waveguide and nonthermal electrons on the Sagdeev potential have been graphically discussed. From the nonlinear equation, the solution for IADLs of small amplitude IAW is obtained and the structure of IADLs is analyzed as a function of the plasma parameters. It is seen that the finite geometry of bounded plasma, nonthermal electrons, the density of negative ions, stream velocity of positive and negative ions have significant contributions to the structure of IADLs. The IADLs may be compressive or rarefactive in plasma depending upon the values of the plasma parameters.

Chapter-10: The influence of nonthermal electrons and ion beams on the IADLs in warm ion plasma has been studied using the SP Method. The expression of Sagdeev potential of an IAW has been derived and the solution of IADLs has been obtained. The critical value of the nonthermal parameter of electrons for the existence of IADLs is also obtained. The profiles of IADLs in the plasma have been drawn taking different values of density and temperature of ion beams, and the nonthermal parameter of electrons obeying the critical condition. It is seen that both compressive and rarefactive IADLs are excited in the plasma depending on the values of nonthermal electrons and ion beam parameters. The width of IADLs is also graphically discussed.

Chapter-11: Using the SP Method, the effects isothermal positrons and nonthermal electrons on the IADLs have been studied in multi-component plasma containing warm positive ions and negative ions with drift motion. The expression of Sagdeev potential has been derived and the solution for IADLs has been obtained from the nonlinear equation. The profiles of IADLs have been drawn taking different values of the parameters of multi-component plasma. It has been observed that both compressive and rarefactive IADLs would be excited in the multi-component plasma having positrons, nonthermal electrons, negative ions and warm positive ions with drift motion... The variation of amplitudes and widths of IADLs have also been discussed graphically for different values of the parameters of multi-component plasma under consideration for our study.

Chapter-12: The effects of relativistically degenerate electrons and positrons on the IADLs in warm ion plasma have been studied theoretically using the SP Method. The expression of Sagdeev potential is derived and from the nonlinear equation, the solution of IADLs is obtained. The structure of IADLs is drawn and discussed for different parameters of the relativistically degenerate plasma. It is seen that IADLs are compressive and the presence of relativistically degenerate electrons and positrons has a significant contribution to excite the IADLs in relativistically degenerate plasma. The variation of thickness of IADLs is also graphically discussed for different values of the parameters of relativistically degenerate plasma.

Future Scope of the Thesis

In the thesis, the IAS and the IADLs are studied using the plasma models of various astrophysical, space and experimental plasma systems. Theoretical results of the thesis would be useful in understanding the generation of different kinds of nonlinear wave structures in the model plasma. The plasma models used in the present thesis can be generalised by the inclusion of the effects of i) Magnetic field, ii) Collisions, iii) In homogeneity in density & temperature and iv) Relativity etc.

Moreover, the KdV solitons, Dressed solitons, Modulation instability, Envelope solitons, can be studied in fully relativistic plasma having superthermal electrons and Tsallis distributed electrons which are found to exist in space and hot astrophysical objects. For the study of modulation instability of the waves in critical case when nonlinear term vanishes for certain value of the plasma parameter, higher-order NLS equation may be derived using standard mathematical methods. The studies of nonlinear propagation of waves in dusty plasma are important in the context of astrophysical plasma. So, following the mathematical methods presented in the thesis, solitons (KdV solitons, dressed solitons, and envelope solitons) and double Layers can be studied for the dust acoustic waves in dusty plasma.

LIST OF PAPERS INCORPORATED IN THESIS

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1. Analytical study on the existence of ion-acoustic solitary waves in a plasma consisting of warm streaming ions and nonthermal electrons, Indian Journal of Physics, Vol.86(5), pp.395-400 (2012).
2. W-type ion-acoustic solitary waves in plasma consisting of cold ions and nonthermal electrons, Indian Journal of Physics, Vol. 90(10), pp 1195-1205 (2016).
3. Ion-acoustic solitary waves in an electron-ion-positron plasma, Indian Journal of Physics, Vol. 86 (6), pp. 545-553 (2012).
4. Nonlinear propagation of ion-acoustic waves in self-gravitating dusty plasma consisting of non-isothermal two-temperature electrons, Indian Journal of Physics, Vol. 91, pp 101-107 (2017).
5. Ion -acoustic solitary waves in a self-gravitating dusty plasma consisting of weak non-isothermal two-temperature electrons, Journal of Applied Physical Science International (JAPSI), Vol. 8(2), pp. 77-90 (2017).
6. Dressed solitons and double layers of ion-acoustic waves in a dusty plasma having ultra-relativistic degenerate two-temperature electrons, Indian Journal of Physics; DOI :10.1007/s12648-021-02059-4 (2021).
7. Nonlinear propagation of ion-acoustic waves in quantum plasma in presence of an ion beam, Laser and Particle Beams , Vol. 37(4), pp. 370- 380(2019).
8. Modulational instability of high frequency surface waves on warm plasma half-space, Canadian Journal of Physics, Vol. 90, pp. 291-297 (2012).
9. Effects of boundary of cylindrical waveguide and nonthermal, electrons on the ion-acoustic double layers in multicomponent bounded plasma, Brazilian Journal of Physics, Vol. 50, pp.272-281(2020).
10. Effects of nonthermal electrons and ion beams on ion-acoustic double layers in warm ion plasma, Indian Journal of Physics, https://doi.org/10.1007/s12648-020-01899-w (2020).
11. Effects of positrons and nonthermal electrons on the ion-acoustic double layers in drifting negative ion plasma, Journal of Applied Physical Science International (JAPSI) Vol.13(3): pp.44-60 (2021).
12. Effects of relativistically degenerate electrons and positrons on the ion-acoustic double layers in warm ion plasma, International Journal of Research in Engineering and Science) (IJRES) Vol. 10(2) pp. 01-11 (2022).

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ORIGINAL PAPER

Analytical study on the existence of ion-acoustic solitary waves in a plasma consisting of warm streaming ions and nonthermal electrons

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Abstract: Theoretical investigations have been made for the formation of ion-acoustic solitary wave structure in a plasma consisting of warm streaming positive ions and nonthermal electrons. Analytical solutions have been obtained with some sufficient and necessary conditions for the existence of ion-acoustic solitary waves. Some critical values of the ion temperature and the nonthermal parameter of electrons have been obtained and presented graphically. It is shown that drift of the ions and the nonthermal electrons have significant effects on the formation of ion-acoustic solitary waves. In presence of the streaming motion of the ions, two distinct modes (fast and slow) of propagation have been obtained. Limiting values of the soliton speed for these modes have been calculated. It is found that the drift motion and ion temperature change significantly the conditions for the existence and properties of ion-acoustic solitons.

Keywords: Ion-acoustic soliton; Nonthermal electrons; Sagdeev potential

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1. Introduction

The study of ion-acoustic solitary waves has received considerable interest because of its important role in the understanding of the nonlinear wave features in the laboratory and space plasmas [1–7]. A two-component plasma with thermally distributed electrons can support only solitary structures with density compression (compressive solitons) and not the solitary structures with density depletion (rarefactive solitons) [8, 9]. In a multispecies plasma consisting of positive and negative ions with thermally distributed electrons it has been predicted that there exists a critical negative ion concentration below which compressive solitons exist and above which rarefactive solitons exist [10]. Recent observational studies on the data from the Freja satellite [11] show the existence of both compressive and rarefactive solitons in the auroral low frequency turbulence. Being motivated by this observation Cairns et al. [12] have considered a plasma consisting of nonthermally distributed electrons and cold ions and have shown that both the compressive and rarefactive solitons can coexist in such nonthermal plasma. Subsequently a number of studies on ion-acoustic solitary waves in plasma consisting of nonthermal electrons have been made [13–16]. In presence of resonant electrons plasma may behave non-isothermally. Resonant electrons, if present, interact strongly with the wave during its evolution and therefore cannot be treated by assuming Boltzmann distribution [17]. Nonthermal electron distribution has been observed in many space and laboratory plasmas, in which wave damping produces an electron tail or there is trapping and acceleration of a substantial number of electrons [8, 9]. The presence of large amplitude waves can also significantly modify the electron and ion distributions [18]. Some authors [19–21] have focused their attention on the study of ion-acoustic solitary waves including the effects of two-temperature electrons, because such plasma can be produced in the laboratory or can be found in space. Streaming motion of ions and electrons may become important in some physical situations such as solar wind plasma, beam plasma interactions and cometary plasmas. Cometary plasmas are mixtures of usual solar wind plasma and the cometary matter, which escaped as neutral gas from the comet's nucleus and became ionized afterwards by

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various physical mechanisms. In these plasmas constituents have their drifts. Kourakis and Shukla [22] have considered the nonlinear ion-acoustic waves in a collisionless three-component plasma consisting of inertial positive ions moving in a background of two thermalized electron populations. The streaming motions of the electrons or ions are expected to introduce new effects [23, 24]. Actually it has been found that the electron and ion distributions can play a crucial role in the formation of nonlinear wave structures. Further, these distributions considerably influence the conditions for the existence of ion-acoustic solitary wave structures. Only a few works have been done on the analytical conditions for the existence of ion-acoustic solitary wave structures in plasma [25–28]. Again these works do not consider the effects of nonthermal electrons. Bhattacharaya and Paul [29] have studied theoretically the effects of nonthermal electrons and cold negative and positive ions on ion-acoustic solitary waves in bounded plasma using pseudo potential technique and obtained both compressive and rarefactive solitons for any value of the nonthermal electron parameter. They have not considered the effects of ion temperature on the conditions for the existence of ion-acoustic solitary wave structures.

The motivation of the present work is to obtain exact analytical solutions of the nonlinear equations governing the propagation of ion-acoustic solitary waves in a plasma consisting of warm streaming positive ions and nonthermal electrons and obtain necessary and sufficient conditions for the existence of ion-acoustic solitons From these conditions we have evaluated numerically the critical values of nonthermal parameter, ion temperature and velocity of ion-acoustic solitary waves and discuss their dependence on various plasma parameters.

2. Formulation

We consider collisionless unmagnetized plasma consisting of warm positive ions with streaming motion in presence of nonthermal electrons. Both the ions and electrons are assumed to be non-relativistic. The system of basic equations governing the one dimensional propagation of ion-acoustic waves in such plasmas are given by [30]

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial u_i}{\partial x} = 0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i \tag{4}$$

where $n_i(n_e)$ is the ion (electron) number density normalized by the equilibrium ion density n_0; $\sigma_i = T_i/T_e$ with $T_i(T_e)$ being ion (electron) temperature; u_i is the ion fluid velocity normalized by the ion-acoustic speed $(k_B T_e/m_i)^{1/2}$ in which k_B is the Boltzmann constant and m_i is ion mass; p_i is the ion pressure normalized by $n_0 k_B T_i$; ϕ is the electrostatic potential normalized by $k_B T_e/e$, e being the magnitude of electronic charge; the space variable x is normalized by the Debye length $(k_B T_e/4\pi n_0 e^2)^{1/2}$ and time t by the ion plasma period $(m_i/4\pi n_0 e^2)^{1/2}$.

The model of nonthermal distribution function used first to study the ion-acoustic solitary structures is non-Maxwellian with the presence of fast moving energetic electrons together with a population of Maxwellian distributed electrons. Such a distribution arises in space plasmas from the force field present there. This type of distribution is called as nonthermal distribution. Following Cairns et al. [12] the electron density n_e is given by

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)e^{\phi} \tag{5a}$$

with,

$$\beta = \frac{4\alpha}{1 + 3\alpha} \tag{5b}$$

where α is a parameter determining the number of nonthermal electrons present in our plasma model.

To obtain a solitary wave solution, we introduce the single independent variable η defined by the relation $\eta = x - Vt$, where V is the velocity of the solitary wave.

Now, using the boundary conditions

$$n_i \to 1', \ u_i \to u_{io}, \ p_i \to 1, \ \phi \to 0 \quad \text{at} \quad |\eta| \to \propto \tag{6}$$

and the charge neutrality condition ($n_{io} = 1$), Eqs. (1–5) in the stationary frame can be integrated to give,

$$n_i = \frac{V - u_{i0}}{V - u_i} \tag{7a}$$

$$p_i = \frac{(V - u_{i0})^3}{(V - u_i)^3} \tag{7b}$$

$$\phi(u_i) = Vu_i - \frac{1}{2}u_i^2 - \frac{3\sigma_i(V - u_{i0})^2}{2(V - u_i)^2} - a_0 \tag{7c}$$

where, $a_0 = -Vu_{i0} + \frac{1}{2}u_{i0}^2 + \frac{3\sigma_i}{2}$.

Using Eqs. (5) and (7), we obtain from Eq. (4),

$$\frac{d^2\phi}{d\eta^2} = \left(1 - \beta\phi + \beta\phi^2\right) e^{\phi} - \frac{V - u_{i0}}{V - u_i} = \mathrm{G}\ (u_i)\ (\text{say}) \tag{8}$$

3. Analytical study

Integrating Eq. (8), we get

$$\left(\frac{d\phi}{du_i}\right)^2 \left(\frac{du_i}{d\eta}\right)^2 = H\,(u_i) - K \tag{9}$$

where, K is an arbitrary constant of integration and

$$H\,(u_i) = 2\int G(u_i)\,\frac{d\phi}{du_i}du_i\,. \tag{10}$$

Now, from Eq. (7c) we obtain

$$\frac{d\phi}{du_i} = (V - u_i)\left[1 - \frac{3\sigma_i(V - u_{i0})^2}{(V - u_i)^4}\right]. \tag{11}$$

Using Eqs. (8), (11) and (10) yields

$$\begin{aligned}H(u_i) = 2\Big[&\{(1 + 3\beta) - 3\beta\phi + \beta\phi^2\}e^{\varphi} \\ &- (V - u_{i0})\,u_i + \frac{\sigma_i(V - u_{i0})^3}{(V - u_i)^3}\Big].\end{aligned} \tag{12}$$

It may be noted that the function H is highly nonlinear and contains a number of parameters including soliton velocity, nonthermal parameter of the electrons and ion temperature. These parameters enter into the function in a complicated way and thus restrict the range of possible solitary wave solutions.

3.1. Necessary and sufficient conditions

For ion-acoustic solitary wave in a nonthermal electron plasma with warm positive ion, the physically admissible solution of Eq. (9) is obtained with some special observations:

(i) $\left(\frac{d\phi}{du_i}\right)^2 \left(\frac{du_i}{d\eta}\right)^2$ must be positive, (13a)

and

(ii) u_i and $\frac{du_i}{d\eta}$ must be bounded. (13b)

From Eqs. (7c) and (8), it is easy to make the following observations:

Observation 1: $G(u_{i0}) = 0$ (14)

Observation 2: $\phi(u_{i0}) = 0$ (15)

Observation 3: If $(V - u_{i0})^2 < 3\sigma_i$, then
(i) $\phi'(u_i) > 0$ for $V < u_i \le u_{i0}$
and (ii) $\phi'(u_i) < 0$ for $u_{i0} \le u_i < V$ (16)

Observation 4: If $(V - u_{i0})^2 > 3\sigma_i$, then
(i) $\phi'(u_i) > 0$ for $V < u_i \le u'$
and (ii) $\phi'\,(u_i) < 0$ for $u' \le u_i < V$ (17)

where u' is given by $\phi'(u') = 0$, that is,

$$u' = V - (3\sigma_i)^{1/4}.\sqrt{V - u_{i0}}\,. \tag{18}$$

Therefore, for physical solution of Eq. (9) we need following requirements;

Requirement **1**: There exists u_{max} or u_{min} such that

$$H\,(u_{i0}) = H\,(u_{max}) = K \quad \text{for } u_{i0} \le u_{max} < u' \tag{19}$$

$$\text{or } H\,(u_{i0}) = H\,(u_{min}) = K \quad \text{for } u' \le u_{min} < u_e \tag{20}$$

Requirement **2**: $H\,(u_i) > K$ for either $u_{i0} \le u_i < u_{max}$
or $u_{min} < u_i \le u_{i0}$. (21)

For real and bounded solution of Eq. (9) the following conditions must be satisfied:

$$\text{Necessary condition: } H''(u_{i0}) > 0 \tag{22}$$

$$\text{Sufficient condition: } H\,(u') - H\,(u_{i0}) < 0\,. \tag{23}$$

By virtue of observations (3) and (4) it follows from the condition (22) that

$$G'\,(u_{i0}) > 0 \text{ for } V > u_{i0} \tag{24}$$

$$G'\,(u_{i0}) < 0 \text{ for } V < u_{i0}\,. \tag{25}$$

Now using Eqs. (8) and (11) we get the following simple conditions for real and bounded solution of Eq. (9):

$$V > u_{i0} + \sqrt{\frac{1}{1 - \beta} + 3\sigma_i} \quad \text{for } V > u_{i0} \tag{26a}$$

$$\text{and } u_{i0} > V + \sqrt{\frac{1}{1 - \beta} + 3\sigma_i} \quad \text{for } V < u_{i0}\,. \tag{26b}$$

For a non-streaming plasma with Boltzmann distributed electrons the condition (26) reduces to the well-known result

$$V > \sqrt{1 + 3\sigma_i}\,. \tag{27}$$

Using Eq. (12) the condition (23) becomes

$$\begin{aligned}&[\{(1 + 3\beta) - 3\beta\gamma + \beta\gamma^2\}\,e^{\gamma} - (V - u_{i0})\,(u' \\ &- \frac{1}{3}\sqrt{3\sigma_i})] < [(1 + 3\beta) - (V - u_{i0})\,u_{i0} + \sigma_i]\,.\end{aligned} \tag{28}$$

For non-streaming plasma with cold ions and Boltzmann distributed electrons conditions (27) and (28) together require that $1 < V < 1.6$ which agrees with the condition established by Sagdeev [31].

3.2. Critical values of plasma parameters for solitary solutions

For the existence of solitary waves in a plasma consisting of warm ions and nonthermal electrons, the conditions (26) must be satisfied for nonthermal parameter (β), soliton velocity (V) and ion temperature (σ_i). The critical values of nonthermal parameter, ion temperature and phase velocity are obtained as

$$\beta_c = 1 - \frac{1}{(V - u_{i0})^2 - 3\sigma_i} \tag{29}$$

$$\sigma_{ic} = \frac{1}{3}\left[(V - u_{i0})^2 - \frac{1}{1-\beta}\right] \tag{30}$$

$$V = V_F = u_{i0} + \left(3\sigma_i + \frac{1}{1-\beta}\right)^{1/2} \tag{31}$$

$$V = V_S = u_{i0} - \left(3\sigma_i + \frac{1}{1-\beta}\right)^{1/2}. \tag{32}$$

It is to be noted that the streaming motion of ions has significant effects on these critical values. In presence of streaming motion of ions two distinct wave modes (fast and slow) are possible. The critical values β_c, σ_{ic}, V_F and V_S are numerically estimated using different values of the plasma parameters and their variations are shown in Figs. 1, 2, 3 and 4. Figure 1 shows the critical values of nonthermal parameter of electrons for different values of ion temperature and stream velocity of ions. It is observed that for a given stream velocity of ions the critical nonthermal parameter decreases with the increase of ion temperature and for a given ion temperature. It also decreases with the increase of stream velocity. So for a given ion temperature with higher stream velocity or for a given stream velocity with higher ion temperature we require less number of nonthermal electrons for the existence of solitary waves. The critical values of ion temperature for different values

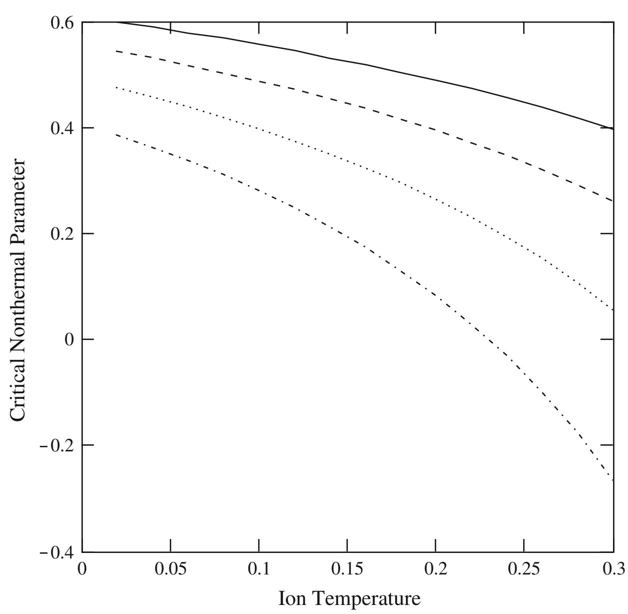

Fig. 1 Variation of critical values of nonthermal parameter (β) with ion temperature (σ_i) for different values of stream velocity (u_{i0}). The *solid*, *dashed*, *dotted* and *dash-dotted* lines correspond to $u_{i0} = 0.0$, 0.1, 0.2 and 0.3 respectively; $p_i = 1$, $V = 1.6$

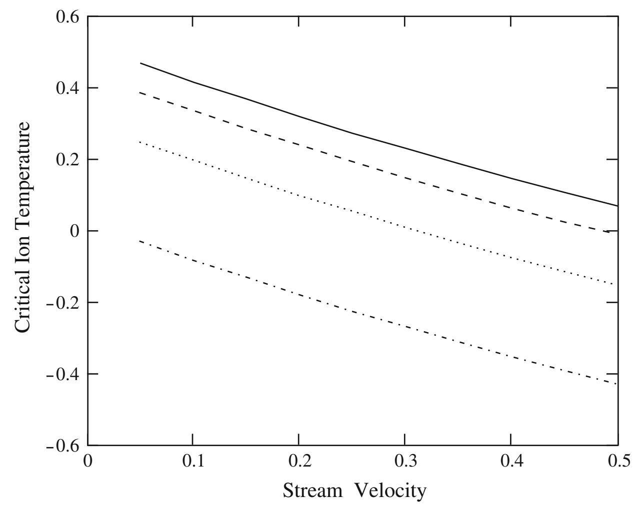

Fig. 2 Variation of critical values of ion temperature (σ_i) with stream velocity (u_{i0}) for different values of nonthermal parameter (β). The *solid*, *dashed*, *dotted* and *dash-dotted* lines correspond to $\beta = 0.0$, 0.2, 0.4 and 0.6 respectively; $p_i = 1$, $V = 1.6$

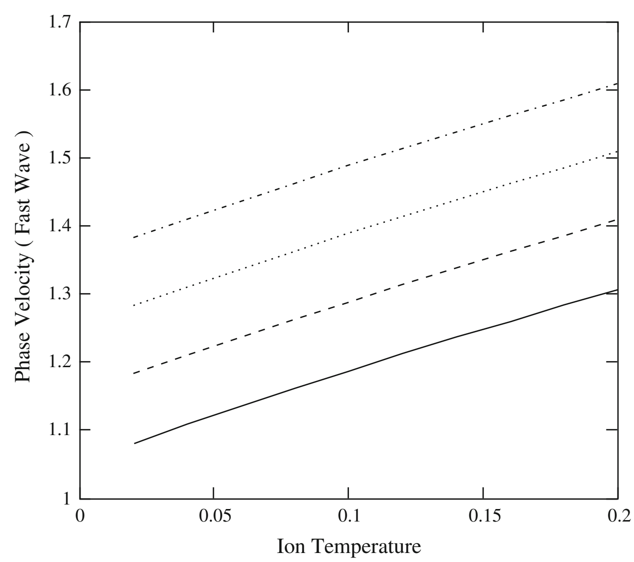

Fig. 3 Variation of phase velocity for fast ion-acoustic wave with ion temperature (σ_i) for different values of stream velocity (u_{i0}). The *solid*, *dashed*, *dotted* and *dash-dotted lines* correspond to $u_{i0} = 0.0$, 0.1, 0.2 and 0.3 respectively; $p_i = 1$, $\beta = 0.1$

of stream velocity of ions and nonthermal parameter of electrons are depicted in Fig. 2. It is seen that for a given nonthermal parameter critical ion temperature decreases with the increase of the stream velocity of ions and for a given stream velocity it also decreases with the increase of nonthermal parameter of electrons. The critical values of the velocity of fast and slow ion-acoustic waves for different values of ion temperature and stream velocity of ions are shown in Figs. 3 and 4 respectively. It is observed that critical velocity of fast wave increases and that of slow waves decreases with the increase of ion temperature and stream velocity of ions. Also through similar numerical estimation it is found that the critical velocity of fast waves increases and that of slow waves decreases with the increase of nonthermal parameter.

4. Sagdeev potential and ion-acoustic solitary waves

We have discussed the conditions for the existence of ion-acoustic solutions in terms of ion fluid velocity (u_i) instead of electrostatic potential ϕ or number density (n_i) of ions. However, expressing G in Eq. (8) as a sole function of ϕ and integrating the resulting equation one can find out the Sagdeev potential and then can use it in the usual way to discuss the properties of solitary wave structures. Eq. (8) can be written in the form

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi} \tag{33}$$

where $\psi(\phi)$ is known as the Sagdeev potential and is given by

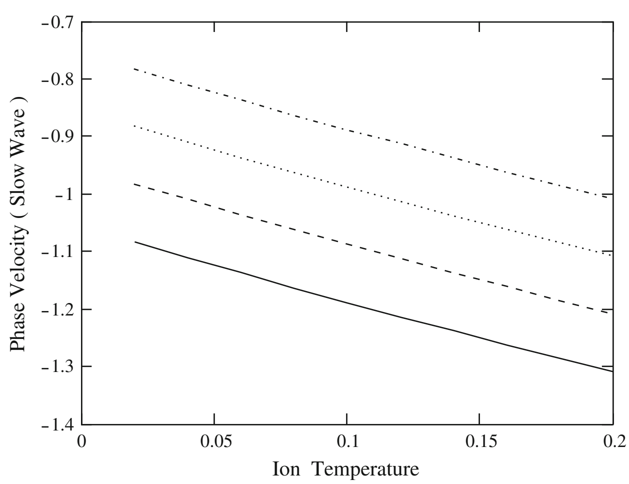

Fig. 4 Variation of phase velocity for slow ion-acoustic wave with ion temperature (σ_i) for different values of stream velocity (u_{i0}). The *solid*, *dashed*, *dotted* and *dash-dotted lines* correspond to $u_{i0} = 0.0$, 0.1, 0.2 and 0.3 respectively; $p_i = 1$, $\beta = 0.1$

$$\begin{aligned}\psi(\phi) &= (1 + 3\beta) - e^{\phi} + \beta(\phi - 1)e^{\phi} \\ &\quad - \beta\left(\phi^2 - 2\phi + 2\right)e^{\phi} \\ &\quad - \frac{1}{6\sqrt{3\sigma_i}}\left[\left\{\left(V - u_{io} + \sqrt{3\sigma_i}\right)^2 - 2\phi\right\}^{\frac{3}{2}}\right. \\ &\quad - \left\{\left(V - u_{io} - \sqrt{3\sigma_i}\right)^2 - 2\phi\right\}^{\frac{3}{2}} \\ &\quad \left. - \left(V - u_{io} + \sqrt{3\sigma_i}\right)^3 + \left(V - u_{io} - \sqrt{3\sigma_i}\right)^3\right].\end{aligned} \tag{34}$$

For soliton solution, the Sagdeev potential $\psi(\phi)$ must satisfy the condition

$$\left.\frac{\partial^2\psi}{\partial\phi^2}\right|_{\phi=0} < 0$$

which implies the inequality

$$\frac{1}{(V - u_{io})^2 - 3\sigma_i} < 1 - \beta\,. \tag{35}$$

The inequality (35) gives the condition for the existence of a potential well. It may be noted that the inequality (35) leads to the same critical values of the non-thermal parameter, ion temperature and phase velocity as obtained earlier in Sect. 3. In order to examine the effects of stream velocity of ions (u_{io}), ion temperature (σ_i) and nonthermal electron on the properties of the ion-acoustic solitary wave structure, we have numerically studied the behaviour of the Sagdeev potential $\psi(\phi)$ as given by Eq. (34) for different values of σ_i, u_{io} and nonthermal parameter β. It is found that the dip in the potential structure decreases with the increase in nonthermal parameter, keeping all other parameters same. This is shown in Fig. 5. Also it has been found that the dip in potential structure decreases with the increase in the ion temperature or with the increase of stream velocity of ions when other parameters are kept constant.

5. Summary and concluding remarks

We have made an analytical study for the existence of ion-acoustic solitary waves in collisionless, unmagnetized plasma consisting of warm streaming ions and nonthermal electrons. Some necessary and sufficient conditions for solitary wave solutions of the nonlinear equations governing the dynamics of the wave are also obtained. We have obtained expressions for the critical values of nonthermal parameter of electrons, ion temperature and velocity of the wave for the existence of solitary wave structure. Dependence of these quantities on different plasma parameters has been depicted graphically and discussed. An important

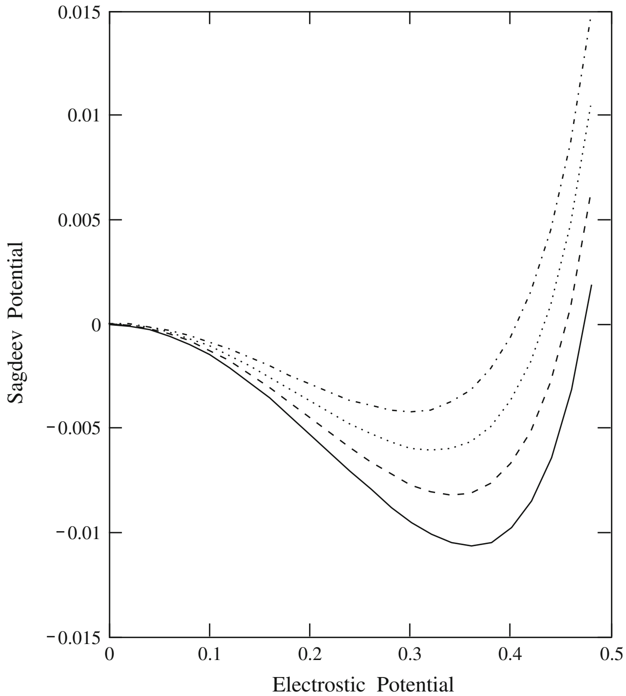

Fig. 5 Behaviour of Sagdeev potential for different values of nonthermal parameter (β). The *solid*, *dashed*, *dotted* and *dash-dotted lines* correspond to $\beta = 0.0$, 0.04, 0.08 and 0.12 respectively; $V = 1.4$, $u_{i0} = 0.17$ and $\sigma_i = 0.018$

observation is that in presence of streaming ions two distinct wave modes (fast and slow) are possible. In order to examine the effects of stream velocity of ions, ion temperature and nonthermal electrons on the structure of ion-acoustic solitons we have also studied graphically the behaviour of Sagdeev potential. It is shown that the stream velocity of ions, ion temperature and nonthermal electrons have considerable effects on changing the properties of the ion-acoustic solitary waves. Finally, we would like to point out that the analysis presented here for a model plasma with warm streaming ions and nonthermal electrons may be of relevance to some space plasmas such as solar wind plasma and cometary plasmas.

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References

[1] K Roy and P Chatterjee *Indian J. Phys*. **85** 1653 (2011)
[2] B Ghosh, S N Paul, C Das, A Sinhamahapatra and I Paul *Indian J. Phys*. **85** 745 (2011)
[3] Z Emami and H R Pakzad *Indian J. Phys*. **85** 1643 (2011)
[4] H R Pakzad *Indian J. Phys*. **84** 867 (2010)
[5] N Nimje, S Dubey and S Ghosh *Indian J. Phys*. **84** 1567 (2010)
[6] R Malik, H K Malik and S C Kaushik *Indian J. Phys*. **85** 1887 (2011)
[7] C Venugopal, M J Kurian, E S Devi, P J Jessy, C P Anilkumar and G Renuka *Indian J. Phys*. **84** 319 (2010)
[8] P K Shukla, R Bingham, R Dendy, H Pecseli and L Stenflo, *J. Phys. (France) IV* **5** C6–19 (1995)
[9] V D Shapiro, G I Soloviev, J M Dawson and R Bingham *Phys. Plasmas* **2** 516 (1995)
[10] R V Reddy and S G Tagare *J. Phys. Soc. Jpn.* **56** 4329 (1987)
[11] P O Dovner, A I Eriksson, R Bostrom and B Holback *Geophys. Res. Lett.* **21** 1827 (1994)
[12] R A Cairns, A A Mamun, R Binghan, R Bostrom, R O Dendy, C M C Nairn, P K Shukla *Geophys. Res. Lett*. **22** 2709 (1995)
[13] A A Mamun *Phys. Rev*. *E* **55** 1852 (1997)
[14] A A Mamun *Euro. Phys. J. D* **11** 143 (2000)
[15] A Bandyapadhyay, K P Das *Physica Scripta* **61** 92 (2000)
[16] A Bandyapadhyay and K P Das *J. Plasma Phys.* **65** 131 (2001)
[17] H Schamel *Plasma Phys.* **14** 905 (1972)
[18] T S Gill, H Kaur and N S Saini *Phys. Plasmas* **10** 3927 (2003)
[19] G C Das, S N Paul and B Karmakar *Phys. Fluids* **29** 2192 (1986)
[20] K P Das, S R Majumdar and S N. Paul *Phys. Rev. E* **51** 4796 (1995)
[21] S R Majumdar, S N Paul and K P Das *J. Plasma Phys.* **64** 297 (2000)
[22] I Kourakis and P K Shukla *J. Phys. A Math. Gen.* **36** 11901 (2003)
[23] G C Das and S N Paul *Phys. Fluids* **28** 823 (1985)
[24] S N Paul, C Das, I Paul, B Bandyopadhyay, S Chattopadhya and S S De *Indian J. Phys*. (In Press)
[25] K K Ghosh, D Ray and S N Paul *Int. J. Theor. Phys.* **31**, 75 (1992)
[26] S Chattopadhyaya, K K. Ghosh and S N Paul *FIZIKA A* **9** 75 (2000)
[27] K K Ghosh, S N Paul and S Chattopadhyay *FIZIKA A* **13** 33 (2005)
[28] K K Ghosh, B Paul, C Das and S N Paul, *J. Phys. A Math. Theor.* **41** 335501 (2008)
[29] S K Bhattacharaya and S N Paul *Indian J. Phys*. **77B** 327 (2003)
[30] G B Whitham *Linear and Nonlinear Wave* (New York: Wiley Interscience) (1974)
[31] R Z Sagdeev *Rev. Plasma Phys*. vol 4 (ed.) M A Leontovich (New York: Consultant Bureau) (1966)

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SHORT RESEARCH COMMUNICATION

W-type ion-acoustic solitary waves in plasma consisting of cold ions and nonthermal electrons

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Abstract: Sagdeev potential approach is used for the study of nonlinear propagation of ion-acoustic waves in plasma consisting of cold positive ions and nonthermal electrons. The nonlinear equation so derived are analysed with the help of Bogoliubov–Mitropolosky method. The profiles of Sagdeev potential solitary waves are evaluated in first-, second- and third- order which are depicted for different values of nonthermal parameter of electrons. It is seen that nonthermal electrons has considerable impact on the shape of ion-acoustic solitary waves in each order. The plasma consisting of cold positive ions and no negative ions can support the formation of compressive as well as W-type solitary waves in second- and third- order for certain value of nonthermal parameter of electrons. The results are new because W-type ion-acoustic solitary wave is found by earlier authors in plasma in presence of negative ions only. The ion-acoustic solitary waves near critical value of nonthermal parameter and arbitrary amplitude solitary waves in presence of nonthermal electrons have also been studied in the paper. Moreover, the solution for ion-acoustic double layers in plasma consisting of nonthermal electrons is obtained. Our results in the paper would be useful to understand the nonlinear wave processes in ionospheric and magnetospheric multicomponent plasma having nonthermal electrons.

Keywords: Ion-acoustic waves; Nonthermal electrons; Pseudo-potential and W-type solitary waves

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1. Introduction

Ion-acoustic solitary waves have been studied by various authors because it is important for wide applications in laboratory and space plasma. In a cold unmagnetized and nondissipative plasma, Washimi and Taniuti [1] have first theoretically studied ion acoustic solitary wave using the reductive perturbation method. Later, considering the effects of ion temperature [2, 3], negative ions [4, 5], two-temperature electrons [6, 7], magnetic field [8, 9] etc. many authors have studied ion acoustic solitary waves and have shown that these plasma parameters have considerable impact on the excitation of solitary waves. But, the presence of non-Maxwellian electrons, non-isothermal electrons [10, 11], super thermal electrons distributions [12, 13] etc., in plasma gives rise to more interesting characteristics in nonlinear propagation of waves, particularly ion-acoustic solitary waves in plasma.

However, non-thermal electrons which are also non-Maxwellian give rises to various interesting behaviours on the ion-acoustic solitary waves in plasma. Cairns et al. [14] have shown that solitary waves are excited in a plasma consisting of non-thermal electrons with an excess of energetic particles and ions. In fact, they are motivated by the recent observation of solitary structures with density depletion made by Freja satellite [15]. Such electron distribution with enhanced population of energetic electrons has been observed in the magnetosphere. Non-thermal distribution is a common feature of auroral zone [16]. This type of distribution is common to many space and laboratory plasmas in which wave damping produces electron tail. Considering the presence of nonthermal electrons in

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plasma Mamun [17, 18], Bandhopadhyay and Das [19–21] and other authors have theoretically studied ion-acoustic solitary waves. In a bounded plasma, Bhattacharaya et al. [22] have investigated the effects of non-thermal electrons on ion-acoustic solitary waves using pseudo-potential technique and have shown that both compressive and rarefactive solitons would be excited for any value of nonthermal electron parameter in plasma. Paul et al. [23] have obtained exact analytical solutions of the nonlinear equations governing the propagation of ion-acoustic solitary waves in a plasma consisting of warm streaming positive ions and nonthermal electrons with necessary and sufficient conditions for the existence of ion-acoustic solitary waves. The critical values of nonthermal parameter, ion temperature and velocity of ion- acoustic solitary waves have been numerically evaluated and discussed their dependence on the plasma parameters. They have analyzed the structure of the solitary waves as a function of the non-thermal electron parameter which measures the deviation from the thermalized state. Later, Ghosh et al. [24] have theoretically investigated the effect of nonthermal electrons and warm negative ions on ion-acoustic solitary waves in multicomponent drifting plasma.

However, from the experimental observations [25–28] it is found that theoretically predicted values of the amplitude, width, and velocity of the solitary waves do not obey the experimental results. To remove the discrepancy between theoretical and experimental results researchers realized the necessity of considering higher-order nonlinear and dispersive terms in the propagation of solitary waves in a plasma, Ichikawa et al. [29] were the first to examine such higher-order effects by the reductive perturbation method for ion-acoustic solitons in plasma consisting of cold ions. Considering the temperature of ions in plasma, Lai [30] has investigated the higher-order effect on the ion-acoustic solitary waves. Kodama and Taniuti [31] have reconsidered this problem and have shown by the method of renormalization how to eliminate the secular terms appearing in higher-order terms of the expansion. Considering the effect of higher order nonlinearity ion-acoustic solitary waves correct up to second order in a plasma have been studied by various authors [32–37]. Taking the electrons as nonisothermal, Kalita and Bujarbarua [32] have investigated the propagation of higher-order ion-acoustic solitary waves in a plasma. Tagare and Reddy [33] have considered the effect of higher-order nonlinearity on ion-acoustic solitary waves in a plasma consisting of negative ions and nonisothermal electrons. Subsequently, Moslem [34] have studied higher-order contributions to ion-acoustic solitary waves in warm multicomponent plasma with an electron beam. Paul et al. [35] have considered the electron-inertia and drift motion and on the higher order ion acoustic solitary waves in a negative ion plasma. To get better theoretical results to match the experimentally observed value, Das et al. [36] and his co-workers [37] have considered the effect of higher-order nonlinear and dispersive terms up to the third order approximation and have studied ion-acoustic solitary waves in a plasma.

However, no author has studied ion acoustic solitary waves critically in presence of the nonthermal electrons in plasma specially considering higher order nonlinearity and dispersive effects. So, in the present we are motivated to investigate theoretically the ion acoustic solitary waves correct up to third order having nonthermal electrons in cold plasma. To get higher-order effects on the solitary waves we have applied the pseudopotential method which is different from the standard method adopted in previous investigations. Starting from the governing equations, an integrated form of the equations in terms of the pseudo-potential is derived and then perturbation solutions of this equation up to third- order approximation for ion acoustic solitary waves in plasma are obtained using the Bogoliubov–Mitropolosky method [38]. It is seen that compressive as well as rarefactive W-type solitary waves would be excited in the third order in plasma consisting of cold positive ions and nonthermal electrons.

2. Basic equations

We consider an unmagnetized collisionless plasma consisting of cold ions and nonthermal electrons. The governing equations for such a plasma system can be written in the following dimensionless form:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} + \frac{\partial \varphi}{\partial x} = 0. \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i \tag{3}$$

where n_i and n_e are, respectively, the number densities of ions and electrons; u_i is the ion-fluid velocity; ϕ is the electrostatic potential. The quantities n_i, n_e, u_i and φ are made dimensionless by n_0, n_0, $(k_B T_e/T_e m_i.m_i)^{1/2}$ and $k_B T_e/T_e e.e$, respectively, where n_0 is the unperturbed electron or ion density, m_i is the mass of the ion, and k_B is Boltzmann constant.

The non-thermal electron distribution function is assumed as [14]

$$f_e(v) = \frac{n_o}{(1+3p)\sqrt{2\pi V_e^2}}\left[1 + p\left(\frac{v}{V_e^2} - 2\phi\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{v}{V_e^2} - 2\phi\right)\right] \tag{4}$$

where $p(\geq 0)$ is a parameter that determines the proportion of the fast energetic electrons, v is the velocity of electrons in phase space and is normalized by the average thermal speed $V_e = \sqrt{K_B T_e/m_e}$ and m_e is the mass of electron.

From Eq. (4) the normalized electron density n_e is obtained as

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)e^{\phi} \tag{5a}$$

where

$$\beta = \frac{4p}{1+3p} \tag{5b}$$

β measures the deviation from thermalized state. The parameter β cannot exceed a certain range $(0 \leq \beta \leq 1.3)$, beyond which p is negative which is not acceptable. When $\beta \to 0$, Eq. (5a) gives the Boltzmann distribution of electrons.

3. Sagdeev potential and conditions for solitary wave solution

For a solitary-wave solution we assume that the dependent variables depend on a single independent variable $\xi = x - Vt$, where V is the velocity of the solitary wave. Using the boundary conditions $n \to 1$, $\phi \to 0$ as $\xi \to \pm\infty$ we obtain from Eqs. (1)–(3),

$$u_i = V\left(1 - \frac{1}{n_i}\right) \tag{6}$$

$$n_i = \frac{V}{\sqrt{V^2 - 2\varphi}} \tag{7}$$

$$\frac{d^2\varphi}{d\xi^2} = (1 - \beta\varphi + \beta\varphi^2)\exp(\varphi) - \frac{1}{\sqrt{1 - \frac{2\varphi}{V^2}}} \tag{8}$$

Eq. (8) can be written as

$$\frac{d^2\varphi}{d\xi^2} = -\frac{d\psi}{d\varphi} \tag{9}$$

Ψ is the Sagdeev potential and it is given by

$$\psi(\phi) = (1+3\beta) - e^{\phi} + \beta(\phi - 1)e^{\phi} - \beta\left(\phi^2 - 2\phi + 2\right)e^{\phi} + V^2\left(1 - \sqrt{1 - \frac{2\varphi}{V^2}}\right) \tag{10}$$

For the solitary wave solution, Sagdeev potential $\psi(\phi)$ in Eq. (10) must satisfy the following conditions:

(a) $\psi(\phi)|_{\phi=0} = \left.\frac{\partial\psi}{\partial\phi}\right|_{\phi=0} = 0$

(b) $\psi(\phi) = 0$ for some $\phi = \phi_m$ where ϕ_m is some maximum value of ϕ.

(c) $\psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$.

In addition to that, the condition for the localized solitary wave solution is

$$\left.\frac{\partial^2\psi}{\partial\phi^2}\right|_{\phi=0} < 0$$

which implies inequality

$$\frac{1}{V^2} < 1 - \beta \tag{11}$$

Inequality (Eq. 11)) gives the condition for the existence of a potential well.

From Eq. (12) the critical values of velocity $(V = V_c)$ and non-thermal parameter $(\beta = \beta_c)$ are obtained as

$$V_c = \sqrt{\frac{1}{1-\beta}} \tag{12}$$

and

$$\beta_c = \left(1 - \frac{1}{V^2}\right) \tag{13}$$

4. Solitary wave solutions

Expanding the right-hand side of Eq. (9) in ascending powers of φ we get

$$\frac{d^2\varphi}{d\xi^2} = \Delta_1\varphi + \Delta_2\varphi^2 + \Delta_3\varphi^3 + \Delta_4\varphi^4 + \ldots. \tag{14a}$$

where,

$$\Delta_1 = (1-\beta) + \frac{1}{V^2}\Delta_2 = \frac{1}{2}\left(1 - \frac{3}{V^4}\right)\Delta_3 = \left(\frac{1}{6} + \frac{\beta}{2} - \frac{5}{2V^6}\right), \text{ and } \Delta_4 = \left(\frac{1}{24} - \frac{\beta}{3} - \frac{35}{8V^8}\right) \tag{14b}$$

An integral of Eq. (14a) satisfying the conditions $d\varphi/dX \to 0$, $\varphi \to 0$ as $\xi \to \pm\infty$ is

$$\frac{\varepsilon}{2}\left(\frac{d\varphi}{dX}\right)^2 = \frac{1}{2}\Delta_1\varphi^2 + \frac{1}{3}\Delta_2\varphi^3 + \frac{1}{4}\Delta_3\varphi^4 + \frac{1}{5}\Delta_4\varphi^5 + \ldots \tag{15}$$

where we have stretched the ξ coordinate according to the relation

$$X = \in^{1/2}\xi. \tag{16}$$

The parameter ε is dimensionless and small $\varepsilon < 1$. It measures the dispersion and nonlinear effects.

We now make the following perturbation expansions for φ and V.

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \ldots \tag{17a}$$

$$V = V_0 + \varepsilon V^{(1)} + \in^2 V^{(2)} + \ldots \tag{17b}$$

where, $\phi^{(1)}$, $\phi^{(2)}$, $\phi^{(3)}$ are the first-,second- and third order perturbed values of ϕ and $V^{(1)}$, $V^{(2)}$, $V^{(3)}$ are the perturbed values of velocity V.

Now, for linear dispersion relation, $(\Delta_1)_0 = 0$.

Therefore,

$$1-\beta-\frac{1}{V_0^2}=0,\ \text{i.e.} V_0=\sqrt{\frac{1}{(1-\beta)}}. \tag{18}$$

With the expansion of V given by (17b) the expansions for $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 become

$$\begin{aligned}
\Delta_1 &= \delta_1^{(0)}+\varepsilon\delta_1^{(1)}+\varepsilon^2\delta_1^{(2)}+\varepsilon^3\delta_1^{(3)}+\cdots\\
\Delta_2 &= \delta_2^{(0)}+\varepsilon\delta_2^{(1)}+\varepsilon^2\delta_2^{(2)}+\cdots\\
\Delta_3 &= \delta_3^{(0)}+\varepsilon\delta_3^{(1)}+\cdots\\
\Delta_4 &= \delta_4^{(0)}+\varepsilon\delta_4^{(1)}+\varepsilon^2\delta_4^{(2)}+\cdots
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
&\delta_1^{(0)}=0,\ \delta_1^{(1)}=2V_0^{-3}V^{(1)},\\
&\delta_1^{(2)}=2V_0^{-3}V^{(2)}-3\,V_0^{-4}V^{(1)^2},\\
&\delta_1^{(3)}=2V_0^{-3}V^{(3)}-6V_0^{-4}V^{(1)}V^{(2)}+4V_0^{-5}V^{(1)3},\\
&\delta_2^{(0)}=\frac{1}{2}(1-3V_0^{-4})\\
&\delta_2^{(1)}=6V_0^{-5}V^{(1)},\\
&\delta_2^{(2)}=6V_0^{-5}V^{(2)}-15\,V_0^{-6}V^{(1)^2}\\
&\delta_3^{(0)}=\left(\frac{1}{6}+\frac{\beta}{2}-\frac{5}{2}V_0^{-6}\right),\\
&\delta_3^{(1)}=15V_0^{-7}V^{(1)},\\
&\delta_4^{(0)}=\left(\frac{1}{24}-\frac{\beta}{3}-\frac{35}{8}V_0^{-8}\right).
\end{aligned} \tag{20}$$

Substituting the expansions (17a) in (15) we get a sequence of equations for the $\varphi^{(i)}$'s and the equation for $\varphi^{(i)}$ at each order becomes a first-order inhomogeneous differential equation for $i>1$. For linear dispersion relation, $(\Delta_1)_0=\delta_1^{(0)}=0$.

4.1. Equation at the first- order and its solution

In the first order i.e. in lowest order which is at the order of ε^3 we obtain the following equation for $\phi^{(1)}$,

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2=\frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2}+\frac{1}{3}\delta_2^{(0)}\varphi^{(1)^3} \tag{21}$$

The solution of which is the first order K-dV soliton

$$\phi^{(1)}=l_0\sec h^2\eta \tag{22}$$

where

$$l_0=-\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}}=3\bar{V}_0V^{(1)} \tag{22a}$$

$$\bar{V}_0=(3-V_0^4)^{-1} \tag{22b}$$

$$\eta=\frac{\sqrt{\delta_1^{(1)}}}{2}X=\left(\frac{V_0^{-3}V^{(1)}}{2}\right)^{1/2}X \tag{22c}$$

$\phi^{(1)}$ vanishes at infinity and it attains a maximum value at $\eta=0$ giving the amplitude of first order solitary wave l_0, $l_0=\alpha_{01}$ (*say*).

The width 'D' of the solitary wave is defined as half the value of width of pulse at a height of 0.42 of its amplitude.

The first order width D_1 *of* the solitary wave is

$$D_1=\left(\frac{2}{V_0^{-3}V^{(1)}}\right)^{1/2}\eta_1 \tag{22d}$$

where η_1 is the positive root of the equation, $\tanh^2\eta_1=0.58$.

4.2. Equation at the second order and its solution

At the second order we get the following equation for $\varphi^{(2)}$using (16) and (22a)

$$\frac{d\phi^{(2)}}{d\eta}-\frac{\phi_{\eta\eta}^{(1)}}{\phi_\eta^{(1)}}\phi^{(2)}=\frac{\delta_1^{(2)}}{V_0V^{(1)}}\frac{\phi^{(1)^2}}{\phi_\eta^{(1)}}+\frac{2\delta_2^{(1)}}{3V_0V^{(1)}}\frac{\phi^{(1)^3}}{\phi_\eta^{(1)}}+\frac{\delta_3^{(0)}}{2V_0V^{(1)}}\frac{\phi^{(1)^4}}{\phi_\eta^{(1)}} \tag{23}$$

Removing the secular term, solution of (23) is obtained as

$$\phi^{(2)}=\left[l_1+l_2\tan h^2\eta\right]\sec h^2\eta \tag{24}$$

where

$$l_1=\frac{2l_0^2}{\delta_1^{(1)}}\left[\frac{1}{3}\delta_2^{(1)}+\frac{1}{2}l_0\delta_3^{(0)}\right],\quad l_2=-\frac{l_0^3}{2}\frac{\delta_3^{(0)}}{\delta_1^{(1)}},l_0=-\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}}, \tag{24a}$$

and $V^{(2)}=\frac{3(V^{(1)})^2}{2V_0}$

The amplitude of second order solitary wave is (at $\eta=0$) $l_1=\alpha_{02}$ (say),

The second order width D_2 of the solitary wave is

$$D_2=\left(\sqrt{\frac{2}{V_0^{-3}V^{(1)}}}\right)\eta_2 \tag{24b}$$

where η_2 is the positive root of the equation

$$(l_2/l_1)\tanh^4\eta_2 - [1+2(l_2/l_1)]\tanh^2\eta_2 + [(l_2/l_1)+0.58] = 0 \tag{24c}$$

l_1 and l_2 are given above.

4.3. Equation at the third order and its solution

We obtain the following equation for $\varphi^{(3)}$ at the third order, in which we use relation (16) and introduce the new variable η given by (22a),

$$\frac{d\phi^{(3)}}{d\eta} - \frac{\phi^{(1)}_{\eta\eta}}{\phi^{(1)}_{\eta}}\phi^{(3)} = \frac{2}{V_0V^{(1)}\phi^{(1)}_{\eta}}\left[\frac{1}{2}\delta^{(1)}_1\phi^{(2)^2} + \frac{1}{2}\delta^{(3)}_1\phi^{(1)^2} + \delta^{(0)}_2\phi^{(1)}\phi^{(2)^2} + \delta^{(1)}_2\phi^{(1)^2}\phi^{(2)} + \frac{1}{3}\delta^{(2)}_2\phi^{(1)^3} + \delta^{(0)}_1\phi^{(1)^3}\phi^{(2)} + \frac{1}{4}\delta^{(1)}_3\phi^{(1)^4} + \frac{1}{5}\delta^{(0)}_4\phi^{(1)^5} - \frac{1}{4}V_0V^{(1)}\left(\frac{d\phi^{(2)}}{d\eta}\right)^2\right] \tag{25}$$

Removing the secular term, solution of (25) for $\phi^{(3)}$ is obtained as

$$\phi^{(3)} = \left(l_3 + l_4\tan h^2\eta + l_5\tan h^4\eta\right)\sec h^2\eta \tag{26a}$$

where,

$$l_3 = \frac{(l_1+l_2)^2}{2l_6^2}\left[\frac{\delta^{(1)}_1}{2l_0} + \delta^{(0)}_2\right] + \frac{l_0(l_1+l_2)}{2l_6^2}\left[\delta^{(1)}_2 + \frac{l_0}{3}\delta^{(2)}_2\right] + \frac{l_0^2}{2l_6^2}\left[(l_1+l_2)\delta^{(0)}_3 + \frac{l_0}{4}\delta^{(1)}_3 + \frac{l_0^2}{5}\delta^{(0)}_4\right]$$

$$l_4 = l_2(l_1+l_2)\left[\frac{4}{l_0} + \frac{\delta^{(0)}_2}{l_6^2}\right] + \frac{l_2}{2l_6^2}\left[l_0\delta^{(1)}_2 + \frac{l_2}{2l_0}\delta^{(1)}_1\right] + \frac{l_0^2}{2l_6^2}\left[(l_1+2l_2)\delta^{(0)}_3 + \frac{l_0}{4}\delta^{(1)}_3 + \frac{2}{5}l_0^2\delta^{(0)}_4\right]$$

$$l_5 = -\frac{l_0^2}{6l_6^2}\left[l_2\delta^{(0)}_3 + \frac{l_0^2}{5}\delta^{(0)}_4\right] - l_2^2\left[\frac{4}{3l_0} + \frac{\delta^{(0)}_2}{6l_6^2}\right]$$

$$l_6^2 = \frac{1}{4}\delta^{(1)}_1$$

After simplification we get

$$l_3 = \frac{6V_0^3V^{(3)}}{5}a_2,\ l_4 = \frac{6V_0^3V^{(3)}}{5}b_2,\ l_5 = \frac{6V_0^3V^{(3)}}{5}c_2, \tag{26b}$$

$$a_2 = -\frac{27\times15V_0^2}{32} - \frac{27\times13V_0^2}{10} + \frac{81\times7V_0^2}{8} - 9,$$

$$b_2 = -\frac{81\times7\times9V_0^2}{32} - \frac{27\times13V_0^2}{32} + \frac{81\times19V_0^2}{8},$$

$$c_2 = -\frac{27\times7V_0^2}{32} + \frac{9\times13V_0^2}{10},$$

and $V^{(3)} = \frac{5}{2}\frac{(V^{(1)})^3}{V_0^2}$

The amplitude of third order solitary wave is l_3, $l_3 = \varphi_{03}\ (say)$, $\alpha_{03} = \frac{6}{5}a_2V_0^3V^{(3)} = 3a_2V_0(V^{(1)})^3$.

So using the perturbed values of φ at first, second and third order i.e. $\phi^{(1)}$, $\phi^{(2)}$, and $\phi^{(3)}$ given by (22), (24) and (26), respectively, the first, second and third order solitary wave solutions Φ_1, Φ_2, Φ_3 with corrections become

$$\Phi_1 = \varphi^{(1)} = l_0\sec h^2\eta \tag{27a}$$

$$\Phi_2 = \varphi^{(1)} + \varphi^{(2)} = \alpha_1\left[1+\alpha_1'\tanh^2\eta\right]\sec h^2\eta \tag{27b}$$

$$\Phi_3 = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} = \alpha_3[1+\alpha_2\tan h^2\eta + \alpha_4\tan h^4\eta]1 + \sec h^2\eta \tag{27c}$$

where,

$$\alpha_1 = l_0 + l_1,\ \alpha_1' = \frac{l_2}{\alpha_1}\alpha_2 = \frac{l_4 - l_2}{\alpha_3},\ \alpha_4 = \frac{l_5}{\alpha_3},\ \alpha_3 = l_0 + l_1 + l_3, \tag{27d}$$

$l_0, l_1, l_2, l_3, l_4, l_5$ are given above.

The amplitude of solitary waves Φ_{01}, Φ_{02}, Φ_{03} are given by

$$\Phi_{01} = l_0 = \alpha_{01},\ \Phi_{02} = l_0 + l_1 = \alpha_{01} + \alpha_{02},\ \Phi_{03} = \alpha = l_0 + l_1 + l_3 = \alpha_{01} + \alpha_{02} + \alpha_{03} \tag{28}$$

The width D_3 of third- order solitary wave is obtained as

$$D_3 = \left(\sqrt{\frac{2}{V_0^{-3}V^{(1)}}}\right)\eta_3 \tag{29}$$

where η_3 can be obtained from the positive root of the following equation of η,

$$\alpha_4\tanh^6\eta + (\alpha_2 - \alpha_4)\tanh^4\eta + (1-\alpha_2)\tanh^2\eta - 0.58 = 0 \tag{30}$$

α_2 and α_4 in (30) are given above.

The phase velocity V_1, V_2 and V_3 of the solitary waves correct up to third order becomes,

$$V_1 = V_0 + V^{(1)}, \tag{31a}$$

$$V_2 = V_0 + V^{(1)} + V^{(2)} = V_0\left[1 + \frac{V^{(1)}}{V_0} + \frac{3}{2}\left(\frac{V^{(1)}}{V_0}\right)^2\right] \tag{31b}$$

$$V_3 = V_0 + V^{(1)} + V^{(3)} = V_0\left[1 + \frac{V^{(1)}}{V_0} + \frac{3}{2}\left(\frac{V^{(1)}}{V_0}\right)^2 + \frac{5}{2}\left(\frac{V^{(1)}}{V_0}\right)^3\right] \tag{31c}$$

Assuming $\gamma = V_0 V^{(1)}$ which can be considered as a parameter and in terms of this parameter the Mach numbers become

$$M_1 = \frac{V_1}{V_0} = [1+\gamma], \quad M_2 = \frac{V_2}{V_0} = \left[1 + \gamma + \frac{3\gamma^2}{2}\right], \quad M_3 = \frac{V_3}{V_0} = \left[1 + \gamma + \frac{3\gamma^2}{2} + \frac{5\gamma^3}{2}\right] \tag{32}$$

5. Special cases

5.1. Solitary waves in critical case

In critical case the nonlinear coefficient vanishes, i.e. $\delta_2^{(0)} = 0$. Under this condition the solitary wave will not be excited in plasma. But near critical value of phase velocity V_0 (i.e. when $\delta_2^{(0)} \approx 0$), solitary wave will be excited with *sechη* profile instead of usual $sech^2\eta$ profile. To study the solitary wave near critical state $\delta_2^{(0)} \approx 0$ the following perturbation expansion for ϕ should be used,

$$\varphi = \varepsilon^{1/2}\varphi^{(1)} + \varepsilon\varphi^{(2)} + \varepsilon^{3/2}\varphi^{(3)} + \cdots \tag{33}$$

Substituting (16), (17b) and (33) in (21) and equating the coefficients of ε^2 from both sides we get

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2} + \delta_2^{(0)}\varphi^{(1)^2}\varphi^{(2)} + \frac{1}{4}\delta_3^{(0)}\varphi^{(1)^4} \tag{34}$$

Near critical situation when $\delta_2^{(0)} \approx 0$, Eq. (34) is reduced to

$$\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \delta_1^{(1)}\varphi^{(1)^2} + \frac{1}{2}\delta_3^{(0)}\varphi^{(1)^4} \tag{35}$$

Eq. (35) has the solution for solitary waves in the form

$$\varphi^{(1)} = \lambda \sec h\eta' \tag{36}$$

where

$$\lambda^2 = -\frac{\delta_3^{(0)}}{2\delta_1^{(1)}}, \quad \eta' = \frac{1}{2}\left(\sqrt{\delta_1^{(1)}}\right)X \tag{37}$$

In critical case ($\delta_2^{(0)} = 0$) we obtain from (20), $V_0 = \sqrt[4]{3}$. Using (18) the critical value of β is 0.432. At $\beta \approx 0.432$, the solution of first order solitary waves in plasma with cold ions and nonthermal electrons is given by (36).

5.2. Arbitrary amplitude solitary waves

To find the arbitrary amplitude solitary waves up to second order we take the terms up to φ^3 of Eq. (14a) and obtain

$$\frac{d^2\varphi}{d\xi^2} = S_1\varphi + S_2\varphi^2 + S_3\varphi^3 \tag{38}$$

where $S_1 = \Delta_1$, $S_2 = -\Delta_2$, $S_3 = \Delta_3$.

For first-order solutions for arbitrary amplitude solitary waves, the terms up to ϕ^2 of the right hand side of Eq. (38) are taken into account and the solution is obtained as

$$\varphi_1 = \frac{3S_1}{2S_2}\sec h^2\left(\sqrt{\frac{S_1}{4}}\eta\right) \tag{39}$$

For second-order solutions for arbitrary amplitude solitary waves, the terms up to ϕ^3 of the right hand side of Eq. (38) are considered and the solution is obtained as

$$\varphi_2 = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}\left[2\cosh^2\left(\sqrt{\frac{S_1}{4}}\eta\right) - 1\right]} \tag{40}$$

The amplitudes of first- and second-order arbitrary amplitude solitary waves are given by

$$\varphi_{01} = \frac{3S_1}{2S_2} \text{ and } \varphi_{02} = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}}, \tag{41}$$

respectively.

The corresponding widths of the solitary waves are

$$D_1 = \frac{2}{\sqrt{S_1}} \text{ and } D_2 = D_1\cosh^{-1}\left(\frac{1.381S_2}{\sqrt{4S_2^2 - 18S_1S_3}} + 1.6905\right)^{1/2} \tag{42}$$

5.3. Double layers

It is known that the double-layers in plasma are formed when Sagdeev potential ψ satisfies the following conditions

(i) $\psi = 0$ at $\varphi = 0$ and $\varphi = \varphi_{\max}(= \varphi_m)$ (say),
(ii) $\frac{\partial\psi}{\partial\varphi} = 0$ at $\varphi = 0$ and $\varphi = \varphi_m$
and (iii)

$$\frac{\partial^2\psi}{\partial\varphi^2} < 0 \text{ at } \varphi = \varphi_m. \tag{43}$$

To obtain the double layer solution from Eq. (14a) we integrate it and then use the stretching ξ- coordinates as $X = \varepsilon\xi$ Thus we obtain

$$\frac{\varepsilon^2}{2}\left(\frac{d\phi}{dX}\right)^2 = \frac{1}{2}\Delta_1\phi^2 + \frac{1}{3}\Delta_2\phi^3 + \frac{1}{4}\Delta_3\phi^4 + \frac{1}{5}\Delta_4\phi^5 + \cdots \tag{44}$$

We also assume the perturbation expansion of V as

$$V = V_0 + \varepsilon^2 V^{(1)} + \epsilon^3\ V^{(2)} + \cdots \tag{45}$$

In order to consider higher order nonlinearity both quadratic and cubic, we take the co-efficient of ϕ^3 in Eq. (44) as $\frac{\varepsilon}{3}\Delta_2$. Then using Eqs. (17a) and (45) in Eq. (44) we take coefficient of ε^4 on both sides. Thus the first order equation for $\phi^{(1)}$ is obtained as

$$\frac{1}{2}\left(\frac{d\phi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{1)}\,(\phi^{(1)})^2 + \frac{1}{3}\delta_2^{(0)}(\phi^{(1)})^3 + \frac{1}{4}\delta_3^{(0)}(\phi^{(1)})^4 \tag{46}$$

The double layer solution of Eq. (46) is obtained as

$$\varphi_d^{(1)} = P[1 - \tanh(X/\mu)] \tag{47}$$

where, $P = -\frac{1}{3}\frac{\delta_2^{(0)}}{\delta_3^{(0)}}$, P is the amplitude of double layers; $\mu = \frac{1}{P}\sqrt{2/\delta_3^{(0)}}$, μ being the thickness of double layers. It is important to note that the term $\delta_3^{(0)}$ should be positive for the excitation of double layers in plasma.

The double layers in plasma may be compressive or rarefactive. For the compressive double- layers $P > 0$ and for rarefactive double-layers $P < 0$.

6. Results and discussion

To observe the existence of solitary waves in plasma, we have plotted Sagdeev potential ψ (ϕ) against electrostatic potential ϕ for different values nonthermal parameter (β) of electrons. During our numerical calculations, we have chosen the values of nonthermal parameter in such a way that the condition given in Eq. (12) is satisfied. Moreover, the values of nonthermal parameter are suitable for laboratory as well as space plasma. According to Cairns et al. [14] the value of nonthermal parameter β is $0 \le \beta \ge 1.3$. But from the condition obtained in our present paper it is seen that for existence of solitary waves the value of nonthermal parameter β of electrons should be less than unity, otherwise the phase velocity V of the ion-acoustic wave will be imaginary which is unphysical. So, for all values of β suggested by Cairns et al. [14] the solitary waves will not be excited at all in the plasma having nonthermal electrons.

Following the condition (12), we have plotted Sagdeev potential ψ (ϕ) given by (10) against electrostatic potential ϕ (shown in Fig. 1) taking the values of β (unit- less) up to 0.2. The potential-well in Fig. 1 indicates that ion-acoustic

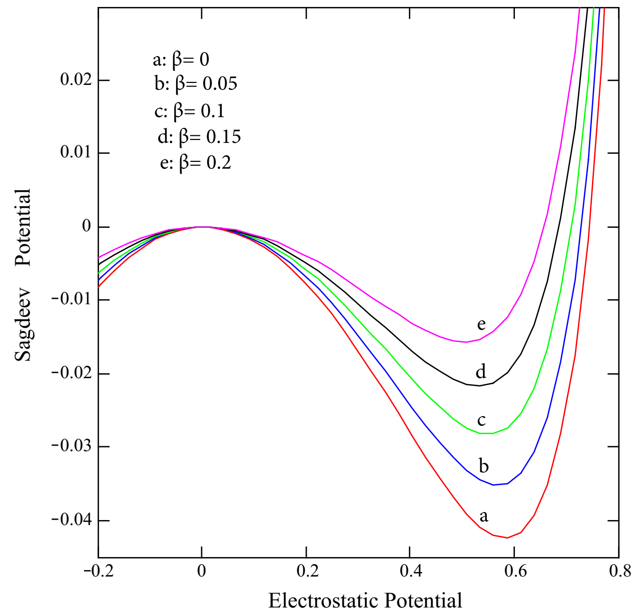

Fig. 1 Sagdeev potential profile for different values of nonthermal parameter β. The curves labeled with *a, b, c, d, e* refer to five different values of β = 0, 0.05, 0.1, 0.15 and 0.2 with V = 1.3

solitary waves will exist in plasma consisting of cold positive ions and nonthermal electrons.

The structures of solitary waves in plasma are shown in Figs. 2(a)–2(c). The perturbed potentials $\phi^{(1)}$, $\phi^{(2)}$ and $\phi^{(3)}$ of solitary waves given by Eqs. (22), (24) and (26a) at first-, second- and third- order for nonthermal parameter $\beta = 0.2$ and $\beta = 0.6$ are shown in Fig. 2(a). It is seen that solitary wave (i) at first- order is compressive for $\beta = 0.2$ and rarefactive for $\beta = 0.6$, (ii) at second- order it is compressive and wavy for $\beta = 0.2$ and rarefactive and wavy for $\beta = 0.6$; (iii) at third order it is rarefactive for $\beta = 0.6$ but it is compressive $\beta = 0.2$. In Fig. 2(b), corrected second- and third-order electrostatic potentials of solitary waves Φ_2 $(= \phi^{(1)} + \phi^{(2)})$ given by Eq. (27b) and Φ_3 $(=\phi^{(1)} + \phi^{(2)} + \phi^{(3)})$ given by (27c) including Φ_1 $(=\phi^{(1)})$ given by Eq. (27a) for nonthermal parameter $\beta = 0.2$ and 0.6 are shown. It is seen that corrected solitary waves are compressive for $\beta = 0.2$ but the solitary waves are rarefactive for $\beta = 0.6$. It is interesting to note that corrected third order solitary wave for $\beta = 0.6$ is W- shaped which is generally observed for the second order solitary waves in a negative ion plasma [33] without the nonthermal electrons. Since, third-order solitary waves are seen to be W-shaped due to presence of nonthermal electrons, its variations for different values of β ($\beta = 0.1, 0.5, 0.6$ and 0.9) are shown in Fig. 2(c). It is seen that W-shape of third- order solitary waves is prominent for large values of β.

For different values of β, (i) the amplitudes of corrected first-, second- and third-order solitary waves [Φ_{10}, Φ_{20}, Φ_{30}

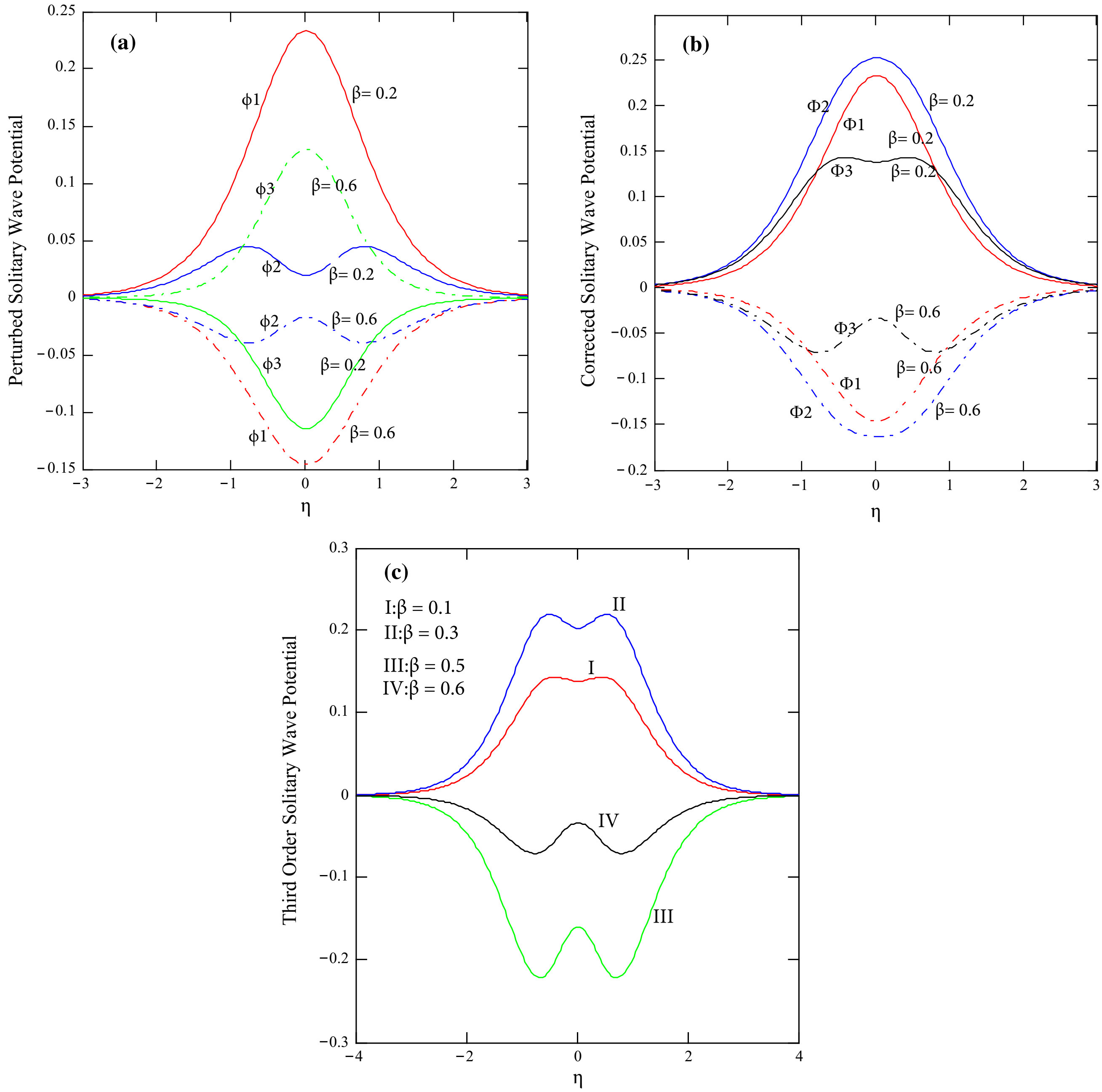

Fig. 2 Profiles of (**a**) first-, second- and third-order perturbed solitons (ϕ_1, ϕ_2, ϕ_3)—solid curves for $\beta = 0.2$ and dadotted curves for $\beta = 0.6$; (**b**) first-, second- and third-order corrected solitons (Φ_1, Φ_2, Φ_3)—solid curves for $\beta = 0.2$ and dadotted curves for $\beta = 0.6$; (**c**) W-type third-order corrected solitons (Φ_3)—the curves labeled with I, II, III, IV refer to $\beta = 0.1$, 0.3, 0.5 and 0.6; [$\Phi_1 = \phi_1$, $\Phi_2 = \phi_1 + \phi_1$, $\Phi_3 = \phi_1 + \phi_2 + \phi_3$]

given by (28)] are shown in Fig. 3(a) and (ii) the widths [D_1 and D_2 given by Eqs. (22d) and (24b)] for perturbed first- and second-order solitary waves are shown in Fig. 3(b). It is seen from Fig. 3(a) that third order amplitude is greater than second order amplitude and second-order amplitude is greater than first-order amplitude, Moreover, all amplitudes are increased with the increase of β. Fig. 3(b) shows that both first-order and second-order widths are decreased with the increase of nonthermal parameter β. The width of second order solitary wave is greater than the width of first order solitary wave for all possible values of β.

It is known that solitary wave will not be excited at critical value of nonthermal parameter ($\beta_c = 0.432$) since at this situation the nonlinear term $\delta_2^{(0)} = 0$ of Eq. (20) will be vanished. However, it has been shown in Eq. (35) that

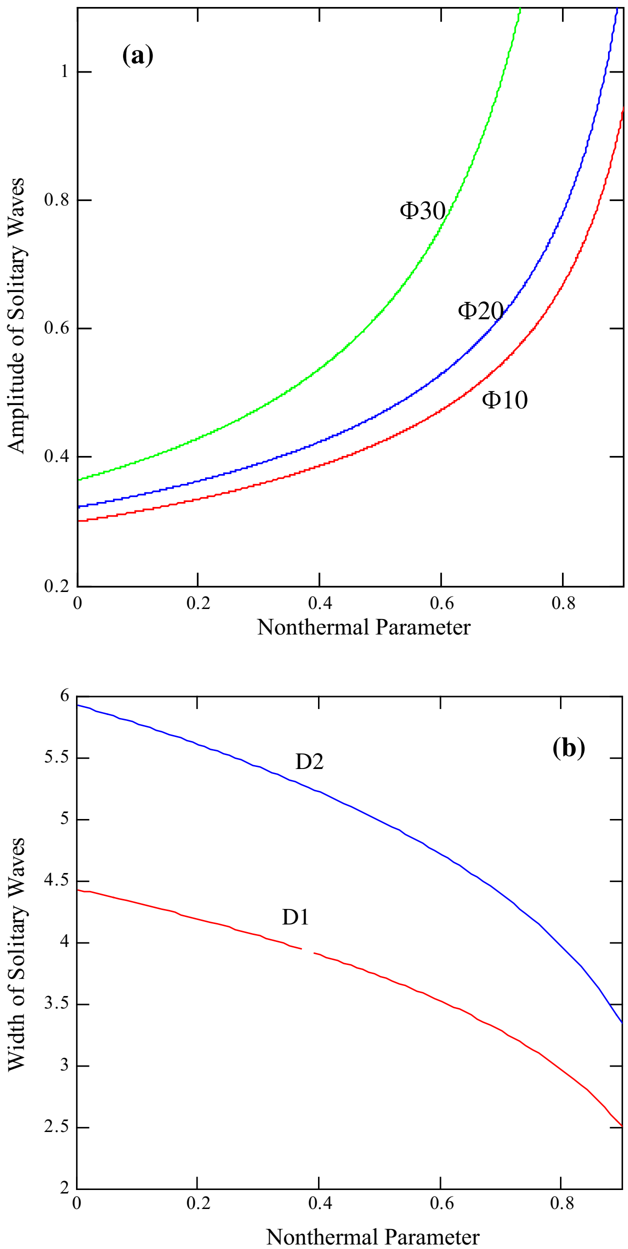

Fig. 3 Variations of (**a**) first-, second- and third-order amplitudes (Φ_{01} Φ_{02}, Φ_{03}) of solitons; (**b**) first- and second-order width (D_1, D_2) of solitons for different values of β

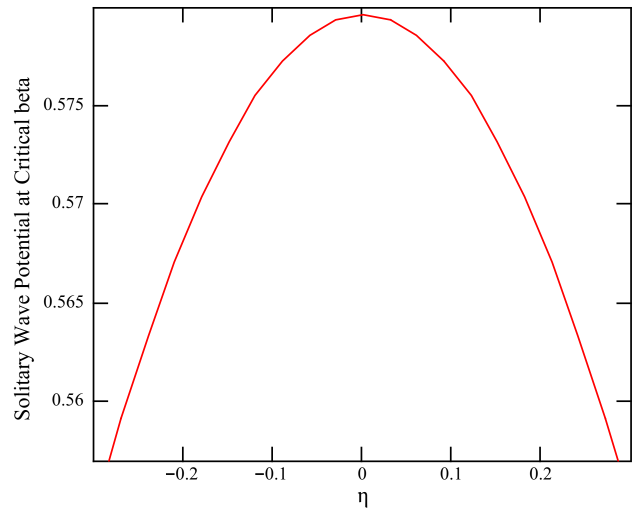

Fig. 4 Profile of ion- acoustic solitary wave near critical value of β

near critical value of β solitary wave will be excited in the plasma with sech(η) profile instead of $\text{sech}^2(\eta)$ profile. The profile of solitary waves near critical value of β is shown in Fig. 4 in which the solitary wave is compressive in nature.

It is important to mention that solitary waves discussed here are for the small-amplitude waves. Arbitrary amplitude solitary wave has been discussed in Sect. 5.2. The solution of arbitrary amplitude first- and second-order ion acoustic solitary waves in plasma having nonthermal electrons are given by Eqs. (39) and (40). Profiles of the arbitrary amplitude solitary waves for $\beta = 0.1$, 0.3 and 0.9. are shown in Figs. 5(a) and 5(b). From Fig. 5(a) it is seen that first-order solitary waves are rarefactive for $\beta = 0.1$, 0.3, but for $\beta = 0.9$ solitary wave is compressive. Similarly, from Fig. 5(b) it is observed that second-order solitary waves are compressive for $\beta = 0.1$, 0.3, but for $\beta = 0.9$ solitary wave is rarefactive. It is important to see that for arbitrary amplitude waves there are no W-type solitary waves in plasma having nonthermal electrons.

Ion-acoustic double layers may also be excited in plasma. Using Eq. (47) the profiles of ion acoustic double layers are drawn (Fig. 6) from which the effects of nonthermal electrons on the double layers are clearly understood. It is observed that ion acoustic double layers are strong for large values of the nonthermal parameter β of electrons in plasma.

7. Conclusions

An integrated form of governing equations in a plasma consisting of cold positive ions and nonthermal electrons has been derived and expressed in terms of pseudopotential for the study of higher order ion-acoustic solitary waves. Nonlinear equation derived from the Sagdeev potential are solved with the help of Bogoliubov–Mitropolosky method for first-, second- and third- order ion acoustic solitary waves and their profiles are depicted for different values of nonthermal parameter of electrons. It is seen that nonthermal electrons has important role on the shape of solitary waves in each order, particularly in third order where W-shaped solitary wave is excited.

Our present method for the study of higher order ion acoustic solitary wave is advantageous than other

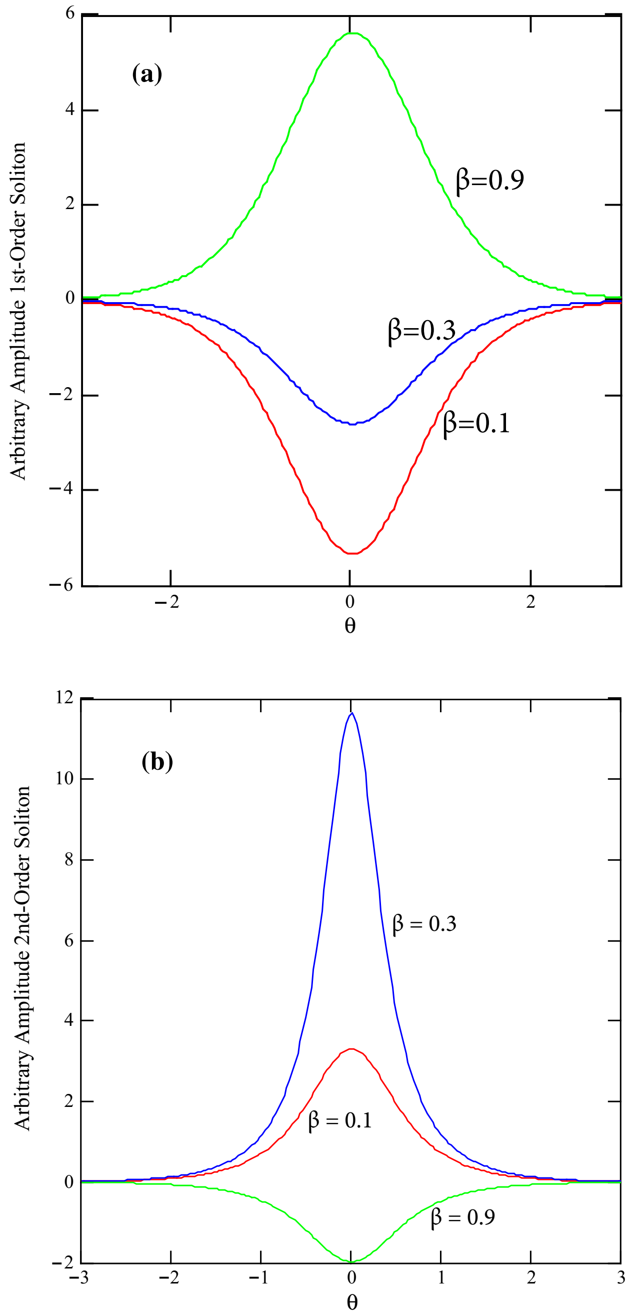

Fig. 5 Profiles of arbitrary amplitude (**a**) first-order and (**b**) second-order solitons for $\beta = 0.1$, 0.3 and 0.9 with $V = 1.4$

mathematical method, because the equation at each order becomes a first-order inhomogeneous differential equation which is easier to solve. But, for the study of solitary waves in plasma using the standard reductive perturbation method [1] we have to solve inhomogeneous second-order differential equation at each order.

Most important and significant achievement in this paper is the W-type ion-acoustic solitary waves in cold plasma in presence of nonthermal electrons when third

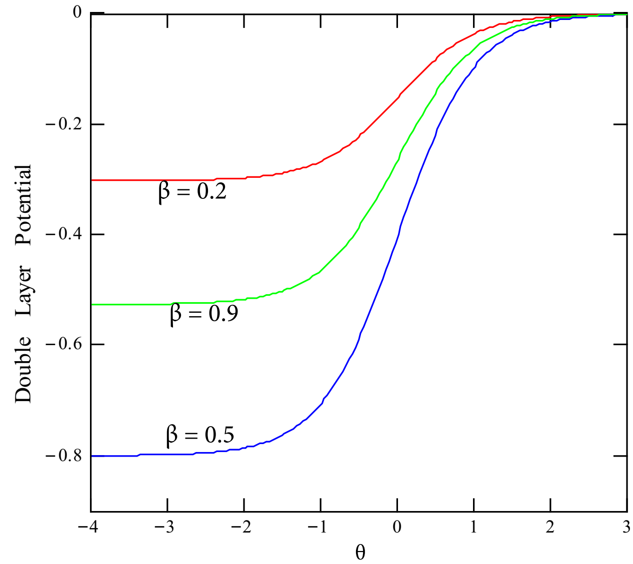

Fig. 6 Profiles of ion-acoustic double layers for three different values of $\beta = 0.2$, 0.5 and 0.9

order nonlinear and dispersive effects are considered. But, if we consider the effect ion temperature in plasma including nonthermal electrons, we will get more significant results on the third order ion acoustic solitary waves.

It is known that the solution of second-order K-dV equation in a negative ion plasma gives a W-type solitary wave [37]. In multicomponent plasma consisting of nonthermal electrons and negative ions, W-type solitary waves are also excited [24]. So, the studies of third-order ion acoustic solitary wave in multicomponent plasma with warm ions, nonthermal electrons, negative ions, positrons etc. using our present mathematical technique would become important and would be useful to understand the nonlinear wave processes in ionospheric and magnetospheric multicomponent plasma.

References

[1] H Washimi and T Taniuti *Phys. Rev. Lett.* **17** 966 (1966)
[2] S G Tagare *Plasma Phys.* **15** 1247 (1973)
[3] R K Roychoudhury and S Bhattacharya *Can. J. Phys.* **65** 699 (1987)
[4] G C Das *IEEE Trans. Plasma Sci.* **3** 168 (1975)
[5] S Chattopadhyaya and S N Paul *Afr. Rev. Phys.* **7** 289 (2012)
[6] B N Goswami and B Buti *Phys. Lett. A* **57** 149 (1976)
[7] K K Ghosh, B Paul, C Das and S N Paul *J. Phys. A Math. Theor.* **41** 335501 (2008)
[8] L C Lee and J R Kan *Phys. Fluids* **24** 430 (1981)
[9] K K Ghosh, S N Paul and S Chattopadhyaya *FIZIKA A* **13** 33 (2004)
[10] H Schamel *Plasma Phys.* **14** 905 (1972)
[11] H Schamel *J. Plasma Phys.* **9** 377 (1973)

[12] N Saini, I Kourakis and M A Hellberg *Phys. Plasmas* **16** 062903 (2009)

[13] M R Rauhani and Z E Abbasi *Iran. J. Phys. Res.* **12** 84 (2013)

[14] R A Cairns, A A Mamun, R Bingham, R Bostrom, R O Dendy, C M C Nairn and P K Shukla *Geophys. Res. Lett.* **22** 2709 (1995)

[15] D S Hall, C P Chaloner, D A Bryant, D R Lepine and V P Tritakis *J. Geophys. Res.* **96** 7869 (1991)

[16] R A Cairns, A A Mamun, R Bingham and P K Shukla *Phys. Scr.* **63** 80 (1996)

[17] A A Mamun *Phys. Rev. E* **55** 1852 (1997)

[18] A A Mamun *Euro. Phys. J*. **11** 143 (2000)

[19] A Bandyapadhyay and K P Das *J. Plasma Phys*. **62** 255 (1999)

[20] A Bandyapadhyay and KP Das *Phys. Scr.* **61** 92 (2000)

[21] A Bandyapadhyay and K P Das *J. Plasma Phys.* **68** 285 (2002)

[22] S K Bhattacharaya, S N Paul and B Chakraborty *Indian J. Phys*. **77B** 327 (2003)

[23] I Paul, B Ghosh, G Pakira, S N Paul and S Chattopadhyay *Indian J. Phys*. **86** 395 (2012)

[24] B Ghosh, S Banerjee and S N Paul *Indian J. Pure Appl. Phys*. **51** 488 (2013)

[25] H Ikezi, R J Tayler and D R Baker *Phys. Rev. Lett.* **25** 11 (1970)

[26] H Ikezi *Phys. Fluids* **16** 1668 (1973)

[27] M Q Tran *Phys. Scr.* **20** 317 (1979)

[28] K E Lonngren *Plasma Phys.* **25** 943(1983)

[29] Y H Ichikawa, T Matsuhasi and K Konno *J. Phys. Soc. Jpn*. **41** 1382 (1976)

[30] C S Lai *Can. J. Phys.* **57** 490 (1979)

[31] Y Kodama and T Taniuti *J. Phys. Soc. Jpn.* **45** 298 (1978)

[32] M K Kalita and S Bujarbarua *Can. J. Phys.* **60** 392 (1982)

[33] S G Tagare and R V Reddy *J. Plasma Phys*. **35** 279 (1986)

[34] W M Moslem *J. Plasma Phys*. **63** 139 (2000)

[35] S N Paul, S Chattopadhyay, S K Bhattacharya and B Bera *Pramana* **60** 1217 (2003)

[36] K P Das and S R Majumdar *Can. J. Phys.* **69** 822 (1991)

[37] S R Majumdar, A Roychowdhury, S K Bhattacharyya and S N Paul *J. Plasma Phys.* **47** 451 (2005)

[38] N N Bogoliubov and Y A Mitropolsky *Asymptotic Methods in the Theory of Non-Linear Oscillations* (New York: Gordon and Breach) (1961)

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ORIGINAL PAPER

Ion acoustic solitary waves in an electron–ion–positron plasma

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Abstract: Large amplitude ion-acoustic solitary waves have been theoretically studied in a non-collisional plasma consisting of warm positive ions, positrons and isothermal electrons using pseudo-potential (effective potential) method. The expressions of phase velocities of slow- and fast- modes of the wave have been derived and graphically analyzed. The critical values of densities of the positrons and the ions, Mach number and positron temperature for the excitation of ion-acoustic solitary wave have also been obtained. The solutions of the first-order and next higher order solitary waves have been obtained and the profiles of the solitary waves have been drawn taking different values of the plasma parameters. It has been observed that both compressive and rarefactive solitary waves are being excited in presence of positrons in the plasma. The variation of amplitudes of the solitary waves has been shown graphically for different values of density and temperature of positrons, drift velocity and temperature of the ions.

Keywords: Ion-acoustic solitary waves; Electron–ion–positron plasma; Sagdeev potential; Ion-acoustic double layers

PACS Nos.: 52.35Fp; 52.35Mw; 52.35Sb

1. Introduction

During the last two decades, a lot of works on theoretical and experimental investigation of solitons and double layers have been done by various authors considering the plasma as two-component(electrons–ions) [1–3], three-component (electrons, positive and negative ions) [4–7] and other varieties (electrons, positive ions and charged dust particles [8–13]. But, in recent years, wave propagation in astrophysical plasma are being investigated in presence of positrons together with electrons and ions [14–17]. It is believed that electron–positron plasma has its origin in the early stage of universe [18–20] and this has important contribution on the physical processes in galactic nuclei [21], pulsar magnetosphere [22], polar caps of neutron star [23]. It has been observed that nonlinear waves in electron–ion–positron (e–i–p) plasma behave differently from the plasma consisting of electrons and ions [24]. To understand the nature of some characteristics of the waves in e–i–p plasma, Shukla [25] has derived the nonlinear Schrodinger equation and studied modulational instability of the wave. He has reported that the presence of positrons in the plasma has important bearings on radio emission from pulsar magnetosphere. Subsequently, Stenflo *et al.* [26] have studied nonlinear propagation of electromagnetic waves in magnetized electron–positron plasma through the derivation of the coupled equation governing electromagnetic radiation and cold electrostatic oscillation. Some works of ion acoustic solitons in a plasma consisting of cold and hot electrons and positrons have been carried out by Pillay and Bharuthram [27]. They have found that both compressive and rarefactive solitons are being excited in the plasma. Later, Popel *et al.* [28] have shown that the presence of positronin multispecies plasma can result in the reduction of the ion-acoustic soliton amplitudes. Burman and Roychowdhury [29] and Salahudin *et al.* [30] have studied the formation of envelope soliton in a collisionless unmagnetized electron–positron plasma deriving the non-linear Schrodinger equation. They have shown that small

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percentage ions in electron–positron plasma can change the dynamics of a system significantly. Subsequently, Shukla *et al*. [31], Alinejad [32], Ghosh and Bharuthram [33], Massod *et al*. [34] have studied nonlinear propagation of ion acoustic waves in an e–i–p plasma with trapped electrons and some interesting results are obtained. Recently, Pakzad [35] has theoretically investigated ion acoustic solitary waves in a weakly relativistic plasma with non-thermal electrons, positrons and warm ions. He has shown that relativistic effect has considerable contribution on the excitation of solitary waves in presence of stream velocity of the ions which was first shown by Das and Paul [36].

However, it is known that in a non-collisional plasma Landau damping is very much important on the propagation of waves in plasma [37–40]. Ott and Sudan [41] have considered the effects of Landau damping on the ion acoustic solitary waves in plasma neglecting the particle trapping effect under the assumptions that the particle trapping time is much greater than the Landau damping time. Subsequently, Roychowdhury *et al*. [42], Bandyopadhyay and Das [43] have studied ion-acoustic solitary waves in Landau damped plasma. Recently, Ghosh and Bharuthram [44] have theoretically investigated the ion-acoustic solitary wave in e–i–p plasma considering Landau damping and neglecting the particle trapping effect in e–i–p plasma using the reductive perturbation method. They have found that Landau damping causes the solitary amplitude to decay with time. They have also shown that both linear wave phase velocity and soliton amplitude with increase of positron density in the absence of Landau damping whereas these are increased with positron temperature in such plasma.

In the stated literature, the influence of drift motion of the ions on the ion-acoustic solitary waves in e–i–p plasma has not been explored. Moreover, previous authors have used the reductive perturbation method for their studies of ion-acoustic solitary waves in e–i–p plasma.

However, pseudo potential method has been used by Popel *et al*. [28], Pakzad [45, 46] and other authors for theoretical investigation of ion-acoustic solitary waves in plasma with electrons, positrons and ions. In the present paper, pseudo-potential method has been used for theoretical investigation of ion-acoustic solitary waves in plasma with isothermal electrons and positrons including warm drifting ions.

2. Formulation

Unmagnetised and collissionless three-component plasma consisting of electrons, positrons and singly charged warm positive ions has been considered. Their respective number densities are n_e, n_p and n_i. At equilibrium, $n_{i0} + n_{p0} = n_{e0}$, where n_{e0}, n_{p0} and n_{i0} are the equilibrium densities. Normalizing the densities by n_{e0}, the equilibrium condition becomes $\chi = n_{p0}/n_{e0} = 1 - n_{i0}$.The temperature of ion (T_i) is assumed to be much smaller than the temperature of electrons (T_e) and positrons (T_p) i.e. $T_i \ll T_e, T_p$ and so the effect of Landau damping can be neglected [47].

The normalized densities of electrons and positrons are given by the Boltzmann distribution,

$$n_e = \exp(\varphi) \tag{1}$$

$$n_p = \chi \exp(-\beta\varphi) \tag{2}$$

where, φ is the electrostatic potential, $\beta = T_e/T_p$.

For the ions, the dynamics of the plasma are governed by the following equations of continuity and momentum transfer,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{3}$$

and

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} = -\frac{\partial \varphi}{\partial x} \tag{4}$$

The pressure equation for the ions is taken as

$$\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial v_i}{\partial x} = 0 \tag{5}$$

For the stated background, Poisson's equation yields

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_p - n_i \tag{6}$$

In Eqs. (3)–(6), v_i is the ion-fluid velocity, p_i is the ion-fluid pressure and $\sigma_i = T_i/T_e$. The velocity v_i is normalized by the characteristic velocity $(\kappa T_e/m)^{1/2}$, the length by the Debye-length $(\kappa T_e/4\pi\, e^2\, n_{e0})^{1/2}$ and the time by the inverse of plasma frequency $(4\pi n_{e0}\, e^2/m)^{-1/2}$. The potential is normalized to $\kappa T_e/e$, T_e and κ being the electron temperature and Boltzmann's constant, respectively. Other parameters have their usual meaning. It is to be stated that the variables in the plasma are normalized in such a way so that the above equations will appear in totally dimensionless form.

For the derivation of Sagdeev potential in e–i–p plasma, the standard mathematical technique is followed [48]. It is assumed that the basic equations are supplemented by the following boundary conditions:

As $|x| \to \infty$

(i) $n_i \to n_{i0}$
(ii) $\varphi \to 0$
(iii)

$$v_i \to v_{i0} \tag{7}$$

(iv) $p_i \to p_{i0}$

To study the existence of ion-acoustic solitons in the electron–positron plasma by using Sagdeev's pseudo-potential approach, the usual transformation $\eta = x - Vt$ is chosen, where V is the velocity of ion-acoustic wave.

Using Eq. (7) in Eqs. (3)–(5), the density of positive ion is obtained from the following equation,

$$\frac{3\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}^3}n_{\rm i}^4 - \left[(V - v_{\rm i0})^2 + \frac{3\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}} - 2\varphi\right]n_{\rm i}^2 + n_{\rm i0}^2(V - v_{\rm i0})^2 = 0 \tag{8}$$

Eq. (8) yields

$$n_{\rm i} = \pm\frac{n_{\rm i0}^{3/2}}{\sqrt{6\sigma_{\rm i}p_{\rm i0}}}\left[\left(V^2 + a_0 - 2\varphi\right) \pm\sqrt{\left\{\left(V^2 + a_0 - 2\varphi\right)^2 - \frac{12\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}}(V - v_{\rm i0})^2\right\}}\right]^{1/2} \tag{8a}$$

where,

$$a_0 = v_{\rm i0}^2 - 2Vv_{\rm i0} + \frac{3\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}}$$

To satisfy the boundary conditions (7). The positive value of negative branch of ion density is taken into account i.e. ion density is

$$n_{\rm i} = \frac{n_{\rm i0}^{3/2}}{\sqrt{6\sigma_{\rm i}p_{\rm i0}}}\left[\left(V^2 + a_0 - 2\varphi\right) -\sqrt{\left\{\left(V^2 + a_0 - 2\varphi\right)^2 - \frac{12\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}}(V - v_{\rm i0})^2\right\}}\right]^{1/2} \tag{9}$$

Or

$$n_{\rm i} = \frac{n_{\rm i0}^{3/2}}{2\sqrt{3\sigma_{\rm i}p_{\rm i0}}}\left[(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}\right] \tag{9a}$$

where

$$A_1 = (V - v_{\rm i0})^2 + \frac{3\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}} \tag{10}$$

$$A_2^2 = \frac{12\sigma_{\rm i}p_{\rm i0}}{n_{\rm i0}}(V - v_{\rm i0})^2 \tag{11}$$

Now, using Eqs. (1), (2) and (9) in Eq. (6) we obtain

$$\frac{\partial^2\varphi}{\partial\eta^2} = \exp(\varphi) + \chi\exp(-\beta\varphi) + \frac{n_{\rm i0}^{3/2}}{2\sqrt{3\sigma_{\rm i}p_{\rm i0}}}\left[(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}\right] \tag{12}$$

Eq. (12) is now written as

$$\frac{d^2\varphi}{d\eta^2} = -\frac{d\psi}{d\varphi} \tag{12a}$$

where $\psi(\varphi)$ is the Sagdeev potential [49].

Now, using the boundary conditions (7) we get from Eqs. (12) and (12a) after integration

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2 + \psi(\varphi) = 0 \tag{12b}$$

where

$$\psi(\varphi) = \psi_e(\varphi) + \psi_p(\varphi) + \psi_i(\varphi) \tag{13}$$

$$\psi_e(\varphi) = 1 - \exp(\varphi) \tag{14}$$

$$\psi_p(\varphi) = \frac{\chi}{\beta}[1 - \exp(-\beta\varphi)] \tag{15}$$

$$\psi_i(\varphi) = \frac{n_{\rm i0}^{3/2}}{6\sqrt{3\sigma_{\rm i}p_{\rm i0}}}\left[(A_1 - A_2 - 2\varphi)^{3/2} - (A_1 + A_2 - 2\varphi)^{3/2} - (A_1 - A_2)^{3/2} + (A_1 + A_2)^{3/2}\right] \tag{16}$$

It is seen that $\psi(\varphi)$ of Eq. (13) reduces to the Sagdeev potential given by Eq. (10) of Popel *et al.* [28] when $\sigma_{\rm i} = 0$ and $v_{\rm i0} = 0$ and the notations for the plasma parameters of [28] are used. It should be remembered that putting $\sigma_{\rm i} = 0$ and $v_{\rm i0} = 0$ directly in Eqs. (13) or (15) the Eq. (10) of Popel *et al.* [28] would not be recovered. In e–i–p plasma consisting of cold and non-drifting ions, $n_{\rm i}$ should be calculated from our Eq. (8) putting $\sigma_{\rm i} = 0$ and $v_{\rm i0} = 0$ and this value of $n_{\rm i}$ should be used for calculating $\psi(\varphi)$.

3. Condition for the existence of soliton solution

The form of the pseudo-potential $\psi(\varphi)$ would determine whether a soliton like solution of Eq. (12b) will exist or not. The conditions for the existence of soliton solution are

(i)

$$\psi(\varphi) = 0 \text{ at } \varphi = 0 \text{ and } \varphi = \varphi_c \tag{17a}$$

(ii)

$$\frac{d\psi}{d\varphi} = 0 \text{ at } \varphi = 0 \tag{17b}$$

and

(iii)

$$\frac{d^2\psi}{d\varphi^2} < 0 \text{ at } \varphi = 0 \tag{17c}$$

It is to be mentioned that for $\varphi_c < \varphi < 0$ rarefactive solitary wave will exist and for $0 < \varphi < \varphi_c$ compressive solitary wave will be excited. Now, using Eq. (13) in Eq. (17c), we obtain

$$n_{i0}^{3/2}\left[\frac{1}{\sqrt{A_1-A_2}}-\frac{1}{\sqrt{A_1+A_2}}\right]<(1+\chi\beta)\sqrt{12\sigma_i p_{i0}} \quad (18)$$

This is the condition for the existences of a potential well in e–i–p plasma.

It is to be mentioned that

$$\psi(\varphi_c)\geq 0 \quad (19)$$

also gives the existence of solitary wave solution, where $\varphi_c = \min.\,(\varphi_{c1}, \varphi_{c2})$.

From Eqs. (17) and (18) is apparent that ψ becomes complex if the following inequalities are satisfied:

$$A_1 - A_2 - 2\varphi < 0 \quad (20)$$

and

$$A_1 + A_2 - 2\varphi < 0 \quad (21)$$

From (20) and (21) we get successively

$$\varphi_{c1} = \varphi > \frac{1}{2}(A_1 - A_2) \quad (22)$$

$$\varphi_{c2} = \varphi > \frac{1}{2}(A_1 + A_2) \quad (23)$$

But it is clearly seen that

$$\varphi_{c1} < \varphi_{c2}$$

or

$$\varphi_c = \min.(\varphi_{c1}, \varphi_{c2}) = \varphi_{c1} = \frac{1}{2}\left[(V - v_{i0}) - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right]^2 \quad (24)$$

for which $\psi(\varphi_c) \geq 0$.

From Eq. (18) it is found that ion acoustic solitary waves will exist in an e–i–p plasma only when the inequality is satisfied. The critical values of the phase velocity (V_p) of ion acoustic solitary waves are obtained from

$$n_{i0}^{3/2}\left[\frac{1}{\sqrt{A_1-A_2}}-\frac{1}{\sqrt{A_1+A_2}}\right]-(1+\chi\beta)\sqrt{12\sigma_i p_{i0}} = 0 \quad (25)$$

From Eq. (25) we get after using $n_{i0} = 1 - \chi$,

$$V_\mathrm{p} = v_{i0} \pm \left[\frac{(1-\chi)}{(1+\chi\beta)} + \frac{3\sigma_\mathrm{i} p_{i0}}{(1-\chi)}\right]^{1/2} \quad (25a)$$

When positrons are not present in a plasma ($\chi = 0$) and only electrons and nondrifting ions are present ($n_{i0} = n_{e0}$), Eq. (25a) gives the phase velocity of ion-acoustic wave $V_\mathrm{p} = \sqrt{1 + 3\sigma_\mathrm{i}}$, which is the well known value of the phase velocity of the ion acoustic solitary wave in plasma having warm ions obtained by Tagare [50].

Eq. (25a) shows that the phase velocity of the ion acoustic wave has two values, one is fast mode (V_pF) and other is slow mode (V_pS),

$$V_\mathrm{pF} = v_{i0} + \left[\frac{(1-\chi)}{(1+\chi\beta)} + \frac{3\sigma_\mathrm{i} p_{i0}}{(1-\chi)}\right]^{1/2} \quad (26a)$$

$$V_\mathrm{pS} = v_{i0} - \left[\frac{(1-\chi)}{(1+\chi\beta)} + \frac{3\sigma_\mathrm{i} p_{i0}}{(1-\chi)}\right]^{1/2} \quad (26b)$$

From Eq. (25), the critical values of the positron density (χ_C), positron temperature (β_C), ion temperature (σ_{iC}) and ion density (n_{i0C}) may be obtained for which ion acoustic solitary waves will exist in plasma in presence of positrons. The critical values of these parameters are:

$$\chi_C = \frac{1}{2(1+\beta M_\mathrm{i}^2)}\left[\gamma \pm \left\{\gamma^2 - 4\left(1+\beta M_\mathrm{i}^2\right)\left(1+3\sigma_\mathrm{i} p_{i0} - M_\mathrm{i}^2\right)\right\}^{1/2}\right] \quad (27a)$$

$$\beta_C = \frac{1}{\chi}\left[\frac{(1-\chi)^2 - M_\mathrm{i}^2(1-\chi) + 3\sigma_\mathrm{i} p_{i0}}{M_\mathrm{i}^2(1-\chi) - 3\sigma_\mathrm{i} p_{i0}}\right] \quad (27b)$$

$$\sigma_{iC} = \frac{(1-\chi)}{3p_{i0}}\left[M_\mathrm{i}^2 - \frac{(1-\chi)}{(1+\chi\beta)}\right] \quad (27c)$$

$$n_{i0C} = \frac{1}{2}\left[M_i^2(1+\chi\beta) \pm \left\{M_i^4(1+\chi\beta)^2 - 12\sigma_i p_{i0}(1+\chi\beta)\right\}^{1/2}\right] \quad (27d)$$

where,

$\gamma = 2 - (1-\beta)M_\mathrm{i}^2 - 3\beta\sigma_\mathrm{i} p_{i0}$

$M_\mathrm{i} = V - v_{i0}$

4. Solitary wave solution

An analytical solution of Eq. (12) for the ion-acoustic solitary waves can be obtained by expanding the r.h.s. in terms of φ keeping the terms up to second order, third order or even next higher orders. After a straight forward calculation one can get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 + R\varphi^3 + \ldots\ldots \quad (28)$$

where,

$$P = 1 + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_\mathrm{i} p_{i0}}}\left(\frac{1}{G_1^{1/2}} - \frac{1}{G_2^{1/2}}\right) + \beta\chi$$

$$Q = -\frac{1}{2}\left[1 + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_\mathrm{i} p_{i0}}}\left(\frac{1}{G_1^{3/2}} - \frac{1}{G_2^{3/2}}\right) - \beta^2\chi\right] \quad (29)$$

$$R = \frac{1}{2}\left[\frac{1}{3} + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_\mathrm{i} p_{i0}}}\left(\frac{1}{G_1^{5/2}} - \frac{1}{G_2^{5/2}}\right) + \frac{1}{3}\beta^3\chi\right]$$

$G_1 = A_1 + A_2$

$G_2 = A_1 - A_2$

Now, for small amplitude solitary waves, the terms up to φ^2 in (28) are taken into consideration, neglecting next higher order terms of φ i.e. φ^3, φ^4, etc. So, we get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 \tag{30}$$

Using the standard mathematical technique, the solution of Eq. (30) for the small amplitude ion-acoustic solitary waves is obtained as:

$$\varphi_1 = \frac{3P}{2Q}\sec h^2\theta \tag{31}$$

where, $\theta = [(P/4)^{1/2}\eta]$. The amplitude and the width of the solitary waves are

$$\varphi_{01} = \frac{3P}{2Q} \tag{32}$$

and

$$\delta_1 = \frac{2}{\sqrt{P}}, \tag{33}$$

respectively.

For large values of φ, the terms unto φ^3 are taken into account and Eq. (29) is written in the following form

$$\left(\frac{d\varphi}{d\eta}\right)^2 = \alpha_1\varphi^2 - \alpha_2\varphi^3 + \alpha_3\varphi^4 \tag{34}$$

where $\alpha_1 = P$, $\alpha_2 = (2Q/3)$ and $\alpha_3 = -(R/2)$

Integrating (34), we get the solution for large amplitude ion acoustic solitary waves given by

$$\varphi_2 = \frac{2\alpha_1}{\left(\alpha_2^2 - 4\alpha_1\alpha_3\right)^{1/2}[2\cosh^2\theta - 1] + \alpha_2} \tag{35}$$

The amplitude and the width of the large amplitude ion acoustic solitary waves in a plasma consisting of positrons are:

$$\varphi_{02} = \frac{2\alpha_1}{\left(\alpha_2^2 - 4\alpha_1\alpha_3\right)^{1/2} + \alpha_2} \tag{36}$$

and

$$\delta_2 = \frac{2}{\sqrt{\alpha_1}}\cosh^{-1}\left[\left\{\frac{0.6905\alpha_2}{(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}} + 1.6905\right\}\right] \tag{37}$$

5. Results and discussion

The characteristic of the Sagdeev potential (Effective potential) given by the expression (13) will determine the possibility of the existence of ion acoustic soliton in an e–i–p plasma. In fact, the soliton will exist in the plasma if the condition $\left|\frac{d^2\psi}{d\varphi^2}\right|_{\varphi=0} < 0$ of Eq. (17) is satisfied. It is to be remembered that ion-acoustic solitary waves will be excited in e–i–p plasma only for some critical values of positron density (χ_c), positron temperature (β_c),ion density (n_{i0c}) and ion temperature σ_{ic}. The critical values of these plasma parameters for the excitation of ion acoustic solitons in e–i–p plasma may be obtained for different values of the plasma parameters using the expressions (27a–d) which are derived from the condition of soliton solution.

In model plasma consisting of positrons, the Sagdeev potential Ψ has been numerically estimated and plotted taking different values of positron density, ion drift, positron temperature and ion temperature. Figure 1 shows the Sagdeev potential for different values of positron density. It is seen that Sagdeev potential is negative in e–i–p plasma (having $\chi = 0.17 - 0.32$, $\sigma_\iota = 0.001$, $\beta = 0.001$, $V = 1.6$, $v_{i0} = 0.5$ and $p_{i0} = 1$) which means plasma particles are trapped and there is every possibility of the excitation of ion acoustic soliton in an e–i–p plasma. It is also seen that the depth of Sagdeev potential increases as χ (positron density) increases. It indicates the trapping of more and more particles in e–i–p plasma having large number of positrons.

We have also studied numerically the dependence of positron temperature (β), ion drift velocity (v_{i0}) and ion temperature (σ_ι) on the Sagdeev potential (Ψ).It has been found that when the temperature of positrons (T_p) is small the possibility of trapping of particles is more i.e. the amplitude of the solitary will be large. Similarly it is also

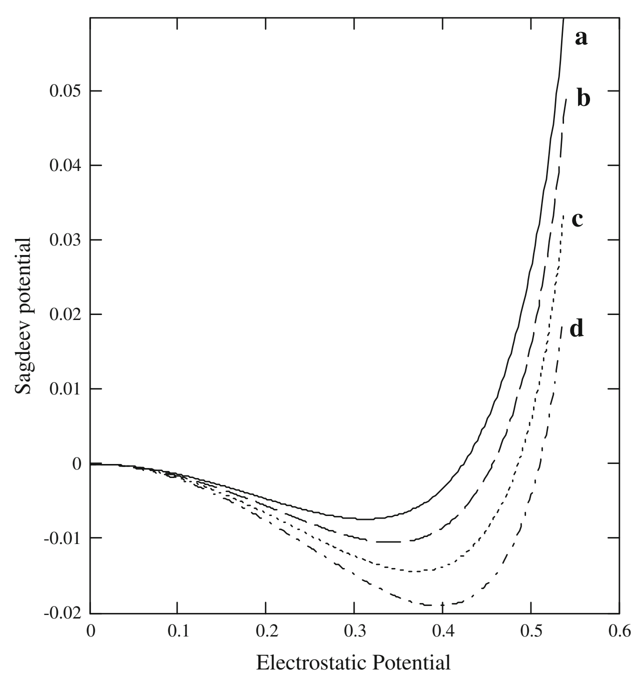

Fig. 1 Sagdeev potentials for different values of positron density (χ): The *solid line* (a), *dashed line* (b), *dotted line* (c) and *dadotted line* (d) represent for $\chi = 0.17$, 0.22, 0.27 and 0.32 respectively; when $\sigma_\iota = 0.001$, $\beta = 0.001$, $V = 1.6$, $v_{i0} = 0.5$ and $p_{i0} = 1$

found that the amplitude of the solitary wave is large for small values of ion drift velocity and ion temperature.

The critical value of the phase velocity for the excitation of ion acoustic soliton in e–i–p plasma is obtained from the expression (25) which gives two values of the phase velocity, one is fast mode (V_F) and the other is slow mode (V_S) given by Eqs. (26a) and (26b). The variations of V_F and V_S with the positron density (χ) and ion drift velocity (v_{i0}) are shown in Fig. 2. In an e–i–p plasma having for ion drift velocity (v_{i0}) $= 0.0 - 0.8$, $\sigma = 0.05$, $\beta = 10$, $V = 1.6$ and $p_{i0} = 1$, it is seen that V_F decreases but V_S increases with the increase of χ. Also, it has been found numerically that V_F decreases but V_S increases with the increase of positron temperature.

It is to be noted that ion-acoustic solitary waves will be excited in e–i–p plasma only for certain restricted regions of plasma parameters. The critical values of various plasma parameters for the excitation of ion acoustic solitons in e–i–p plasma may be obtained by using the expressions (27a–27d) which are derived from the condition of soliton solution.

The effects of the plasma parameters, e.g. positron density positron temperature, ion drift velocity or Mach number, and ion temperature on the ion acoustic solitary waves can be understood from the profiles of solitary waves. The profiles of solitary waves for different values positron density in e–i–p plasma having $\chi = 0.1-0.4$. $\sigma_i = 0.05$, $\beta = 0.9$, $V = 1.6$, $v_{i0} = 0.7$, $p_{i0} = 1$ are shown in Fig. 3. It is seen that positron density is the deciding factor for the excitation of compressive or rarefactive solitary waves in the plasma. Both the small amplitude and large amplitude solitary waves are rarefactive in nature when $\chi = 0.1$. But, these solitary waves become compressive when $\chi = 0.3$ and 0.4.

From a similar study for different values of positron temperatures in e–i–p plasma having $\beta = 0.8$ and 2.4, $\chi = 0.2$, $\sigma_i = 0.001$, $V = 1.6$, $v_{i0} = 0.1$, $p_{i0} = 1$, it has been found that small amplitude solitary waves will be rarefactive when $\beta = 0.8$ and compressive when $\beta = 2.4$. But the large amplitude solitary waves are compressive for both values of β.

From the profiles of the solitary waves for different ion drift velocity (v_{i0}) in e–i–p plasma having $v_{i0} = 0.8$, 0.95 and 1.1, $\chi = 0.25$, $\sigma_i = 0.01$, $\beta = 2$, $V = 1.6$, $p_{i0} = 1$, it has been found that small amplitude solitary waves are compressive when $v_{i0} = 0.8$ and rarefactive when $v_{i0} = 1.1$. But, the large amplitude solitary waves become rarefactive when $v_{i0} = 0.95$ and 1.1.

A similar study for different ion temperature in e–i–p plasma shows that small amplitude solitary wave is compressive when $\sigma_i = 0.35$, but it is rarefactive when $\sigma_i = 0.65$ and 0.95. It is interesting to find that large amplitude solitary waves will be only compressive in nature.

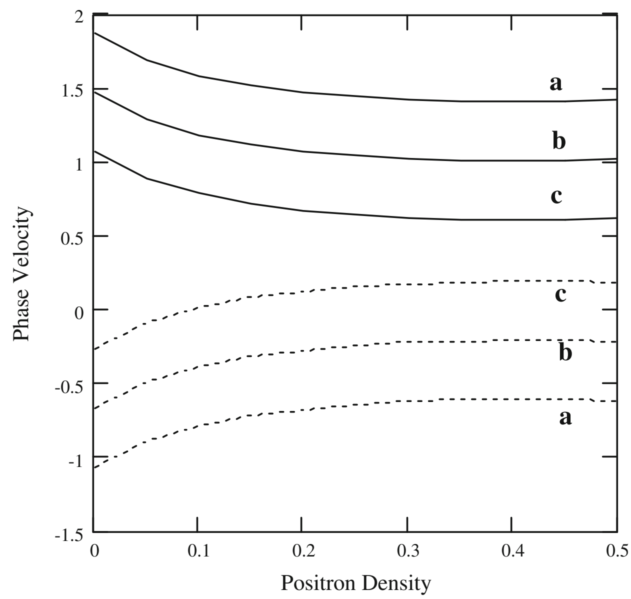

Fig. 2 Variation of phase velocity of the wave for different values of positron density (χ) and ion drift velocity v_{i0}: The *solid line* and *dotted line* represent for fast- and slow-ion acoustic wave respectively; The lines (*a*), (*b*) and (*c*) represent for ion drift velocity $v_{i0} = 0$, 0.4 and 0.8 respectively; when $\sigma_i = 0.05$, $\beta = 10$, $V = 1.6$ and $p_{i0} = 1$

Thus we find that amplitude of the solitary waves depend on the values of plasma parameters. The variations of amplitude with positron density for different values of positron temperature are shown in Fig. 4. It is observed that amplitudes are large for large values of β i.e. when the electron temperature is much higher than positron temperature. Similarly we have found that the amplitude increases with the increase of positron density in e–i–p plasma.

6. Summary and concluding remarks

In the present paper, ion-acoustic solitary waves have been theoretically studied in a multicomponent plasma consisting of warm electrons, ions and positrons using the pseudo-potential method neglecting Landau damping. The expressions of (i) Sagdeev potential, (ii) phase velocity of fast- and slow-ion acoustic mode, (iii) critical values of positron density, (iv) positron temperature, (v) ion temperature and (vi) ion density for the excitation of ion acoustic solitary wave in e–i–p plasma have been derived. The solitons for the lowest order (small amplitude) and higher order (large amplitude) are obtained from the nonlinear equation derived from the Sagdeev potential. It is observed that both compressive and rarefactive solitons may be excited in presence of the positrons in a plasma having warm and drifting ions. It is seen that positron

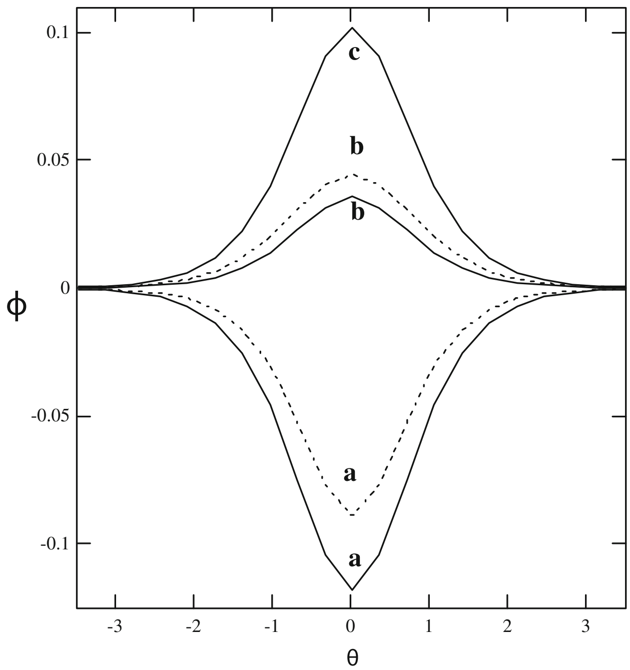

Fig. 3 The profiles of solitary waves for different values of positron density (χ): The *solid lines* and *dotted lines* represent for small amplitude and large amplitude solitary waves. The *lines* showed from bottom to top (a), (b) and (c) represent for positron densities (χ) = 0.1, 0.3 and 0.4 respectively; when $\sigma_i = 0.05$, $\beta = 0.9$, $v_{i0} = 0.7$, $V = 1.6$ and $p_{i0} = 1$

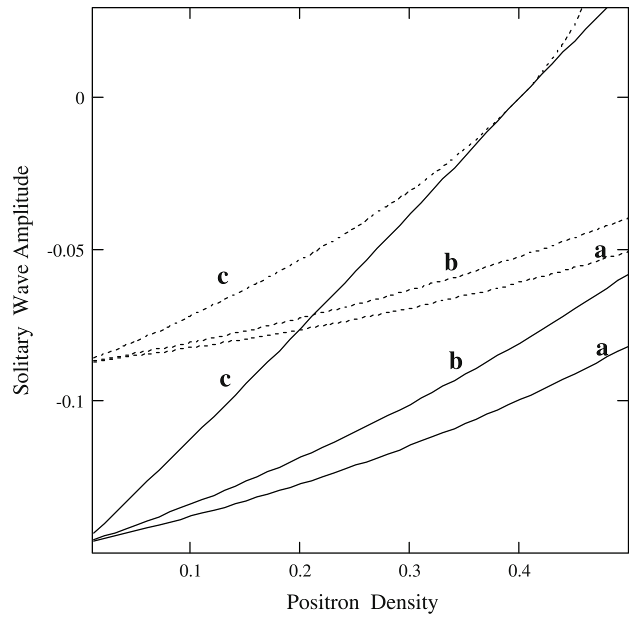

Fig. 4 Variation of amplitude of solitary waves for different values of positron density (χ) and positron temperatures (β). The *solid lines* and *dotted lines* represent for small amplitude and large amplitude solitary waves respectively. The lines showed by (a), (b) and (c) represent for β = 0.1, 1 and 5 respectively; when $V = 1.6$, $v_{i0} = 1.1$, $\sigma_i = 0.01$, and $p_{i0} = 1$

density, positron temperature, ion temperature and ion drift velocity have considerable impact on the excitation of solitons in e–i–p plasma. Interesting results which are found are: (i) first-order ion-acoustic solitons in e–i–p plasma may be compressive and rarefactive depending upon the values of the plasma parameters, but second order ion-acoustic solitons is mainly compressive in nature in e–i–p plasma. The amplitude of solitary waves is larger for large values positron density. But, Popel *et al*. [28] has shown that the amplitude of solitary waves is reduced due to presence of positrons in a multispecies plasma

However, from Eq. (34) we may also obtain double-layer solution for the ion-acoustic waves in e–i–p plasma. The double-layers are formed

when

(i) $\psi = 0$ at $\varphi = 0$ and $\varphi = \varphi_{\max} = \varphi_m$ (say)

(ii) $\frac{\partial\psi}{\partial\varphi} = 0$ at $\varphi = 0$ and $\varphi = \varphi_m$

and

(iii) $\frac{\partial^2\psi}{\partial\varphi^2} < 0$ at $\varphi = \varphi_m$

The double-layer solution s obtained from Eq. (34) is

$$\varphi_d = \frac{1}{2}\rho(g)\varphi_m(1 - \tanh\theta) \tag{38}$$

where $\theta = (-G/24)^{1/2}\varphi_m(\eta - \eta_0)$, $\varphi_m = -(2g/G)$, $g = 2Q$ and $G = -6R$

For compressive double layers, $\rho(g) = +1$ for $g > 0$. When $\rho(g) = -1$ for $g < 0$, the rarefactive double-layer are excited.

From Eq. (38) it is observed that positron density, positron temperature, ion temperature and ion drift velocity have significant roles on the excitation of ion acoustic double layers in e–i–p plasma. In Fig. 5, the profiles of double-layers in e–i–p plasma having $\sigma_i = 0.2$, $\beta = 0.5$, $V = 1.6$, $v_{i0} = 1.5$, $p_{i0} = 1$ and $\chi = 0.2, -0.45$ are shown. It is clear from the figure that seen rarefactive double-layers are excited in an e–i–p plasma when χ is 0.2, 0.3 and 0.35. But, the compressive double-layers are excited in such plasma when χ is 0.45.

A similar study for different drift velocity of the ions in e–i–p plasma having $\sigma_i = 0.003$, $\chi = 0.4$, $\sigma_i = 0.04$, $\beta = 5$, $V = 1.6$ and $p_{i0} = 1$, shows that compressive double layers are formed when v_{i0} is 0.0 and 0.3. But, the double-layers are rarefactive in such plasma when v_{i0} is 0.6 and 0.9.

Also we have found that for an e–i–p plasma having $\sigma_i = 0.04$, $\chi = 0.5$, $V = 1.6$, $v_{i0} = 0.3$ and $p_{i0} = 1$, the double layers are compressive when β is 0.5 and 2.5 and double-layers are rarefactive when β is 4.5 and 6.5.

In an e–i–p plasma consisting of non-drifting cold ions, only compressive solitons are observed by Popel et al. [28].

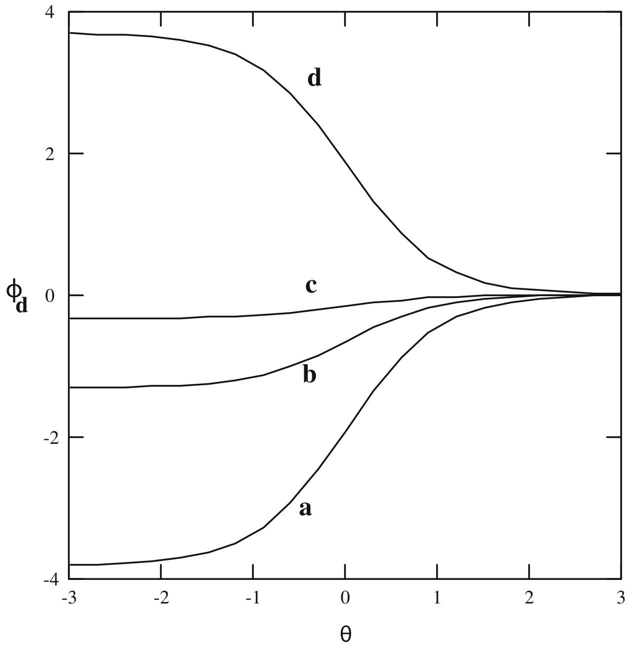

Fig. 5 The profiles of double-layers for different values of positron density (χ): The lines showed by (*a*), (*b*), (*c*) and (*d*) represent for χ = 0.2, 0.3, 0.35 and 0.45 respectively; when $\beta = 0.5$, $\sigma_i = 0.2$, $V = 1.6$ and $v_{i0} = 1.5$ and $p_{i0} = 1$

But, considering cold and hot electrons and positrons: Pillay and Bharuthram [27] have obtained both compressive and rarefactive solitary waves and double layers in plasma. However, these authors [27, 28] have not considered the drift motion of the ions. Moreover, above authors have not critically studied the effects of positron density, positron temperature and drift velocity of ions on solitary waves and double layers in e–i–p plasma. In the present paper, we consider the drifting ions in e–i–p plasma and have shown that Sagdeev potential is considerably influenced by the ion drift velocity and other plasma parameters. It is also seen that ion-acoustic solitary waves and double layers are compressive as well as rarefactive in nature. It is known that double layers are formed due to net potential difference of electrostatic fields and any particle traveling through the region of double layers is directly associated by this potential differences. So, double-layers act as the possible source of acceleration of charged particles in active galactic nuclei [21], pulsar magnetosphere [22] and neutron stars [23] where positrons exists.

However, in our present investigation, Landau damping is neglected for the study of ion acoustic solitary waves double layers in e–i–p plasma. It is expected that some more interesting results will be obtained for the ion-acoustic solitary waves and double layers, if the effect of Landau damping in the plasma is taken into account. It is to be noted that previous workers mainly have used the reductive perturbation method and the multiple scale method for their studies of solitons and double-layers in plasma. We think, it is difficult to study these nonlinear phenomena considering Landau damping in plasma using the Sagdeev potential method. Probably, for this reason, previous workers have preferred perturbation method or multiple scale method instead of Sagdeev potential method for their studies of these nonlinear phenomena in Landau damped plasma.

References

[1] H Washimi and T Taniuti *Phys. Rev. Lett*. **17** 996 (1966)
[2] K K Ghosh, B Paul, C Das and S N Paul *J. Phys. A Math. Theor*. **41** 335501 (2008)
[3] B Ghosh, S Chandra and S N Paul *Phys. Plasmas* **18** 012106 (2011)
[4] S Chattopadhyaya, S Bhattacharyya and S N Paul *Indian J. Phys*. **76B** 53 (2002)
[5] S K Bhattacharya, S N Paul and B Chakraborty *Indian J. Phys*. **77B** 327 (2003)
[6] S R Majumdar, A Roychowdhury and S N Paul *Physica Scripta* **69** 355 (2004)
[7] S Chattopadhyay, S N Paul and D Ray *FIZIKA* (Zagreb) **A18** 89 (2009)
[8] S N Paul, K K Mondal and A Roychowdhury *Phys. Lett.* **A257** 165 (1999)
[9] K K Mondal, A Roychowdhury and S N Paul *Phys. Rev.E* **65** 16404 (2002).
[10] K Roy and P Chatterjee *Indian J. Phys*. **85** 1653 (2011)
[11] P Mottaghizadeh and P Eslami *Indian J. Phys*. **86** 71 (2012)
[12] Z Emami and H R Pakzad *Indian J. Phys*. **85** 1643 (2011)
[13] H R Pakzad *Indian J.Phys*. **84** 867 (2010)
[14] N L Satalishvili, J L Javakeshvili and H Kaya *Astrophys. Space Sci*. **250** 109 (1997)
[15] P K Shukla and L Stenflo *Astrophys. Space Sci*. **209** 323 (1993)
[16] S Mahmood, A Mustaq and H Saleem *New J. Phys*. **5** 28(2003)
[17] H Saleem *Phys. Plasmas* **13** 034503 (2006)
[18] W Misner, K S Thorne and J A Wheeler *Gravitation,* (San Francisco: Freeman) (1980)
[19] M J Rees, G W Gibbons, S W Hawkins and S Siklas (eds.) *The Very Earlier Universe* (Cambridge: Cambridge University Press) (1983)
[20] Y B Zeldovich and I Novikov *Relativistic Astrophysics* (Chicago: Universitry of Chicago) (1983)
[21] H R Millar and P J Wilte *Active Galactic Nuclei* (Berlin: Springer) 202 (1987)
[22] P A Sturrock and K B Baker *Astrophys. J*. **234** 612 (1979)
[23] F C Michel *Theory of Neutron Star Magneto-sphere* (Chicago: Chicago University Press) (1991)
[24] F B Rozzato *J. Plasma* Phys. **40** 289 (1988)
[25] P K Shukla *Astrophys. Space Sci*. **114** 381 (1985)
[26] L Stenflo, P K Shukla and M Y Yu *Astrophys. Space Sci*. **117** 303 (1985)
[27] R Pillay and R Bharuthram *Astrophys. Space Sci*. **188** 592 (1992)
[28] S I Popel, S V Vladimirov and P K Shukla *Phys.Plasmas* **2** 716 (1995)
[29] S Burman and A Roychowdury *Fizika* **A8** 83 (1999)
[30] M Salahudin, H Saleem and M Siddiq *Phys. Rev*. E **66** 03407 (2002)

[31] P K Shukla, J T Mendonca and R Bingham *Phys.Scr*. **T113** 133 (2004)
[32] H Alinejad, S Sovanian and J Mahmoodi *Phys. Plasmas* **13** 012304 (2006)
[33] S Ghosh and R Bharuthram *Astrophys. Space Sci*. **189** 121 (2008)
[34] W Massod,N Jehnab,A M Mirza and P H Sakanaka *Phys. Lett*. **A372** 4279 (2008)
[35] N Nimje, S Dubey and S Ghosh *Indian J. Phys*. **84** 1567 (2010)
[36] G C Das and S N Paul *Phys. Fluids* **28** 823 (1985)
[37] L D Landau *J. Phys*. (USSR) **10** 25 (1946)
[38] I Alexeff, W D Jones and D Montgomery *Phys. Rev. Lett*. **19** 422 (1967)
[39] S A Shah and H Saleem *Phys. Plasmas* **16** 022111 (2009)
[40] S N Paul, C Das, B Paul, S K. Bhattacharya and B. Ghosh *Bulg. J. Phys*. **37** 16 (2010)
[41] F Ott and R N Sudan *Phys. Fluids* **13** 1432 (1970)
[42] A Roychowdhury, G Pakira and S N Paul *Physica* **C151** 518 (1988)
[43] A Bandyopadhyay and K P Das *Phys. Plasmas* **9** 465 (2002)
[44] S Ghosh and R Bharuthram *Astrophys. Space Sci*. doi: 10.1007/s10519-0-10-04336 (2010)
[45] H R Pakzad *Phys. Lett*. **A373** 847 (2009)
[46] H R Pakzad *Astrophys. Space Sci*. **331** 175 (2010)
[47] H K Anderson, N D'Angelo, P Michelson and P Nelson *Phys. Rev. Lett*. **19** 149 (1967)
[48] P Chatterjee and R Roychowdhury *Phys.Plasmas* **2** 1352 (1995)
[49] R Z Sagdeev *Rev. Plasma Phys.* (ed.) M A Leontovich (New York: Conslutant Bureau) **4** 23 (1966)
[50] S G Tagare *Phys. Plasmas* **15** 1247 (1973)

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ORIGINAL PAPER

Nonlinear propagation of ion-acoustic waves in self-gravitating dusty plasma consisting of non-isothermal two-temperature electrons

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Abstract: Nonlinear propagation of ion-acoustic waves in self-gravitating multicomponent dusty plasma consisting of positive ions, non-isothermal two-temperature electrons and negatively charged dust particles with fluctuating charges and drifting ions has been studied using the reductive perturbation method. It has been shown that nonlinear propagation of ion-acoustic waves in gravitating dusty plasma is described by an uncoupled third order partial differential equation which is a modified form of Korteweg–deVries equation, in contraries to the coupled nonlinear equations obtained by earlier authors. Quasi-soliton solution for the ion-acoustic solitary wave has been obtained from this uncoupled nonlinear equation. Effects of non-isothermal two-temperature electrons, gravity, dust charge fluctuation and drift motion of ions on the ion-acoustic solitary waves have been discussed.

Keywords: Ion-acoustic waves; Gravitating plasma; Non-isothermal two-temperature electrons; Modified Korteweg–deVries equation; Quasi-soliton

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1. Introduction

It is known that dusty plasmas exist in various environments like cometary tails, planetary rings, asteroids, magnetosphere, lower ionosphere, interstellar and circumstellar clouds, laboratory devices etc. [1–5]. In dusty plasma, the dust particles get negatively charged due to attachment of the background electrons and ions on the surface via collisions [6]. The charge on a dust particle does not remain fixed but depends on plasma properties, electron and ion currents flowing into or out of dust grains, photoemission etc. In dusty plasma, the existence of dust-acoustic wave (DAW) is first predicted theoretically by Rao et al. [7] with the dust grains providing the inertia and the pressure of inertialess electrons and ions providing the restoring force. Subsequently, propagation of dust-acoustic and ion-acoustic waves have been studied by various authors and have obtained interesting results [8–12]. In experiments, propagation of waves in dusty plasma have been studied by Nakamura and Sarma [13] and others [14]. However, most interesting features in dusty plasma are the solitary waves and the shock waves and a lot of works have been reported in last few years considering different parameters e.g. two-temp electrons [15], non-isothermal electrons and ions [16], nonthermal electrons and ions [17, 18], super thermal electrons [19], magnetic field [20, 21], variable dust charge [22, 23], inhomogeneity [24], finite geometry [25, 26] etc. for the applications in various fields. It is to be noted that when the dust grains are of considerable size the gravitational effects of dust grains become significant and important which modify the linear and nonlinear propagation of waves in dusty plasma though the effect is certainly negligible for the electrons and ions. Misra et al. [27] and others [28–30] have theoretically investigated the solitary waves in gravitating dusty plasma from which it is seen that gravitational effect is important on the excitation of solitary waves.

However, in earlier works [6], charge fluctuations on dust have been treated in an ad hoc fashion. In the

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continuity equations of electrons and ions, sink terms have not been considered. As a consequence the charge conservation equation is not consistent with the charge variation equation. The dynamical equation for charge fluctuation become consistent with the charge continuity equation if the sink terms in continuity equation is considered for the loss of electrons and ions due to attachment to the dust particles. Paul et al. [31] have considered the sink term in the continuity equations only for ions and have derived an uncoupled third order partial differential equation in contraries to the coupled nonlinear equations obtained by earlier authors [32]. From the modified KdV equation the solution of ion acoustic solitary waves have been obtained and the effects of isothermal two-temperature electrons, gravity and dust charge fluctuations etc. on the solitary waves in self-gravitating dusty plasma have been discussed.

But, the effects of two-temperature non-isothermal electrons [33] on the ion acoustic solitary waves in gravitating dusty plasma with the fluctuating dust-charges have not yet been studied by any author. So, in this paper, we have studied the existence and characteristics of ion-acoustic solitary waves in self-gravitating dusty plasma consisting of warm positive ions, non-isothermal two-temperature electrons and dust particles with dust charge variation. Using the reductive perturbation method we have derived an uncoupled third order partial differential equation which gives the quasi-soliton solution of ion acoustic wave in gravitating dusty plasma.

2. Basic equations

We consider an unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and non-isothermal two-temperature electrons. The dust particles are mainly charged by the attachment of the ions and electrons to the dust grains. Electrons are initially attached to the dust particles at a faster rate than the ions because the electrons have lighter mass and faster motion than the ions. As a result the dust particles get negatively charged. Once the dust particles have enough negative charge the chance of an electron being attached to a negatively charged dust grain becomes much smaller than the chance of an ion being attached to the dust grain. So we may assume that the charge fluctuation of dust grains is due to the attachment of positive ions only. Further, we consider that the self gravitating effect in dusty plasma influences both the linear and nonlinear modes of propagating ion acoustic waves. Under these assumptions, the set of normalized basic equations governing the plasma dynamics are [31]:

For ions:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = -\beta_i n_i \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} - \beta_i (v_i - v_d) \tag{2}$$

$$\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial v_i}{\partial x} = 0 \tag{3}$$

For dust particles:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \tag{4}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} + \frac{\sigma_d}{\mu_d n_d}\frac{\partial p_d}{\partial x} = -\frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} - \beta_i (v_d - v_i) \tag{5}$$

$$\frac{\partial p_d}{\partial t} + v_d \frac{\partial p_d}{\partial x} + 3p_d \frac{\partial v_d}{\partial x} = 0 \tag{6}$$

Poisson's equations:

$$\frac{\partial^2 \phi}{\partial x^2} = n_{ec} + n_{eh} - n_i + Z_d n_d \tag{7}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\omega_{gi}^2}{m_i \omega_i^2} \sum_{s=e,i,d} m_s n_s = X_i \sum_{s=i,d} m_s n_s \tag{8}$$

At equilibrium the charge neutrality condition is

$$n_i^{(0)} = n_e^{(0)} + Z_d n_d^{(0)} = n_{ec}^{(0)} + n_{eh}^{(0)} + Z_d n_d^{(0)} \tag{9}$$

the subscript $s = i$ represents for ions, $s = d$ for dust particles; m_s, n_s, v_s, q_s and p_s denote respectively the mass, number density, velocity, charge and scalar pressure of the sth species of the plasma: n_{ec}, n_{eh} are number density of electrons at low and high temperatures, Z_d is the number of electronic charge attached on the grains; $\mu_d = m_d/m_i$, $\sigma_i = T_i/T_{ef}$, $\sigma_d = T_d/T_{ef}$, $T_{ef} = T_{ec}T_{eh}/(\mu T_{eh} + \nu T_{ec})$, $X_i = \omega_{gi}^2/m_i\omega_i^2$, $\omega_{gi}^2 = 4\pi G m_i n_i$; $n_{i0}, n_{ec0}(n_{eh0})$ and n_{d0} are respectively the equilibrium number densities of ions, electrons and dust particles; G is the universal gravitational constant: φ and ψ are respectively the electrostatic and gravitational potentials; T_{ec} and T_{eh} are the low- and high-temperatures of electrons; μ and ν equilibrium values of electrons at low and high temperatures; other symbols have their usual meanings.

The parameter β_i is the attachment co-efficient (frequency) of ions to the dust particles. The right hand side of Eq. (1) is the sink term which describes the loss in the ion density due to attachment of ions to the dust particles. The attachment coefficient of ions may vary due to the change in ion current falling on the dust grains, and also due to the change in the densities of ions and dust grains. Since, β_i has the collision like behavior, the last terms in the right hand side of Eqs. (2) and (4) are also be like collisional effect between ions and dust particles [31].

In the above equations we have normalized distance x by Debye length $\lambda_D = (\boldsymbol{k_B T_e}/4\pi \boldsymbol{e}^2 \boldsymbol{n}_{i0})^{1/2}$, time t by the inverse of ion plasma frequency $\omega_i^{-1} = (\boldsymbol{m_i}/4\pi \boldsymbol{e}^2 \boldsymbol{n}_{i0})^{1/2}$, all pressures by ion plasma pressure $n_i^{(0)}\, k_B T_i$, φ by $k_B T_e/e$, ψ by $k_B T_e/m_i$, and all number densities by n_{i0}. $\sigma_i = T_i/T_{ef}$.

For the nonisothermal electrons at two different temperatures the number density of electrons is given by [33]

$$n_{ec} = \mu\left[1 + \frac{\varphi}{\mu+\nu\beta} + \frac{4(1-\beta_l)}{3\pi^{1/2}}\left(\frac{\varphi}{\mu+\nu\beta}\right)^{3/2} + \frac{1}{2}\left(\frac{\varphi}{\mu+\nu\beta}\right)^2 - \frac{8(1-\beta_l^2)}{15\pi^{1/2}}\left(\frac{\varphi}{\mu+\nu\beta}\right)^{5/2} + \frac{1}{6}\left(\frac{\varphi}{\mu+\nu\beta}\right)^3 - \cdots\right] \tag{10}$$

$$n_{eh} = \nu\left[1 + \frac{\beta\varphi}{\mu+\nu\beta} + \frac{4(1-\beta_h)}{3\pi^{1/2}}\left(\frac{\beta\varphi}{\mu+\nu\beta}\right)^{3/2} + \frac{1}{2}\left(\frac{\beta\varphi}{\mu+\nu\beta}\right)^2 - \frac{8(1-\beta_h^2)}{15\pi^{1/2}}\left(\frac{\beta\varphi}{\mu+\nu\beta}\right)^{5/2} + \frac{1}{6}\left(\frac{\beta\varphi}{\mu+\nu\beta}\right)^3 - \cdots\right] \tag{11}$$

where

$$\beta = T_{ec}/T_{eh},\quad \beta_l = T_{el}/T_{elt},\quad \beta_h = T_{eh}/T_{eht},\quad \mu = n_{ec0},\quad \nu = n_{eh0},\quad b_l = \frac{(1-\beta_l)}{\pi^{1/2}},\quad b_h = \frac{(1-\beta_h)}{\pi^{1/2}},$$
$$b_l^{(1)} = \frac{(1-\beta_l^2)}{\pi^{1/2}},\quad b_h^{(1)} = \frac{(1-\beta_h^2)}{\pi^{1/2}},\quad \mu = n_{ec}^{(0)},\quad \nu = n_{eh}^{(0)}.$$

Therefore,

$$n_e = n_{ec} + n_{eh} = 1 + \varphi - \frac{4}{3}\left(\frac{\mu b_l + \nu b_h \beta}{\mu+\nu\beta}\right)^{3/2}\varphi^{3/2} + \frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\varphi^2 - \frac{8}{15}\frac{(\mu b_l^{(1)} + \nu b_h^{(1)}\beta^{5/2})}{(\mu+\nu\beta)^{5/2}}\varphi^{5/2} + \cdots \tag{12}$$

in which T_{ef} is the temperature for the free electrons and T_{et} is the temperature for the trapped electrons.

3. Modified KdV equation

For the derivation of the nonlinear equation i.e. K–dV equation governing the nonlinear dynamics of the wave, we make the usual stretching of the space co-ordinates and time,

$$\xi = \varepsilon^{1/4}(x - Vt) \quad \text{and} \quad \tau = \varepsilon^{3/4} t \tag{13}$$

where, V is the linear phase velocity, ε is a dimensionless expansion parameter which is very small $\varepsilon \ll 1$ measuring the dispersion and nonlinear effects. Further, we assume the following perturbation expansion for the field variables:

$$X = X_0 + \varepsilon X_1 + \varepsilon^{3/2} X_2 + \varepsilon^2 X_3 + \cdots,\quad \beta_i = \varepsilon^{3/4}\overline{\beta_i} \tag{14}$$

3.1. Phase velocity of ion-acoustic wave

Using Eqs. (13) and (14) in Eqs. (1)–(8) and equating the coefficients of the lowest order of ε we obtain

$$A_1\psi_1 = B_1\phi_1 \tag{15a}$$

$$A_2\psi_1 = B_2\phi_1 \tag{15b}$$

where

$$A_1 = \frac{n_{i0}}{\lambda_1^2} - \frac{Z_d n_{d0}}{\lambda_2^2},\quad A_2 = \frac{w_{gi}^2 a_2}{\lambda_1^2} - \frac{w_{gd}^2 Z_d}{\lambda_2^2},$$
$$B_1 = 1 - \frac{a_2 n_{i0}}{\lambda_1^2} - \frac{Z_d b_2 n_{d0}}{\lambda_2^2},$$
$$B_2 = -\left(\frac{w_{gi}^2}{\lambda_1^2} + \frac{w_{gd}^2 b_2}{\lambda_2^2}\right),\quad \lambda_1 = V_1^2 - 3a_1 n_{i0},$$
$$\lambda_2 = V_2^2 - 3b_1 n_{d0},\quad V_1 = V - v_{i0},$$
$$V_2 = V - v_{d0},\quad w_{gi}^2 = 4\pi G m_i n_{i0},\quad w_{gd}^2 = 4\pi G m_d n_{d0}$$
$$a_1 = \sigma_i/m_i,\quad b_1 = q_i/m_i,\quad a_2 = \sigma_d/m_d,\quad b_2 = -Z_d \tag{16}$$

and the mass of electron m_e is neglected.

From Eqs. (15a) and (15b) we obtain a relation between ψ_1 and φ_1 given by,

$$\psi_1 = \pm\left(\frac{B_1B_2}{A_1A_2}\right)^{1/2}\varphi_1 = \pm C_1\varphi_1 \tag{17a}$$

or,

$$\varphi_1 = \pm\left(\frac{A_1A_2}{B_1B_2}\right)^{1/2}\psi_1 = \pm C_2\psi_1 \tag{17b}$$

where $C_1 = \left(\frac{B_1B_2}{A_1A_2}\right)^{1/2}$, $C_2 = \left(\frac{A_1A_2}{B_1B_2}\right)^{1/2}$

From Eqs. (15a) and (15b) the linear dispersion relation for the ion-acoustic wave in self-gravitating dusty plasma is derived as

$$A_1B_2 - A_2B_1 = 0 \tag{18}$$

Simplifying Eq. (18) we obtain the linear dispersion relation,

$$c_4V^4 + c_3V^3 + c_2V^2 + c_1V + c_0 = 0 \tag{19}$$

where

$$\begin{aligned}
c_4 &= 2\\
c_3 &= -4(v_{i0} + v_{d0})\\
c_2 &= 2\left[2\left(v_{i0}^2 + v_{d0}^2\right) - 3\left(b_1\omega_{gi}^2 + a_1\omega_{gd}^2\right)\right]\\
c_1 &= 4\left[\left(v_{i0}^3 + v_{d0}^3\right) + 3\left(a_1v_{i0}^2\omega_{gd}^2 + b_1v_{d0}^2\omega_{gi}^2\right)\right]\\
c_0 &= \left(v_{i0}^2\omega_{gd}^2 + v_{d0}^2\omega_{gi}^2\right) - 6\left(a_1v_{i0}^2\omega_{gd}^2 + b_1v_{d0}^2\omega_{gi}^2\right)\\
&\quad + 9\left(a_1^2\omega_{gd}^2 + b_1^2\omega_{gi}^2\right) - (1-b_2)\left(n_{i0}\omega_{gd}^2 + Z_dn_{d0}\omega_{gi}^2\right)
\end{aligned} \tag{20}$$

Solving Eq. (19) we obtain four values of V given by

$$\begin{aligned}
V &= \frac{1}{2}[(c_3 + d_1) + \sqrt{(d_1 + c_3) - 4(d_2 + d_3)}\\
V &= \frac{1}{2}[(c_3 + d_1) - \sqrt{(d_1 + c_3) - 4(d_2 + d_3)}\\
V &= \frac{1}{2}[(c_3 - d_1) + \sqrt{(d_1 - c_3) - 4(d_2 - d_3)}\\
V &= \frac{1}{2}[(c_3 - d_1) - \sqrt{(d_1 - c_3) - 4(d_2 - d_3)}
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
d_1^2 &= c_3^2 - c_2 + 2d_2\\
d_1d_3 &= c_1 + c_3d_2\\
d_3^2 &= d_2^2 - c_0\\
d_2 &= \left[\frac{1}{2}\left(d_4 + \sqrt{(d_4^2 - d_5)}\right]^{1/3} + \left[\frac{1}{2}\left(d_4 - \sqrt{(d_4^2 - d_5)}\right]^{1/3} + \frac{1}{6}c_2\\
d_4 &= \frac{1}{6}c_2(c_0 + c_1c_3) + \frac{1}{108}c_2^3 - \frac{1}{2}\left(c_0c_2 - c_0c_3^2 - c_1^2\right)\\
d_5 &= \frac{4}{729}\left[3(c_0 + c_1c_3) + \frac{1}{4}c_2^2\right]^2
\end{aligned}$$

For the dusty plasma having cold ions and cold dust particles (i.e. $\sigma_i = 0$, $\sigma_d = 0$) and for equal stream velocity of ions and dust particles (i.e. $v_{i0} = v_{d0} = v_0$), the phase velocity Eq. (19) of linear propagating wave becomes

$$V = \pm\left[v_0 \pm \left\{(1-b_2)\left(n_{i0}\omega_{gd}^2 + Z_dn_{d0}\omega_{gi}^2\right)\Big/\left(\omega_{gi}^2 + \omega_{gd}^2\right)\right\}^{1/4}\right] \tag{22}$$

The negative value of V in Eq. (22) is neglected. Since b_2 is negative as shown in Eq. (16), there are only two real and positive values of V in presence of stream velocity, one is fast-mode and the other is slow-mode. The fast- and slow-mode exist when $v_{i0\neq0}$. It is to be noted that the two-temperature electrons and its non-isothermality have no influence on the phase velocity V of linear ion-acoustic wave.

3.2. Derivation of modified K–dV equation

Now, to derive the K–dV equation we use Eqs. (13) and (14) in Eqs. (1)–(8) and then equating the coefficients of $\varepsilon^{3/2}$ we obtain

$$\frac{\partial n_{i1}}{\partial\tau} + \bar{\beta}_in_{i1} = (V - v_{i0})\frac{\partial n_{i2}}{\partial\xi} - n_{i0}\frac{\partial v_{i2}}{\partial\xi} \tag{23a}$$

$$\begin{aligned}\frac{\partial v_{i1}}{\partial\tau} + \bar{\beta}_i(v_{i1} - v_{d1}) &= (V - v_{i0})\frac{\partial v_{i2}}{\partial\xi} - a_1n_{i0}\frac{\partial p_{i2}}{\partial\xi}\\ &\quad - a_2\frac{\partial\phi_2}{\partial\xi} - \frac{\partial\psi_2}{\partial\xi}\end{aligned} \tag{23b}$$

$$\frac{\partial p_{i1}}{\partial\tau} = (V - v_{i0})\frac{\partial p_{i2}}{\partial\xi} - 3\frac{\partial v_{i2}}{\partial\xi} \tag{23c}$$

$$\frac{\partial n_{d1}}{\partial\tau} = (V - v_{d0})\frac{\partial n_{d2}}{\partial\xi} - n_{d0}\frac{\partial v_{d2}}{\partial\xi} \tag{24a}$$

$$\begin{aligned}\frac{\partial v_{d1}}{\partial\tau} + \bar{\beta}_i(v_{d1} - v_{i1}) &= (V - v_{d0})\frac{\partial v_{d2}}{\partial\xi} - b_1n_{d0}\frac{\partial p_{d2}}{\partial\xi}\\ &\quad - b_2\frac{\partial\phi_2}{\partial\xi} - \frac{\partial\psi_2}{\partial\xi}\end{aligned} \tag{24b}$$

$$\frac{\partial p_{d1}}{\partial\tau} = (V - v_{i0})\frac{\partial p_{d2}}{\partial\xi} - 3\frac{\partial v_{d2}}{\partial\xi} \tag{24c}$$

$$\frac{\partial^2\varphi_1}{\partial\xi^2} = \varphi_2 - \frac{4}{3}\frac{(\mu b_l + \nu\beta b_h)^{3/2}}{(\mu + \nu\beta)^{3/2}}\varphi^{3/2} - n_{i2} + Z_dn_{d2} \tag{25}$$

and

$$\frac{\partial^2\psi_1}{\partial\xi^2} = 4\pi G(m_in_{i2} + m_dn_{d2}) \tag{26}$$

To derive the K–dV equation from Eqs. (23)–(26) we first eliminate $v_{i2}, v_{d2}, p_{i2}, p_{d2}$ after using the values of $n_{i1}, v_{i1}, p_{i1}, n_{d1}, v_{d1}, p_{d1}$ and find $\partial n_{i2}/\partial\xi$ and $\partial n_{d2}/\partial\xi$. Finally from Eq. (25) we obtain the nonlinear equations

$$\begin{aligned}
&\frac{\partial^3\varphi_1}{\partial\xi^3} + \frac{2(\mu b_l + \nu\beta b_h)^{3/2}}{(\mu + \nu\beta)^{3/2}}\varphi_1^{1/2}\frac{\partial\varphi_1}{\partial\xi}\\
&\quad + \left[\frac{n_{i0}}{V_1\lambda_1}\left(\frac{V_1^2}{\lambda_1} + \frac{3a_1n_{i0}}{\lambda_1} + 1\right)\left(a_2\frac{\partial\varphi_1}{\partial\tau} + \frac{\partial\psi_1}{\partial\tau}\right)\right]\\
&\quad + \left[\frac{Z_dn_{d0}}{V_2\lambda_2}\left(\frac{V_2^2}{\lambda_2} + \frac{3b_1n_{d0}}{\lambda_2} + 1\right)\left(b_2\frac{\partial\varphi_1}{\partial\tau} + \frac{\partial\psi_1}{\partial\tau}\right)\right]\\
&\quad + \left[\frac{\overline{\beta}_in_{i0}}{\lambda_1}\left(a_2V_1 + \frac{Z_dV_2}{\lambda_2} + \frac{a_2}{V_1}\right)\right]\varphi_1 + \frac{Z_d^2n_{d0}V_1\overline{\beta}_i}{\lambda_2^2}\varphi_1\\
&\quad - \frac{Z_dn_{d0}V_1\overline{\beta}_i}{\lambda_2^2}\psi_1 - \frac{\overline{\beta}_in_{i0}}{\lambda_1}\left(\frac{V_2}{\lambda_2} + \frac{V_1}{\lambda_1} + \frac{V_2}{\lambda_1}\right)\Bigg]\psi_1\\
&\quad - \frac{Z_dn_{d0}V_1\overline{\beta}_i}{\lambda_1\lambda_2}(a_2\varphi_1 + \psi_1)\\
&= \left[1 - \frac{a_2n_{i0}}{\lambda_1^2} - \frac{Z_db_2n_{d0}}{\lambda_2^2}\right]\frac{\partial\varphi_2}{\partial\xi} - \left[\frac{n_{i0}}{\lambda_1^2} - \frac{Z_dn_{d0}}{\lambda_2^2}\right]\frac{\partial\psi_2}{\partial\xi}.
\end{aligned}$$

$$\frac{\partial^3\varphi_1}{\partial\xi^3}+\left[\frac{2(\mu b_l+\nu\beta b_h)^{3/2}}{(\mu+\nu\beta)^{3/2}}\right]\varphi_1^{1/2}\frac{\partial\varphi_1}{\partial\xi}$$
$$+\left[\frac{a_2 n_{i0}}{V_1\lambda_1}\left(\frac{V_1^2}{\lambda_1}+\frac{3a_1 n_{i0}}{\lambda_1}+1\right)+\frac{b_2 Z_d n_{d0}}{V_2\lambda_2}\left(\frac{V_2^2}{\lambda_2}+\frac{3b_1 n_{d0}}{\lambda_2}+1\right)\right]\frac{\partial\varphi_1}{\partial\tau}$$
$$+\left[\frac{n_{i0}}{V_1\lambda_1}\left(\frac{V_1^2}{\lambda_1}+\frac{3a_1 n_{i0}}{\lambda_1}+1\right)+\frac{Z_d n_{d0}}{V_2\lambda_2}\left(\frac{V_2^2}{\lambda_2}+\frac{3b_1 n_{d0}}{\lambda_2}+1\right)\right]\frac{\partial\psi_1}{\partial\tau}$$
$$+\left[\frac{\overline{\beta}_i n_{i0}}{\lambda_1}\left(a_2 V_1+\frac{Z_d V_2}{\lambda_2}+\frac{a_2}{V_1}\right)+\frac{Z_d^2 n_{d0} V_1\overline{\beta}_i}{\lambda_2^2}-\frac{a_2 Z_d n_{d0} V_1\overline{\beta}_i}{\lambda_1\lambda_2}\right]\varphi_1$$
$$-\left[\frac{Z_d n_{d0} V_1\overline{\beta}_i}{\lambda_2^2}+\frac{\overline{\beta}_i n_{i0}}{\lambda_1}\left(\frac{V_2}{\lambda_2}+\frac{V_1}{\lambda_1}+\frac{V_2}{\lambda_1}\right)+\frac{Z_d n_{d0} V_1\overline{\beta}_i}{\lambda_1\lambda_2}\right]$$
$$\psi_1=\left[1-\frac{a_2 n_{i0}}{\lambda_1^2}-\frac{Z_d b_2 n_{d0}}{\lambda_2^2}\right]\frac{\partial\varphi_2}{\partial\xi}-\left[\frac{n_{i0}}{\lambda_1^2}-\frac{Z_d n_{d0}}{\lambda_2^2}\right]\frac{\partial\psi_2}{\partial\xi}$$

Following usual procedure we have derived the following modified KdV equation from Eqs. (23)–(26) for the electrostatic field in self-gravitating dusty plasma consisting non-isothermal two-temperature electrons:

$$P\frac{\partial\phi_1}{\partial\tau}+Q\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+R\frac{\partial^3\phi_1}{\partial\xi^3}+S\phi_1=0 \tag{27a}$$

Eq. (27a) can also be written as

$$\frac{\partial\phi_1}{\partial\tau}+\overline{Q}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+\frac{1}{2}\overline{R}\frac{\partial^3\phi_1}{\partial\xi^3}+\overline{S}\phi_1=0 \tag{27b}$$

where

$$\overline{Q}=\frac{Q}{P},\quad \overline{R}=\frac{2R}{P},\quad \overline{S}=\frac{S}{P},$$
$$P=\left[\frac{n_{i0}}{V_1\lambda_1}(a_2+C_1)\left(\frac{V_1^2}{\lambda_1}+\frac{3a_1 n_{i0}}{\lambda_1}+1\right)+\frac{Z_d n_{d0}}{V_2\lambda_2}(Z_d+C_1)\left(\frac{V_2^2}{\lambda_2}+\frac{3b_1 n_{d0}}{\lambda_2}+1\right)\right], \tag{28a}$$

$$Q=\frac{2(\mu b_l+\nu\beta b_h)^{3/2}}{(\mu+\nu\beta)^{3/2}} \tag{28b}$$

$$R=1 \tag{28c}$$

$$S=\frac{\overline{\beta}_i n_{i0}}{\lambda_1}\left(a_2 V_1+\frac{Z_d V_2}{\lambda_2}+\frac{a_2}{V_1}\right)+\frac{Z_d^2 n_{d0} V_1\overline{\beta}_i}{\lambda_2^2}-\frac{Z_d n_{d0} V_1\overline{\beta}_i}{\lambda_2^2}C_1 \tag{28d}$$

It is known that the solitary wave is formed due to balance between dispersive and nonlinear effects in the K–dV equation. Relative strength of these two effects determines the characteristic of such solitary wave structure. It is seen that the dispersive term $\overline{R}$ of the K–dV Eq. (27b) depends on the parameters of ions and dust particles in the gravitating plasma (i.e. $n_{i0}, n_{d0}, \mu_d, Z_d, \sigma_i, \sigma_d, \omega_{gi}, \omega_{gd}$) whereas the nonlinear term $\overline{Q}$ depends on these parameters and also on the parameters of non-isothermal electrons (i.e. $\mu, \nu, \beta, b_l, b_h$). Since the coefficients $\overline{R}$ and $\overline{Q}$ play crucial roles on the excitation of solitary waves, it is important to find the dependence of these coefficients on the amplitude, width and velocity of the solitary wave in the gravitating plasma. If $\bar{Q}\approx 0$ i.e. when the nonlinear term in Eq. (27b) vanishes at some critical values of (1) dust density, or (2) gravitational effects for ions and dust particles, or (3) density (or temperature) of two-temperature electrons etc., the ion-acoustic solitary waves will not exist in the dusty plasma. Similarly, for $\overline{R}\approx 0$, the dispersive term in Eq. (27b), there will no excitation of the ion-acoustic solitary waves in dusty plasma. Eq. (27b) shows that there is one more term $\overline{S}\varphi_1$ in the K–dV equation which consists of the attachment coefficient β_i and also the parameters of ions and dust particles in the gravitating plasma.

It is to be mentioned that an uncoupled K–dV equation for the gravitational potential ψ can also be derived for the waves in self-gravitating dusty plasma using the method for the case of electrostatic potential. Then one can examine the possibility of having soliton-like variation of the gravitational potential in dusty plasma.

4. Solitary wave solution

Solving Eqs. (27a) and (27b) the solution for the ion-acoustic wave in a self-gravitating dusty plasma consisting of ions, charged dust particles and non-isothermal two-temperature electrons can be obtained for variable dust charges and also for constant dust charges. We first obtain the solution of ion acoustic solitary waves for dust particles in gravitating dusty plasma. Neglecting the effect of charge fluctuation on the dust grain (i.e. $\beta_i=0$ and so $\overline{S}=0$), the KdV Eq. (27b) becomes

$$\frac{\partial\phi_1}{\partial\tau}+\overline{Q}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+\frac{1}{2}\overline{R}\frac{\partial^3\phi_1}{\partial\xi^3}=0 \tag{29}$$

For having the stationary solutions of Eq. (29) we introduce a new variable $\eta=\xi-\lambda\tau$ and integrate the resulting equation using the boundary

$$\begin{aligned}&\text{(i)}\quad \varphi^{(1)}\to 0,\\ &\text{(ii)}\quad \frac{d\varphi^{(1)}}{d\eta}\to 0,\\ &\text{(iii)}\quad \frac{d^2\varphi^{(1)}}{d\eta^2}\to 0.\end{aligned} \tag{30}$$

as $\eta\to\infty$.

Thus we get the intermediate integral

$$\frac{d\varphi^{(1)}}{d\eta}=\left\{\frac{2\lambda}{\overline{R}}(\varphi^{(1)})^2\left[1-\frac{8\overline{Q}}{15\lambda}(\varphi^{(1)})\right]^{1/2}\right\}^{1/2} \tag{31}$$

and then from Eqs. (29) and (31) we obtain the solution for ion acoustic solitary waves in a self-gravitating dusty plasma given by:

$$\varphi^{(1)} = \varphi_0 Sech^4(\lambda/D) \tag{32}$$

where

$$\varphi_0 = \frac{225}{64}\frac{\lambda^2}{\overline{Q}^2} \text{ (is the amplitude)}, \tag{33a}$$

$$D = \left[\frac{8\overline{R}}{\lambda}\right]^{1/2} \text{ (is the width)}, \tag{33b}$$

and λ is the phase velocity of ion acoustic solitary wave.

In Eq. (33a), it is seen that both λ^2 and $\overline{Q}^2$ are positive for any value of the plasma parameters. So the amplitude φ_0 is positive which indicates that only compressive ion-acoustic solitary waves will be excited in the self-gravitating dusty plasma.

4.1. Quasi-soliton solution

When the variation of dust charges in the dusty plasma is taken into account (i.e. $\beta_i \neq 0$, and so $\overline{S} \neq 0$ or $S \neq 0$) the quasi-soliton solution for ion acoustic wave is obtained from Eq. (27a). To find it, we put $\tau/\boldsymbol{P} = \eta'$ and $\left(\frac{8\boldsymbol{Q}}{15}\right)^2 \phi_1 = \phi$ in Eq. (27a) which gives

$$\frac{\partial \phi}{\partial \eta'} + \frac{15}{8}\phi^{1/2}\frac{\partial \phi}{\partial \xi} + \frac{\partial^3 \phi}{\partial \xi^3} + \boldsymbol{S}\phi = 0 \tag{34}$$

The modified K–dV Eq. (33) has a quasi-soliton solution given by

$$\phi = \boldsymbol{a}(\eta')\boldsymbol{sech}^4\left[\left(\frac{\boldsymbol{a}(\eta')}{16}\right)^{1/4}(\xi - \sqrt{\boldsymbol{a}(n')}n')\right] \tag{35}$$

whose amplitude can be expressed as

$$\boldsymbol{a}(\eta') = \boldsymbol{a}(\eta'_0)\boldsymbol{exp}[-\boldsymbol{S}(\eta' - \eta'_0)] \tag{36}$$

where η'_0 corresponds to some initial value of the variable η'.

Obviously, if $\boldsymbol{S} < 0$ the amplitude of the quasi-soliton increases exponentially indicating instability. On the other hand if $\boldsymbol{S} > 0$ the wave amplitude of solitary wave decays exponentially with the increase in the independent variable η'.

5. Results and discussion

From Eqs. (32) and (34) it is seen that ion acoustic solitary waves will be excited in an unmagnetized self-gravitating dusty plasma having nonisothermal two-temperature electrons for constant dust charges and also for variable dust charges. The amplitude of the solitary waves given by $\varphi_0 = \frac{225}{64}\frac{\lambda^2}{\overline{Q}^2}$ in Eq. (33a) depends on (1) the density and temperature of two-temperature electrons, (2) the densities and temperatures of ions and dust particles, (3) the masses of ions and dust particle and (4) the charge of dust particles. But the width of the solitary wave given by $D = \left[\frac{8\overline{R}}{\lambda}\right]^{1/2}$ in Eq. (33b) depends only on the parameters of ions and dust particles. There is no effect of the two-temperature electrons and the nonisothermal parameter on the width of the solitary waves.

The plasma parameters of electrons, ions and dust particles are seen to be present in the expressions of amplitude and width of the solitary wave in a very complicated way. It is very difficult to see the effect of these plasma parameter separately on the amplitude and width of the wave. However, a close observation indicates that for the increase of temperature and nonisothermal parameter of two-temperature electrons the amplitude of the solitary wave will be decreased and the width will be increased. On the other hand, the effects of gravitational forces of ions and dust particles on the amplitude and width are seen to be more significant than other parameters in the dusty plasma. The amplitude will be drastically reduced due to the gravitational effect, but the width will be increased for the gravitational forces of mainly due to dust particles. Similarly, the variation of phase velocity of the solitary wave with two-temperature electrons and self-gravitation of dusty plasma can be understood from Eqs. (33a) and (33b) if the amplitude or width is known.

The attachment coefficients and the gravitational effects are involved in $\overline{S}$ in the K–dV Eq. (27b) and these parameters will significantly influence the soliton properties including soliton amplitude, speed, width as well as its stability. The amplitude will also be increased for the increase of the dust density and dust charges. For the dust charge variation due to attachment coefficient of ions, the amplitude of ion-acoustic solitary wave will grow or decay exponentially.

The stability of the solitary wave in a self-gravitating dusty plasma is important for understanding various nonlinear wave processes in the plasma. It is found that stability of solitary wave depends on the sign of the coefficient S of the fourth term $S\,\phi$ in the modified KdV equation. It is seen that the attachment coefficient, non-isothermal parameter of two-temperature electrons and gravitational effects are present into the factor S in a complicated way. The stability of the solitary structure will therefore be determined by the relative magnitudes of these effects.

It is also to be noted that the non-isothermality of two-temperature electrons has no effect on the linear properties of the ion-acoustic wave but has significant effects on the nonlinear properties of the wave.

In fact, numerical analysis will give clear physical implications of the gravitational potential, dust density, ion temperature, non-isothermal two-temperature electrons on the ion acoustic solitary waves in a dusty plasma. Inclusion of drift motion of plasma leads to two distinct modes of wave propagation-one having comparatively high phase speed called as 'Fast mode' and the other with lower phase speed called as 'Slow mode'.

6. Conclusions

In this paper, we have theoretically studied the nonlinear propagation of ion-acoustic waves in self-gravitating dusty plasma consisting of positive ions, non-isothermal two-temperature electrons and negatively charged dust particles with fluctuating charges and drifting ions by using the reductive perturbation method. The self-gravitation and charge variation in the dusty modify the structure of the solitary wave e.g. amplitude and width. For the presence of two-temperature electrons the amplitude of the solitary wave is decreased but the width increases. Moreover, for the variable dust charge due to attachment of ions on the dust particles the amplitude will increase or decrease exponentially. However, most important achievement in this paper is that instead of coupled-nonlinear equations (obtained by earlier authors) we have obtained an uncoupled-nonlinear equation for the study of ion acoustic solitary waves in self-gravitating dusty plasma. Our analysis is advantageous for solving the nonlinear equation for getting the solution of solitary waves.

Moreover, the model considered in this paper can be easily extended to study the effects of different types of electron distributions (e.g. nonthermal, superthermal) on the linear and nonlinear propagation of ion-acoustic waves in self-gravitating dusty plasma with dust charge fluctuations.

In our present analysis we have considered the finite temperature of dust particles. In most astrophysical situations dusts are considered as cold, but there are some astrophysical and experimental situations where the dust temperature may become comparable to ion temperature. The temperature of the dust particles is important owing to thermalization with the ions or orbital effects. In fact it has been suggested that the dust temperature in the planetary rings may have a large value compared to the electron temperature.

Recently, the quantum effect in dusty plasma has been considered by many authors for the study of linear and nonlinear propagation of waves [34–36]. So considering both the self-gravitation and quantum effect in dusty plasma, nonlinear propagation of dust-acoustic and ion-acoustic waves can be studied from which more interesting results will come out applicable in dense astrophysical plasma.

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References

[1] G K Goertz *Rev. Geophys.* **27** 271 (1989)
[2] D A Mendis and M Rosenberg *Ann. Rev. Astron. Astrophys.* **32** 419 (1994)
[3] F Verheest *Space Sci. Rev.* **77** 267 (1996)
[4] P K Shukla and A A Mamun *Introduction to Dusty Plasma Physics* (Bristol: Institute of Physics Publishing) (2002)
[5] S K Maharaj et al *Phys. Plasmas* **71** 345 (2005)
[6] M R Jana, A Sen and P K Kaw *Phys. Rev. E* **48** 3930 (1993)
[7] N N Rao, P K Shukla and M Y Yu *Planet Space Sci.* **38** 543 (1990)
[8] K Alam, A Roychowdhury and S N Paul *Int. J. Theor. Phys.* **38** 757 (1999)
[9] K K Mondal, A Roychowdury and S N Paul *Phys. Rev. E* **65** 016404 (2001)
[10] S Burman, S N Paul and A Roychowdury *Phys. Plasmas* **9** 3752 (2002)
[11] A P Misra, A Roychowdury and S N Paul *Pramana J. Phys.* **63** 579 (2004)
[12] M S A Khan, S Sultana and M Salimullah *Int. J. Sci. Res.* **2** 132 (2013)
[13] Y Nakamura and A Sarma *Phys. Plasmas* **8** 3921 (2001)
[14] H F Liu et al *Adv. Space Res.* **51** 2368 (2013)
[15] S S De, B Paul, G Pakira and S N Paul *Proceedings of 3rd International Conference on Computers and Devices for communication (CODEC-06)*, Institute of Radio Physics and Electronics, Calcutta University, Calcutta, December 18–20, p 668 (2006)
[16] O Rahman et al *Int. J. Astron. Astrophys.* **4** 119 (2014)
[17] F Verheest, M A Hellberg and W A Hereman *Phys. Rev. E* **86** 036402 (2012)
[18] E Eslami and R Baraz *AIP Adv.* **4** 027108 (2014)
[19] C R Choi, K W Min and T N Rhee *Phys. Plasmas* **18** 092901 (2011)
[20] M Shahmonsouri and H Alinezad *Phys. Plasmas* **20** 033704 (2013)
[21] A P Misra and Y Wang *Commun. Nonlinear Sci. Numer. Solut.* **22** 1360 (2015)
[22] H R Pakzad and K Javidan *Chaos Solitons Fractals* **42** 2904 (2009)
[23] S Maitra *Phys. Plasmas* **19** 03217 (2012)
[24] S A Elwakil et al *Adv. Space Res.* **48** 1067 (2014)
[25] X Yang et al *Phys. Rev. E* **87** 063101 (2013)
[26] K Annon *Astrophys. Space Sci.* doi:10.1007/s10509-013-1739-0 (2014)
[27] A P Misra, K R Chowdhury and A R Chowdhury *Phys. Scr.* **69** 44 (2005)
[28] A P Misra and A R Chowdhury *Eur. Phys. J.* **D37** 105 (2006)
[29] S Burman and A Roychowdhuty *Chaos Solitons Fractals* **13** 973 (2002)
[30] G Zhi-Rong et al *Chin. Phys.* **B19** 115203 (2010)
[31] S N Paul et al *WASET*, *Proceedings of International Conference ICFPP* held at Bali, Indonesia on 26–28 October (2011)
[32] S N Paul et al *Czech. J. Phys.* **56** 1453 (2006)
[33] S R Mazumdar, S N Paul and K P Das *J. Plasma Phys.* **64** 297 (2000)
[34] S Ali and P K Shukla *Phys. Plasmas* **13** 022313 (2006)
[35] M Shahmansouri *Pramana J. Phys.* **80** 295 (2013)
[36] B Ghosh, S N Paul, and S. Banerjee *World Acad. Sci. Eng. Technol. Int. J. Math. Comput. Phys. Quantum Eng.* **8**(6) 970 (2014)

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ION ACOUSTIC SOLITARY WAVES IN A SELF-GRAVITATING DUSTY PLASMA CONSISTING OF WEAK NONISOTHERMAL TWO-TEMPERATURE ELECTRONS

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author SNP designed the study, wrote the protocol and interpreted the data and also supervised the work. Authors AC, IP and GP anchored the field study, gathered the initial data and performed preliminary data analysis. Authors AC, IP and GP managed the literature searches and produced the initial draft consulting with author SNP. All authors read and approved the final manuscript.

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ABSTRACT

Ion-acoustic waves in a self-gravitating dusty plasma consisting of warm positive ions, weak- nonisothermal two-temperature electrons and negatively charged dust particles having charge fluctuations has been theoretically investigated using the reductive perturbation method. It has been shown that the nonlinear propagation of ion-acoustic waves in such a plasma can be described by an uncoupled third order partial differential equation which is a modified form of Korteweg-deVries (KdV) equation. From this nonlinear equation, solution of the ion acoustic solitary waves in the dusty plasma having weak nonisothermal electrons is obtained. Moreover, the solitary wave solution for very weak nonisothermality of electrons in gravitating plasma is obtained which is similar to the solution in isothermal electrons. The effects of gravity, two-temperature electrons dust charge fluctuations on the ion acoustic solitary waves for weak nonisothermal as well as very weak nonisothermal two-temperature of electrons are discussed with possible applications in astrophysical plasma.

Keywords: Ion-acoustic waves; gravitating plasma; dust charge fluctuation; two-temperature electrons; weak non-isothermal two-temperature electrons; modified Korteweg-deVries equation.

1. INTRODUCTION

In last few years, linear and nonlinear propagation of dust-acoustic and ion-acoustic waves in dusty plasma have been studied by various authors and interesting results are obtained which are not found in ordinary electron-ion plasma [1-6]. In dusty plasma, the existence of dust-acoustic wave (DAW) was first

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predicted theoretically by Rao et al. [7] with the dust grains providing the inertia and the pressure of inertia-less electrons and ions providing the restoring force. Most interesting features in dusty plasma are the solitary waves and the shock waves and a lot of works have been reported in last few years by various authors considering different parameters e.g. two-temp electrons [8,9], nonisothermal electrons and ions [10,11], nonthermal electrons and ions [12-14], super thermal electrons [15-17], magnetic field [18-20], inhomogeneity [21-22], finite geometry [23-26] etc. for the applications in various fields.

But, the self-gravitation of species of dusty plasma has not been taken into account in all those studies. In fact, the plasmas both in laboratory and in space have the gravitation field. In most of the studies of linear waves in collisional dusty plasma in gravitational field have been investigated neglecting the gravitational effects of electrons, ions and neutral gas molecules. Misra et al. [27] and others [28-34] have theoretically investigated the solitary waves in gravitating dusty plasma from which it is seen that gravitational effect is important on the excitation of solitary waves in dusty plasmas.

In dusty plasma, the dust particles get negatively charged due to attachment of the background electrons and ions on the surface via collisions [35,36]. The electrostatic charging of dust grains immersed in plasma is the main feature of dust-plasma interaction in dusty plasma. The charge on a dust particle does not remain fixed but depends on plasma properties, electron and ion currents flowing into or out of dust grains, photoemission etc. In fact the charge on the dust grain can be considered as an extra dynamical variable which controls the motion of dust grains in fluctuating electrostatic fields. But, For the study of wave propagation in dusty plasma, most of the authors have considered the effect of charge fluctuations on dust in the dusty plasma in an ad hoc fashion [37-41]. The dust charge is treated as a dynamical variable which is related to the plasma density, potential, etc. In the continuity equations of electrons and ions, no sink term is not considered. As a consequence the charge conservation equation is not consistent with the charge variation equation. Considering the sink terms in continuity equation for the loss of electrons and ions due to attachment to the dust particles, the dynamical equation for charge fluctuation become consistent with the charge continuity equation. For the study of ion acoustic solitary waves in a self-gravitating dusty plasma Paul et al. [42] have considered the sink terms in the continuity equations for both electrons and ions for the fluctuation of dust charges due to attachments of electrons and ions to the dust particles. They have shown that nonlinear excitation of waves follows a pair of coupled third order partial differential equations which is slightly different from usual case of coupled K-dV system and the solitary wave structure is not possible in such self-gravitating dusty plasma. Later, Paul et al. [43] have considered the sink term in the continuity equation of ions only assuming the dust charge fluctuation is caused by the attachment of ions only to the dust particles. With these assumptions they have derived an uncoupled third order partial differential equation instead of coupled nonlinear equations. They have finally obtained a modified KdV equation in self-gravitating dusty plasma consisting of warm positive ions, isothermal two-temperature electrons and dust particles. From the modified K-dV equation, they have obtained the solution of ion acoustic solitary wave and have discussed the effects of two-temperature electrons, gravity and dust charge fluctuations on the ion acoustic solitary waves. Subsequently, Paul et al. [44] have theoretically studied the ion acoustic solitary waves in a self-gravitating dusty plasma considering the presence nonisothermal electrons in the dusty plasma with fluctuating dust charges and drifting motion of the particles. They have also obtained an uncoupled third order partial differential equation which yield a modified form of KdV equation from which the quasi-soliton solution of ion acoustic wave has been obtained.

However, the effects of two-temperature electrons having weak non-isothermalilty on the ion acoustic solitary waves in a self-gravitating dusty plasma have not yet been studied by any author. So, in this paper, our interest is to study the existence and characteristics of ion-acoustic solitary waves in self-gravitating dusty plasma consisting of warm positive ions, weak non-isothermal two-temperature electrons and negatively charged dust particles with charge variation. Using the reductive perturbation method we have derived uncoupled third order partial differential equations instead of coupled nonlinear equation for the usual nonisothermal and weak nonisothermal electrons in gravitating plasma. From this uncoupled nonlinear equation, the solution of ion acoustic wave has been obtained and the effects of two-temperature electrons, gravity and dust charge fluctuations on the ion acoustic solitary waves have been discussed.

The paper is organized in the following way. In Section-2 we present the hydrodynamic equations for the ions and dust fluids. In Section-3 modified KdV equation has been derived using reductive perturbation method. In section-4 we find the solutions of ion acoustic waves from the modified KdV equation. In section-5, the effects of weak-nonisothermality of electrons are discussed. In Section -6 we have discussed the results with possible applications. Section-7 is devoted for the conclusions.

2. BASIC EQUATIONS

We consider an unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and non-isothermal two-temperature electrons. We assume that the dust particles are mainly charged by the attachment of ions to dust grains. Initially, electrons are attached to dust particles at a faster rate than the ions because the electrons have lighter mass and faster motion than the ions. As a result the dust particles get negatively charged. Once the dust particles have enough negative charge the chance of an electron being attached to a negatively charged dust grain becomes much smaller than the chance of an ion being attached to the dust grain. Ultimately, the charge fluctuation of dust grains is due to the attachment of positive ions only. Further, we consider that the self gravitating effect in dusty plasma influences both the linear and nonlinear modes of propagating ion acoustic waves. Under these assumptions, the normalized basic equations governing the plasma dynamics are [43]:

For ions*:*

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=-\beta_i n_i \tag{1}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial x}-\beta_i(v_i-v_d) \tag{2}$$

$$\frac{\partial p_i}{\partial t}+v_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial v_i}{\partial x}=0 \tag{3}$$

For dust particles:

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d v_d)=0 \tag{4}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}+\frac{\sigma_d}{\mu_d n_d}\frac{\partial p_d}{\partial x}=-\frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial x}-\beta_i(v_d-v_i) \tag{5}$$

$$\frac{\partial p_d}{\partial t}+v_d\frac{\partial p_d}{\partial x}+3p_d\frac{\partial v_d}{\partial x}=0 \tag{6}$$

Poisson's equations:

$$\frac{\partial^2 \phi}{\partial x^2}=n_{ec}+n_{eh}-n_i+Z_d n_d \tag{7}$$

$$\frac{\partial^2 \psi}{\partial x^2}=\frac{\omega_{gi}^2}{m_i\omega_i^2}\sum_{s=e,i,d} m_s n_s = X_i\sum_{s=i,d} m_s n_s \tag{8}$$

At equilibrium the charge neutrality condition is

$$n_i^{(0)}=n_e^{(0)}+Z_d n_d^{(0)}=n_{ec}^{(0)}+n_{eh}^{(0)}+Z_d n_d^{(0)} \tag{9}$$

the subscript $s = i$ represents for ions, $s = d$ for dust particles; m_s, n_s, v_s, q_s and p_s denote respectively the mass, number density , velocity, charge and scalar pressure of the s-th species of the plasma : n_{ec} ,n_{eh} are number density of electrons at low and high temperatures, Z_d is the number of electronic charge attached on the grains; $\mu_d = m_d/m_i$, $\sigma_i = T_i/T_{ef}$, $\sigma_d = T_d/T_{ef}$, $T_{ef}=T_{ec}T_{eh}/(n_{ec0}T_{eh}+n_{eh0}T_{ec})$, $X_i=\omega_{gi}^2/m_i\omega_i^2$, $\omega_{gi}^2=4\pi G m_i n_i$; $n_{i0}, n_{ec0}(n_{eh0})$ and n_{d0} are respectively the equilibrium number densities of ions, electrons and dust particles; G is the universal gravitational constant : φ and ψ are respectively the electrostatic and gravitational potentials; T_{ec} and T_{eh} are the low- and high- temperatures of electrons; T_{ef} is the temperature of free electrons; other symbols have their usual meanings. The parameter β_i is the attachment co-efficient (frequency) of ions to the dust particles. The right hand side of Eq. (1) is the sink term which describes the loss in the ion density due to attachment of ions to the dust particles. The attachment coefficient of ions may vary due to the change in ion current falling on the dust grains, and also due to the change in the densities of ions and dust grains. Since, β_i has the collision like behavior, the last terms in the right hand side of Eqs.(2) and (4) are also be like collisional effect between ions and dust particles [44-45] .

In the above equations we have normalized distance x by Debye length $\lambda_D=(k_B T_e/4\pi e^2 n_{i0})^{1/2}$, time t by the inverse of ion plasma frequency $\omega_i^{-1}=(m_i/4\pi e^2 n_{i0})^{1/2}$, all pressures by ion plasma pressure $n_i^{(0)} k_B T_i$, φ by $k_B T_e/e$, ψ by $k_B T_e/m_i$, and all number densities by n_{i0}. $\sigma_i = T_i/T_{ef}$.

For the nonisothermal electrons at two different temperatures the number density is given by [45].

$$n_{ec} = \mu[1+\frac{\varphi}{\mu+\nu\beta}+\frac{4(1-\beta_l)}{3\pi^{1/2}}(\frac{\varphi}{\mu+\nu\beta})^{3/2}+\frac{1}{2}(\frac{\varphi}{\mu+\nu\beta})^2-\frac{8(1-\beta_l^2)}{15\pi^{1/2}}(\frac{\varphi}{\mu+\nu\beta})^{5/2}+\frac{1}{6}(\frac{\varphi}{\mu+\nu\beta})^5-\cdots] \quad (10)$$

$$n_{eh} = \nu[1+\frac{\beta\varphi}{\mu+\nu\beta}+\frac{4(1-\beta_h)}{3\pi^{1/2}}(\frac{\beta\varphi}{\mu+\nu\beta})^{3/2}+\frac{1}{2}(\frac{\beta\varphi}{\mu+\nu\beta})^2-\frac{8(1-\beta_h^2)}{15\pi^{1/2}}(\frac{\beta\varphi}{\mu+\nu\beta})^{5/2}+\frac{1}{6}(\frac{\beta\varphi}{\mu+\nu\beta})^5-\cdots] \quad (11)$$

Where,

$$\beta = T_{ec}/T_{eh}, \beta_l = T_{el}/T_{elt}, \beta_h = T_{eh}/T_{eht}, \mu = n_{ec0}, \nu = n_{eh0}, b_l = \frac{(1-\beta_l)}{\pi^{1/2}}, b_h = \frac{(1-\beta_h)}{\pi^{1/2}},$$

$$b_l^{(1)} = \frac{(1-\beta_l^2)}{\pi^{1/2}}, b_h^{(1)} = \frac{(1-\beta_h^2)}{\pi^{1/2}}, \mu = n_{ec}^{(0)}, \nu = n_{eh}^{(0)}.$$

and T_{et} is the temperature for the trapped electrons.

Therefore, from (10) and (11) we obtain

$$n_e = n_{ec} + n_{eh}$$
$$= 1+\varphi-\frac{4}{3}(\frac{\mu b_l+\nu b_h\beta}{\mu+\nu\beta})^{3/2}\varphi^{3/2}+\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\varphi^2-\frac{8}{15}\frac{(\mu b_l^{(1)}+\nu b_h^{(1)}\beta^{5/2})}{(\mu+\nu\beta)^{5/2}}\varphi^{5/2}+\cdots\cdots \quad (12)$$

3. MODIFIED KdV EQUATION

For the derivation of the nonlinear equation i.e. K-dV equation governing the nonlinear dynamics of the wave, we make the usual stretching of the space co-ordinates and time,

$$\xi = \varepsilon^{1/4}(x-Vt) \text{ and } \tau = \varepsilon^{3/4}t \quad (13)$$

where, V is the linear phase velocity, ε is a dimensionless expansion parameter which is very small ε << 1 measuring the dispersion and nonlinear effects. Further, we assume the following perturbation expansion for the field variables:

$$\left.\begin{aligned}
n_i &= n_{i0}+\varepsilon n_{i1}+\varepsilon^{3/2}n_{i2}+\varepsilon^2 n_{i3}+\cdots\cdots\\
n_d &= n_{d0}+\varepsilon n_{d1}+\varepsilon^{3/2}n_{d2}+\varepsilon^2 n_{d3}+\cdots\cdots\\
u_i &= u_{i0}+\varepsilon u_{i1}+\varepsilon^{3/2}u_{i2}+\varepsilon^2 u_{i3}+\cdots\cdots\\
u_d &= u_{d0}+\varepsilon u_{d1}+\varepsilon^{3/2}u_{d2}+\varepsilon^2 u_{d3}+\cdots\cdots\\
p_i &= p_{i0}+\varepsilon p_{i1}+\varepsilon^{3/2}p_{i2}+\varepsilon^2 p_{i3}+\cdots\cdots\\
p_d &= p_{d0}+\varepsilon p_{d1}+\varepsilon^{3/2}p_{d2}+\varepsilon^2 p_{d3}+\cdots\cdots\\
\varphi &= \varphi_0+\varepsilon\varphi_1+\varepsilon^{3/2}\varphi_2+\varepsilon^2\varphi_3+\cdots\cdots\\
\psi &= \psi_0+\varepsilon\psi_1+\varepsilon^{3/2}\psi_2+\varepsilon^2\psi_3+\cdots\cdots\\
\beta_i &= \varepsilon^{3/4}\overline{\beta}_i
\end{aligned}\right\} \quad (14)$$

where $\varphi_0 = 0$, $\psi_0 = 0$.

3.1 Linear Dispersion Relation

Using (13) and (14) in Eqs. (1)- (8) and equating the coefficients of the lowest order of ε we obtain

$$Z_1\psi_1 = Y_1\phi_1 \quad (15a)$$

$$Z_2\psi_1 = Y_2\phi_1 \quad (15b)$$

where,

$$Z_1 = \frac{n_{i0}}{\lambda_1^2}-\frac{Z_d n_{d0}}{\lambda_2^2}, \quad Z_2 = \frac{w_{gi}^2 a_2}{\lambda_1^2}-\frac{w_{gd}^2 Z_d}{\lambda_2^2},$$
$$Y_1 = 1-\frac{a_2 n_{i0}}{\lambda_1^2}-\frac{Z_d b_2 n_{d0}}{\lambda_2^2},$$
$$Y_2 = -(\frac{w_{gi}^2}{\lambda_1^2}+\frac{w_{gd}^2 b_2}{\lambda_2^2}) \quad \lambda_1 = V_1^2-3a_1 n_{i0},$$
$$\lambda_2 = V_2^2-3b_1 n_{d0}, \quad V_1 = V-v_{i0}, \quad (16)$$

$$V_2 = V-v_{d0}, w_{gi}^2 = 4\pi G m_i n_{i0}, w_{gd}^2 = 4\pi G m_d n_{d0}$$
$$a_1 = \sigma_i/n_{i0}, a_2 = 1, b_1 = \sigma_d/\mu_d n_{d0}, b_2 = -Z_d/\mu_d.$$

and the mass of electron m_e is neglected.

From (15a) and (15b) we obtain the relation between ψ_1 and φ_1 given by,

$$\psi_1 = \pm\left(\frac{Y_1Y_2}{Z_1Z_2}\right)^{1/2}\varphi_1, i.e.\psi_1 = \pm\gamma_1\varphi_1 \tag{17a}$$

Or, $$\varphi_1 = \pm\left(\frac{Z_1Z_2}{Y_1Y_2}\right)^{1/2}, i.e.\varphi_1 = \pm\gamma_2\psi_1 \tag{17b}$$

Where,

$$\gamma_1 = \left(\frac{Z_1Z_2}{Y_1Y_2}\right)^{1/2}, \gamma_2 = \left(\frac{Y_1Y_2}{Z_1Z_2}\right)^{1/2}$$

From (15a) and (15b) the linear dispersion relation for the ion acoustic wave propagating through the self-gravitating dusty plasma having nonisothermal plasma is obtained as

$$Y_1Z_2 - Y_2Z_1 = 0 \tag{18}$$

Simplifying (18) we obtain the linear dispersion relation in the following form

$$(a_2\omega_{ig}^2)\lambda_1^2\lambda_2^4 - (Z_d\omega_{dg}^2)\lambda_1^4\lambda_2^2 - (n_{io}\omega_{ig}^2)\lambda_2^4 - (Z_dn_{d0}\omega_{dg}^2)\lambda_1^4 -$$

$$(a_2^2n_{i0}Z_d\omega_{ig}^2 + a_2n_{do}\omega_{ig}^2 - n_{i0}\omega_{dg}^2 - n_{d0}Z_d\omega_{dg}^2)\lambda_1^2\lambda_2^2 = 0 \tag{19}$$

For cold ions and cold dust in the dusty plasma (a_1 = 0 , b_1 = 0) and equal stream velocity of ions and dust particles (v_{i0} =0 = v_{i0}) the phase velocity of ion acoustic wave becomes

$$V = \left[v_{i0} \pm \frac{(1-b_2)(n_{i0}\omega_{gd}^2 + Z_dn_{d0}\omega_{gi}^2)}{(\omega_{gi}^2 + \omega_{gd}^2)}\right]^{1/4} \tag{20}$$

Since b_2 is negative as shown in (16), there are only two real and positive values of V in presence of stream velocity, one is fast- mode and the other is slow- mode. The fast- and slow- mode exist when $v_{i0} \neq 0$. It is to be noted that the two-temperature electrons and its non-isothermality have no influence on the phase velocity V of linear ion-acoustic wave.

3.2 Derivation of Modified K-dV Equation

Now, to derive the modified K-dV equation we use Eqs.(13) and (14) in Eqs. (1)-(8) and then equating the coefficients of $\varepsilon^{3/2}$ we obtain

$$\frac{\partial n_{i1}}{\partial\tau} + \bar{\beta}_i n_{i1} = (V - v_{i0})\frac{\partial n_{i2}}{\partial\xi} - n_{i0}\frac{\partial v_{i2}}{\partial\xi} \tag{21a}$$

$$\frac{\partial v_{i1}}{\partial\tau} + \bar{\beta}_i(v_{i1} - v_{d1}) = (V - v_{i0})\frac{\partial v_{i2}}{\partial\xi} - a_1n_{i0}\frac{\partial p_{i2}}{\partial\xi} - a_2\frac{\partial\phi_2}{\partial\xi} - \frac{\partial\psi_2}{\partial\xi} \tag{21b}$$

$$\frac{\partial p_{i1}}{\partial\tau} = (V - v_{i0})\frac{\partial p_{i2}}{\partial\xi} - 3\frac{\partial v_{i2}}{\partial\xi} \tag{21c}$$

$$\frac{\partial n_{d1}}{\partial\tau} = (V - v_{d0})\frac{\partial n_{d2}}{\partial\xi} - n_{d0}\frac{\partial v_{d2}}{\partial\xi} \tag{22a}$$

$$\frac{\partial v_{d1}}{\partial\tau} + \bar{\beta}_i(v_{d1} - v_{i1}) = (V - v_{d0})\frac{\partial v_{d2}}{\partial\xi} - b_1n_{d0}\frac{\partial p_{d2}}{\partial\xi} - b_2\frac{\partial\phi_2}{\partial\xi} - \frac{\partial\psi_2}{\partial\xi} \tag{22b}$$

$$\frac{\partial p_{d1}}{\partial\tau} = (V - v_{i0})\frac{\partial p_{d2}}{\partial\xi} - 3\frac{\partial v_{d2}}{\partial\xi} \tag{22c}$$

$$\frac{\partial^2\varphi_1}{\partial\xi^2} = \varphi_2 - \frac{4}{3}\frac{(\mu b_l + \nu\beta b_h)^{3/2}}{(\mu + \nu\beta)^{3/2}}\varphi^{3/2} - n_{i2} + Z_dn_{d2} \tag{23}$$

and

$$\frac{\partial^2\psi_1}{\partial\xi^2} = 4\pi G(m_in_{i2} + m_dn_{d2}) \tag{24}$$

To derive the K-dV equation from Eqs (21)-(24) we first eliminate $v_{i2}, v_{d2}, p_{i2}, p_{d2}$ after using the values of $n_{i1}, v_{i1}, p_{i1}, n_{d1}, v_{d1}, p_{d1}$ and find $\partial n_{i2}/\partial\xi$ and $\partial n_{d2}/\partial\xi$. Finally following usual procedure we have derived the following modified KdV equation from Eqs. (21)- (24) for the electrostatic field in self-gravitating dusty plasma consisting non-isothermal two-temperature electrons:

$$\frac{\partial\phi_1}{\partial\tau} + C\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi} + \frac{1}{2}B\frac{\partial^3\phi_1}{\partial\xi^3} + D\phi_1 = 0 \tag{25}$$

Where

$$C = \frac{\left[\dfrac{2(\mu b_l + \nu\beta b_h)^{3/2}}{(\mu+\nu\beta)^{3/2}}\right]}{\left[\dfrac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\dfrac{V_1^2}{\lambda_1}+\dfrac{3a_1n_{i0}}{\lambda_1}+1)+\dfrac{Z_dn_{d0}}{V_2\lambda_2}(Z_d+\gamma_2^{1/2})(\dfrac{V_2^2}{\lambda_2}+\dfrac{3b_1n_{d0}}{\lambda_2}+1)\right]}$$

$$B = \frac{1/2}{\left[\dfrac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\dfrac{V_1^2}{\lambda_1}+\dfrac{3a_1n_{i0}}{\lambda_1}+1)+\dfrac{Z_dn_{d0}}{V_2\lambda_2}(Z_d+\gamma_1^{1/2})(\dfrac{V_2^2}{\lambda_2}+\dfrac{3b_1n_{d0}}{\lambda_2}+1)\right]}$$

$$D = \frac{\left[\dfrac{\overline{\beta}_in_{i0}}{\lambda_1}(V_1+\dfrac{Z_dV_2}{\lambda_2}+\dfrac{1}{V_1})+\dfrac{Z_d^2n_{d0}V_1\overline{\beta}_i}{\lambda_2^2}-\dfrac{Z_dn_{d0}V_1\gamma_1^{1/2}\overline{\beta}_i}{\lambda_2^2}\right]}{\left[\dfrac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\dfrac{V_1^2}{\lambda_1}+\dfrac{3a_1n_{i0}}{\lambda_1}+1)+\dfrac{Z_dn_{d0}}{V_2\lambda_2}(Z_d+\gamma_1^{1/2})(\dfrac{V_2^2}{\lambda_2}+\dfrac{3b_1n_{d0}}{\lambda_2}+1)\right]}$$

It is seen that the dispersive term ***B*** of the K-dV equation (25) depends on the parameters of ions and dust particles in the gravitating plasma (i.e. $n_{i0}, n_{d0}, \mu_d, Z_d, \sigma_i, \sigma_d, \omega_{gi}, \omega_{gd}$) whereas the nonlinear term ***C*** depends on these parameters and also on the parameters of non-isothermal electrons (i.e. $\mu, \nu, \beta, b_l, b_h$). Since the coefficients ***B*** and ***C*** play crucial roles on the excitation of solitary waves , it is important to find the dependence of these coefficients on the amplitude, width and velocity of the solitary wave in the gravitating plasma. If $C = 0$ i.e. when the nonlinear term in (25) vanishes at some critical values of i) dust density, or ii) gravitational effects for ions and dust particles, or iii) density (or temperature) of two-temperature electrons etc. , the ion-acoustic solitary waves will not exist in the dusty plasma. Similarly, for B, the dispersive term in (25), there will no excitation of the ion-acoustic solitary waves in dusty plasma. Eq.(25) shows that there is one more term $D\varphi_1$ in the K-dV equation which consists of the attachment coefficient β_i and also the parameters of ions and dust particles in the gravitating plasma.

It is to be mentioned that an uncoupled K-dV equation for the gravitational potential ψ can also be derived for the waves in self-gravitating dusty plasma using the method for electrostatic potential. Then one can examine the possibility of having soliton-like variation of the gravitational potential in dusty plasma.

4. SOLITARY WAVE SOLUTION IN PRESENCE OF NONISOTHERMAL ELECTRONS

Solving Eq. (25) the solution for the ion-acoustic wave in a self-gravitating dusty plasma consisting of ions, charged dust particles and non-isothermal two-temperature electrons can be obtained for variable dust charges and also for constant dust charges. We first obtain the solution of ion acoustic solitary waves for dust particles in gravitating dusty plasma. Neglecting the effect of charge fluctuation on the dust grain (i.e. $\beta_i = 0$ and so D =0), the KdV Eq.(25) is reduced to

$$\frac{\partial\phi_1}{\partial\tau} + C\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi} + \frac{1}{2}B\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{26}$$

For having the stationary solutions of equations (26) we introduce a new variable $\eta = \xi - \lambda\tau$ and integrate the resulting equation using the boundary $i)\varphi_1 \to 0, ii)\dfrac{d\varphi_1}{d\eta} \to 0, iii)\dfrac{d^2\varphi_1}{d\eta^2} \to 0$ as $\eta \to \infty$.

Thus we get the intermediate integral

$$\frac{d\varphi_1}{d\eta} = \left\{\frac{2\lambda}{\overline{R}}(\varphi_1)^2\left[1-\frac{8\overline{Q}}{15\lambda}(\varphi_1)\right]^{1/2}\right\}^{1/2} \tag{27}$$

and then from (27) we obtain the solution for ion acoustic solitary waves in a self-gravitating dusty plasma given by:

$$\varphi_1 = \varphi_0 Sech^4(\lambda / \delta) \tag{28}$$

where, $\varphi_0 = \frac{225}{64}\frac{\lambda^2}{C^2}$ (is the amplitude), $\delta = [\frac{8B}{\lambda}]^{1/2}$ (is the width) and λ is the phase velocity of ion acoustic solitary wave. it is seen that both λ^2 and C^2 are positive for any value of the plasma parameters. So the amplitude φ_0 is positive which indicates that only compressive ion-acoustic solitary waves will be excited in the self-gravitating dusty plasma.

When the variation of dust charges in the dusty plasma is taken into account (i.e. $\beta_i \neq 0$, and so $D \neq 0$) the quasi-soliton solution for ion acoustic wave is obtained from Eq.(25). The modified K-dV Eq. (25) has a quasi-soliton solution given by

$$\varphi_1 = \boldsymbol{a(\eta')\,sec\,h}^4\boldsymbol{[(}\frac{\boldsymbol{a(\eta')}}{16}\boldsymbol{)}^{1/4}\boldsymbol{(}\xi - \sqrt{\boldsymbol{a(\eta')}}\boldsymbol{\eta')]} \tag{29}$$

whose amplitude can be expressed as [46]

$$\boldsymbol{a(\eta') = a(\eta'_0)\,exp[-S(\eta' - \eta'_0)]} \tag{30}$$

where η'_0 corresponds to some initial value of the variable η'.

Obviously, if $\boldsymbol{D < 0}$ the amplitude of the quasi-soliton increases exponentially indicating instability. On the other hand if $\boldsymbol{D > 0}$ the wave amplitude of solitary wave decays exponentially with the increase in the independent variable η'.

5. EFFECT OF SMALL DEVIATIONS OF NONISOTHERMALITY OF TWO-TEMPERATURE ELECTRONS ON THE SOLITARY WAVE SOLUTION

Now, we consider the case of dusty plasma having different effect of nonisothermality of two-temperature electrons. We assume that a small percentage of nonisothermality is present in the plasma i.e. the plasma has also a reasonable percentage of isothermal two temperature electrons. In this case, the above analysis for deriving the K-dV equation will not hold good and recourse must be taken to the original governing Eqs. (1)-(8). This can be done by introducing the nonisothermal parameters b_l and b_h as small quantities in slight different forms; $b_{l,h}$ are assumed to be $b_{l,h} = \varepsilon^{1/2}\, b`_{l,h}$ where $b`_{l,h} > 0$. In this case the scheme of perturbation expansion of the field variables and stretching of the space co-ordinates and time will be different from that considered earlier in (14) and (13). We have to use the scheme of Washimi and Taniuti [47] given by

$$\xi = \varepsilon^{1/2}(x - Vt) \text{ and } \tau = \varepsilon^{3/2} t \tag{31}$$

where, V is the linear phase velocity and ε is a smallness parameter measuring the dispersion and nonlinear effects. Further, the perturbation expansions for the field variables are :

$$\left.\begin{aligned}
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \cdots\cdots \\
n_d &= n_{d0} + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \varepsilon^3 n_{d3} + \cdots\cdots \\
u_i &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \varepsilon^3 u_{i3} + \cdots\cdots \\
u_d &= u_{d0} + \varepsilon u_{d1} + \varepsilon^2 u_{d2} + \varepsilon^3 u_{d3} + \cdots\cdots \\
p_i &= p_{i0} + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \varepsilon^3 p_{i3} + \cdots\cdots \\
p_d &= p_{d0} + \varepsilon p_{d1} + \varepsilon^2 p_{d2} + \varepsilon^3 p_{d3} + \cdots\cdots \\
\varphi &= \varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^3 \varphi_3 + \cdots\cdots \\
\psi &= \psi_0 + \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \varepsilon^3 \psi_3 + \cdots\cdots \\
\alpha_i &= \varepsilon^{5/2} \bar{\alpha}_i
\end{aligned}\right\} \tag{32}$$

where $\varphi_0 = 0$, $\psi_0 = 0$.

Using (31) and (32) in (1)-(8) we obtain from the first order equations in ε. From these equations the linear dispersion relation similar to (19) is obtained and the phase velocity is found to be same. This indicates that percentage of nonisothermality of electrons in plasma does not influence the phase velocity of the solitary wave.

Now, equating the coefficients of ε^2 we obtain from Eqs. (1) - (8) the following equations.

$$\frac{\partial n_{i1}}{\partial \tau} + \frac{\partial}{\partial \xi}(n_{i1} v_{i1}) + \bar{\alpha}_i n_{i0} = (V - v_{i0})\frac{\partial n_{i2}}{\partial \xi} - n_{i0}\frac{\partial v_{i2}}{\partial \xi} \tag{33a}$$

$$\begin{aligned}
&\frac{\partial v_{i1}}{\partial \tau} + v_{i1}\frac{\partial v_{i1}}{\partial \xi} + \bar{\alpha}_i (v_{i0} - v_{d0}) = \\
&(V - v_{i0})\frac{\partial v_{i2}}{\partial \xi} - a_1 n_{i0}\frac{\partial p_{i2}}{\partial \xi} - a_2\frac{\partial \phi_2}{\partial \xi} - a_3\frac{\partial \psi_2}{\partial \xi}
\end{aligned} \tag{33b}$$

$$\frac{\partial p_{i1}}{\partial \tau} = (V - v_{i0})\frac{\partial p_{i2}}{\partial \xi} - 3\frac{\partial v_{i2}}{\partial \xi} \tag{33c}$$

$$\frac{\partial n_{d1}}{\partial \tau} = (V - v_{d0})\frac{\partial n_{d2}}{\partial \xi} - n_{d0}\frac{\partial v_{d2}}{\partial \xi} \tag{33d}$$

$$\frac{\partial v_{d1}}{\partial \tau} + v_{d1}\frac{\partial v_{d1}}{\partial \xi} + \bar{\alpha}_i(v_{d0} - v_{i0}) = (V - v_{d0})\frac{\partial v_{d2}}{\partial \xi} - b_1 n_{d0}\frac{\partial p_{d2}}{\partial \xi} - b_2\frac{\partial \phi_2}{\partial \xi} - b_3\frac{\partial \psi_2}{\partial \xi} \tag{33e}$$

$$\frac{\partial p_{d1}}{\partial \tau} = (V - v_{i0})\frac{\partial p_{d2}}{\partial \xi} - 3\frac{\partial v_{d2}}{\partial \xi} \tag{33f}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 - \frac{4}{3}\frac{(\mu b_l + \nu b_h^3)}{(\mu + \nu\beta)^3}\varphi_1^{3/2} + \frac{\varphi_1^2}{2} - n_{i2} + Z_d n_{d2} \tag{33g}$$

Using (33a) - (33g) the following modified K-dV equation has been derived,

$$\frac{\partial \phi_1}{\partial \tau} + [A\phi_1 + C\phi_1^{1/2})\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + D\phi_1 = 0 \tag{34}$$

Where,

$$A = \frac{\left[\alpha_1\left\{1+\gamma_1^{1/2}\right\}^2 - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2} - Z_d\alpha_2\left\{Z_d + \mu_d\gamma_2^{1/2}\right\}^2\right]}{\left[\frac{n_{i0}}{V_1\lambda_1}(1+\gamma_1^{1/2})(\frac{V_1^2}{\lambda_1} + \frac{3a_1n_{i0}}{\lambda_1} + 1) + \frac{Z_d n_{d0}}{V_2\lambda_2}(Z_d + \gamma_2^{1/2})(\frac{V_2^2}{\lambda_2} + \frac{3b_1n_{d0}}{\lambda_2} + 1)\right]} \tag{35}$$

The coefficients ***B, C*** and D are given above.

Neglecting the dust charge fluctuation (i.e. D = 0) we get the stationary solution of (32) with the boundary condition at $\chi \to 0$. We assume $\chi = \xi - U\tau$, U is the velocity of transformed co-ordinates. Using the technique of Davidson [48] and Das et al. [49], we obtain the following intermediate equation

$$(\frac{\partial \varphi_1}{\partial \chi})^2 = \frac{U}{B}(\varphi_1)^2(1 - \frac{8C}{15}(\varphi_1)^{1/2} - \frac{A}{3U}\varphi_1) \tag{36}$$

Now, we introduce $\phi = M^{-2}$ and the Eq. (36) is written as

$$-[\frac{U}{B}(M^2 - \frac{8C}{15}M - \frac{A}{3U})]^{-1/2}dM = \pm d\chi \tag{37}$$

Again integrating the equation and then putting back to into the equation of ϕ, we get the solution

$$\varphi_1 = [\frac{4C}{15U} + (\frac{16C^2}{225U^2} + \frac{A}{3U})^{1/2}\cosh(\chi/\delta_2)]^{-2} \tag{38}$$

where, $\delta_2 = 2(B/U)^{1/2}$ is the width of the solitary wave.

It is to be noted that the expression $(\frac{16C^2}{225U^2} + \frac{A}{3U})$ is positive if ***A*** is positive. But, this expression may not be positive if ***A*** is negative. The negative values of $(\frac{16C^2}{225U^2} + \frac{A}{3U})$ will give complex values of the amplitude which indicates the non-propagation of wave in the plasma. For a real solitary wave the expression $(\frac{16C^2}{225U^2} + \frac{A}{3U})$ should be positive. It is possible if $(\frac{16C^2}{225U^2})$ > $(\frac{A}{3U})$ for ***A*** < 0. Moreover, the values of ***A*** and ***C*** give some idea about the solitary wave solutions. If C is very smaller than ***A*** ,the solitary wave solution takes the form

$$\varphi_1 = \frac{3U}{A}[1 - \frac{8C^2}{5\sqrt{3}AU}\operatorname{sec}h(\chi/\delta_2) - \frac{16C^2}{75AU}\{1 - 3\operatorname{sec}h^2(\chi/\delta_2)\}]\operatorname{sec}h^2(\chi/\delta_2) \tag{39}$$

If ***C*** is very small and if it is negligibly small then the solution becomes,

$$\varphi_1 = \frac{3U}{A}\operatorname{sec}h^2(\chi/\delta_2) \tag{40}$$

Which is the value of the electrostatic potential of the solitary waves in a plasma having isothermal electrons.

Considering the charge fluctuation of dust particles in plasma i.e. $D \neq 0$ having isothermal two-temperature electrons, the solutions for the solitary waves is obtained by Paul et al. [43,44] given by

$$\varphi_{1S} = \varphi_0 \sec h^2 \theta_S + D\tau \tag{41}$$

Where, $\varphi_0 = \dfrac{1}{3\zeta^2}$, $\theta_s = f(\zeta)$, ζ is a transformation parameter.

Moreover, in the case when ***A*** **i**s very small as compared to ***C*** the solution of the ion acoustic solitary wave in plasma having no charge fluctuation becomes

$$\varphi_1 = \frac{225U^2}{64C^2}[1 - \frac{75AU}{32C^2}\cosh(\chi/\delta_2) \times \sec h^2(\chi/2\delta_2)] \times \sec h^4(\chi/2\delta_2) \tag{42}$$

where the terms of $1/C^6$ are neglected.

As a special case, when ***A*** is negligibly small the above solution becomes

$$\varphi_1 = \frac{225U^2}{64C^2}\sec h^4(\chi/2\delta_2) \tag{43}$$

This is solution of the solitary wave in a plasma having non-isothermal electrons described by the K-dV equation (25) , neglecting the charge fluctuation parameter *D* for the attachment coefficients.

Considering the fluctuation charge of dust particles through attachment coefficients, i.e. when $D \neq 0$, quasi-soliton solution of ion acoustic wave in gravitating plasma having nonisothermal electrons is given by (30).

6. RESULTS AND DISCUSSION

From the modified K-dV equation (25) and its solution (28) and (30) it is seen that ion acoustic solitary waves will be excited in an unmagnetized self-gravitating dusty plasma having nonisothermal two-temperature electrons for constant dust charges and also for variable dust charges. In a dusty plasma with constant dust charge the amplitude of the solitary waves is given by $\varphi_0 = (225/64)(\lambda^2/C^2)$ which shows the dependence of amplitude on i) the densities and temperatures of two-temperature electrons , ions and dust particles, ii) the masses of ions and dust particle and iii) the charge of dust particles. But the width of solitary wave given by $\delta = (8B/\lambda)^{1/2}$ depends only on the parameters of ions and dust particles. But, the width of solitary waves does not depend on the density, temperature and nonisothermal parameter of electrons. On the other hand it is seen that the attachment coefficients and the gravitational effects are involved in *D* in fourth term of K-dV equation (25) and these parameters will significantly influence the soliton properties including soliton amplitude, speed, width as well as its stability. For the dust charge variation due to attachment coefficient of ions , the amplitude of ion-acoustic solitary wave will grow or decay exponentially.

In dusty plasma, weak nonisothermal electrons have a different role on the excitation of solitary wave, In this case, solitary wave has $\sec h^2 -$ type profile instead of $\sec h^4 -$type. The amplitude is given by $\varphi_0 = [\frac{4C}{15U} + (\frac{16C^2}{225U^2} + \frac{A}{3U})^{1/2}]^{-2}$ and width is $\delta = 2(B/U)^{1/2}$.

It is seen that amplitude depends on all the parameters in dusty plasma related to electrons, ions and dust particles including the gravitational effect. But, the electrons have no contribution on the width of solitons.

However, the plasma parameters of electrons, ions and dust particles are seen to be present in the expressions of amplitude and width of the solitary wave in a very complicated way in gravitating dusty plasma with ordinary-nonisothermal and weak-nonisothermal two-temperature electrons.. It is very difficult to see the effect of these plasma parameter separately on the amplitude and width of the solitary wave. A close observation shows that for the increase of temperature and nonisothermal parameter of two-temperature electrons the amplitude of the solitary wave will be decreased and the width will be increased. On the other hand, the effects of gravitational forces of ions and dust particles on the amplitude and width are seen to be more significant than other parameters in the dusty plasma. The amplitude will be drastically reduced due to the gravitational effect, but the width will be increased for the gravitational forces of mainly due to dust particles. In fact, numerical analysis will give clear physical implications of the gravitational potential, dust density, ion temperature, non-isothermal two-temperature electrons on the ion acoustic solitary waves in a dusty plasma. Inclusion of drift motion of plasma leads to two distinct modes of wave propagation- one having comparatively high phase speed called as ‘Fast mode’ and the other with lower phase speed called as ‘Slow mode’. For numerical analysis of amplitude and width of solitary wave in dusty plasma with drifting of ions and dust particles, one should carefully choice "Fast-mode" or Slow-mode" of the phase velocity for their investigation.

The attachment coefficients and the gravitational effects are involved in the modified K-dV equation; these parameters will influence the soliton properties including soliton amplitude, speed, width as well as its stability. The stability of solitary wave in a gravitating dusty plasma is important for understanding various nonlinear wave processes in the plasma. It is found that stability of solitary wave depends on the sign of the coefficient D of the fourth term Dϕ in the modified KdV equation. It is seen that the attachment coefficient, non-isothermal parameter of two-temperature electrons and gravitational effects are present into the factor D in a complicated way. The stability of the solitary structure will therefore be determined by the relative magnitudes of these effects.

7. SUMMARY AND CONCLUDING REMARKS

In this paper, we have theoretically studied the ion-acoustic solitary waves in self-gravitating dusty plasma consisting of positive ions, weak-nonisothermal two-temperature electrons and negatively charged dust particles with fluctuating charges and drifting ions by using the reductive perturbation method. Most important achievement in the paper is that instead of coupled nonlinear equations as obtained by earlier authors we have obtained an uncoupled nonlinear equation for the study of ion acoustic solitary waves in a self-gravitating dusty plasma. However, we have not analyzed our theoretical results numerically for a real dusty plasma of the astrophysical objects.

Our analysis is advantageous for solving the nonlinear equation for getting the solution of solitary waves. The results obtained by previous authors for ion acoustic soliton in gravitating plasma with one component electrons can be easily recovered from our investigation.

In our study, it is seen that because of the extra electron the solitary wave in plasma shows a different role played as a result of the nonlinearity and the dispersive effect of the plasma. The multiple electrons in plasma exhibit a steepening tendency in solitary waves and therefore the solitons may not exist ultimately. Moreover, because of the appearance of large amplitude ion acoustic waves in plasmas, the solitons move with faster velocity, causing the soliton to break into many more solitons. These phenomena are exhibited in the isothermal plasma , whereas the nonisothermal plasma does not have any such singularity. However, the plasma with a small percentage of nonisothermality may introduce such singularity. A close observation indicates that for the increase of temperature and nonisothermal parameter of two-temperature electrons the amplitude of the solitary wave will be decreased and the width will be increased. On the other hand, the effects of gravitational forces of ions and dust particles on the amplitude and width are seen to be more significant than other parameters in the dusty plasma. The amplitude will be drastically reduced due to the gravitational effect, but the width will be increased for the gravitational forces of mainly due to dust particles.

Possible applications of our present study are to understand the nonlinear wave processes in astrophysical plasma because the dusty plasmas exist in various environments like cometary tails, planetary rings, asteroids, magnetosphere, lower ionosphere, interstellar and circumstellar clouds [50-56].

Instead of dusty plasma one may consider gravitational effect in a heavy-ion plasma [57-60] and study the heavy-ion acoustic solitary waves shock structures for constant charge of heavy-ion. It is to be remembered that the dynamics of dusty plasma and heavy ion plasma are completely different. The charge and the mass of dusts in dusty plasma are very large than the heavy ions in dust-less electron-ion plasma. So, the gravitational effects of heavy ions on the solitary waves and double layers (or shock waves) would be insignificant.

We like to mention here that our present investigation is based on the plasma dynamics of non-degenerate plasma. But, in recent years there has been a great interest on the investigation of various collective processes in quantum-dusty plasmas. Quantum effects may become important in a variety of environments when the plasma temperature is low and density is high. The dispersion caused by strong density correlation due to quantum fluctuations can play important role. For a quantum plasma de-Broglie wavelength is comparable to the dimensions of the system.

Recently, the quantum effect in dusty plasma has been considered by many authors for the study of linear and nonlinear propagation of waves [61-69]. So considering both the self-gravitation and quantum effect in dusty plasma, nonlinear propagation of dust-acoustic and ion-acoustic waves can be studied from which more interesting results will be obtained for the applications in dense astrophysical plasma.

Some Special Remarks:

- In a dusty plasma both dust- acoustic and ion-acoustic solitary waves are excited in presence of drift motion . It is true that drift motion of dust particles are slower than the positive ions in dusty plasma. But, the motion of dust particles has important roles on linear and nonlinear propagation of waves in dusty plasma because both fast-mode and slow-mode are generated due to drift motion.
- In our paper we have considered an unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and non-isothermal two-temperature electrons. We have assumed that the dust particles are mainly charged by the attachment of ions to dust grains. Initially, electrons are attached to dust particles at a faster rate than the ions because the electrons have lighter mass and faster motion than the ions. As a result the dust particles get negatively charged. Once the dust particles have enough negative charge the chance of an electron being attached to the negatively charged dust grain becomes very much smaller than the chance of an ion being attached to the dust grain. Ultimately, the charge fluctuation of dust grains is due to the attachment of positive ions only. But, the dust particles remain negatively charged which is varying due to attachment of ions to the dust particles. After a long period of time, the dust particles become neutral in charge and finally become positively charged due to continuous attachment of positive ions. In present paper, we have considered the case when the charge of the dust particles are negative before going to be neutral due to attachment of positive ions. However, when the dust particles become positively charged the mathematical analysis for the study of ion acoustic solitary waves would be same.
- In dusty plasma, it is assumed that the electron current and ion current are the causes of charge fluctuation of dust particles [70]. The variation of dust charge is given by

$$\frac{dQ_{d1}}{dt}+\eta Q_{d1}=\left|I_{e0}\right|\left[\frac{n_{i1}}{n_{i0}}-\frac{n_{e1}}{n_{e0}}\right]$$

where, $\eta=(e\left|I_{e0}\right|/C)[1/T_e+1/\omega_0]$, C is the capacitance of dust grain, $\omega_0=T_i-e\phi_{f0}$, T_i is the ion temperature and $e\phi_{f0}$ is the equilibrium floating potential.

Since, in our paper we have assumed that all electrons are already attached to dust particles i.e. there is no free electron for attachment and only ions are being attached to dust particles causing dust charge variation.

So, I_{e0} =0 , η =0 and $\frac{dQ_{d1}}{dt}$ =0.

Therefore, attachment of ions is the only source for dust charge fluctuation. So, inclusion of attachment coefficient of ions in the continuity equation sand the momentum equations is justified.

Since the electrons are inertia- less, the question of inclusion of sink- term in the continuity equation of electron does not arise.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Alam MK, Roychowdhury A, Paul SN. On the characteristics of wave propagation in a magnetized dusty plasma with streaming of electrons and ions. Astrophys. Space Sc. 1998; 259:15-30.
2. Alam MK, Roychowdhury A, Paul SN. Wave propagation in a magnetized dusty plasma. International J. Theo. Phys. 1999;38:757-776.
3. Mondal KK, Roychowdhury A, Paul SN. Propagation of dust-acoustic waves in a bounded dusty plasma. Phys. Rev. 2001;E65: 016404.
4. Burman S, Paul SN, Roychowdhury A. Stimulated Brillouin scattering in a magnetized dusty plasma with charge fluctuation. Phys. Plasmas. 2002;9:3752-3757.
5. Misra AP, Roychowdury A, Paul SN. Higher order contribution to the propagation characteristics of low frequency transverse waves in a dusty plasma. Pramana. 2004;63: 579- 593.
6. Khan MSA, Sultana S, Salimullah M. Nature of electromagnetic wave in uniform dusty plasma. International Journal. Sci. Res. 2013; 2:132-136.
7. Rao NN, Shukla PK, Yu MY. Dust-acoustic waves in dusty plasmas. Planet Space Sci. 1990;38:543-546.

8. De SS, Paul B, Pakira G. Paul SN. Solitons in a dusty plasma having drift ions and two-temperature electrons. Proc. 3rd International Conf. Computers and Devices for communication (CODEC-06), Institute of Radio Physics and Electronics, Calcutta, University, Calcutta. December 18-20. 2006; 668-672.
9. Tagare SG. Dust-acoustic solitary waves and double layers in dusty plasma consisting of cold dust particles and two-temperature isothermal ions. Phys. Plasmas. 1997;4:3167-3172.
10. Das GC, Sarma J. A new mathematical approach for finding the solitary waves in dusty plasmas. Phys. Plasmas. 1998;5:3918-3923.
11. Rahman O, Bhuyan MDI, Haidar M, Islam J. Dust-acoustic solitary waves in an unmagnetized dusty plasma with arbitrarily charged dust fluid and trapped ion distribution. International J. Astron. and Astrophys. 2014;4: 119 -127.
12. Verheest F, Hellberg MA, Hereman WA.Head-on collisions of electrostatic solitons in nonthermal plasmas. Phys. Rev..2012, E86: 036402.
13. Asgari H, Muniandy SV, Wong CS. Dust-acoustic solitary waves in dusty plasmas with non-thermal ions. Phys. Plasmas. 2013;20: 023705.
14. Eslami E, Baraz R. Rarefactive and compressive soliton waves in unmagnetized dusty plasma with non-thermal electron and ion distribution. A I P Advances. 2014;4:027108.
15. Gogoi R, Roychowdhury R, Khan M. Arbitrary amplitude dust ion acoustic solitary waves and double layers in a kappa distributed electron plasmas. Indian J. Pure & App. Phys. 2011;49:173-179.
16. Choi CR, Min KW, Rhee TN. Electrostatic Korteweg-deVries solitary waves in a plasma with Kappa-distributed electrons. Phys Plasmas. 20211;18:092901.
17. Hossen MR, Ema SA, Mamun AA. Small Amplitude Dust–electron-acoustic shock waves and double layers in a nonextensive complex plasma with viscous electron fluids. IEEE Trans. on Plasma Sci. 2016;44(4):492-500.
18. Shahmonsouri M, Alinezad H. Dust acoustic solitary waves in a magnetized electron depleted superthermal dusty plasma. Phys. Plasmas. 2013;20:033704.
19. Mamun AA. Obliquely propagating dust-acoustic solitary waves in a magnetized dusty plasma. IL Nuvo Cimento. 1998;20:1307-1312.
20. Misra AP, Wang Y. Dust-acoustic solitary waves in a magnetized dusty plasma with nonthermal electrons and trapped ions. Comm. Nonlinear Science and Numerical Solution. 2015;22:1360-1369.
21. Tomar R, Malik HK, Dahiya RP. Reflection of ion acoustic solitary waves in a dusty plasma with variable charge dust. J. Theor. Appl Phys. 2014;8:126.
22. Elwakil SA, Zahran MA, El-Shewy MK, Mowafy AS. Effects of adiabaticity and non-adiabaticity of dust charge fluctuation on the propagation of the dust ion acoustic waves in inhomogeneous dusty plasma.Advances in Space Research. 2011;48:1067-1075.
23. Mamun AA, Shukla PK. Cylindrical and spherical dust ion–acoustic solitary waves. Phys. Plasmas. 2002;9:1468-1470.
24. Misra AP, Chowdhury AR. Dust-acoustic solitary waves in an inhomogeneous magnetized hot dusty plasma with dust charge fluctuations.Phys. Plasmas. 2006;13:062307.
25. Annon K. Effects of spherical geometry and two temperature electrons on dust acoustic waves. Astrophys. Space Sci. 2014;350:211-215.
26. Hossen MR, Ema SA, Mamun AA. Nonlinear dynamics in a nonextensive complex plasma with viscous electron fluids. Chinese Phys. Letts. 2016;33:065203.
27. Misra AP, Roycchowdhury A, Roychowdhury K. Electrostatic acoustic modes in a self-gravitating complex plasma with variable charge impurities. Phys. Lett. 2004;A323:110-121.
28. Misra AP, Roychowdhury A. Acoustic surface waves in a collisional dusty plasma. Phys. Sripta. 2004;69:44-47.
29. Misra AP, Roychowdhury A. Dust acoustic waves in a self-gravitating complex plasma with trapped electrons and nonisothermal ions. Euro Phys. J. 2006;D37:105-113.
30. Burman S, Roychowdhuty A. Solitary waves in self-gravitating dusty plasma. Chaos, Solitons and Fractals. 2002;13:973-979.
31. Verheest F, Shukla PK. Nonlinear waves in multispecies self-gravitating dusty plasmas. Phys. Scr. 1997;55:83-85.
32. Mamun AA, Salahuddin M, Salimulla M Ultra-low-frequency electrostatic waves in a self-gravitating magnetized and inhomo-geneous dusty plasma. Planet. Space Sci. 1998;47:79-83.
33. Yaroshenko VV, Verheest F. Nonlinear low-frequency waves in dusty self-gravitating plasmas. J. Plasma Phys. 2000;64:359-370.
34. Zhi-Rong G, Zeng-Qiang Y, Bao-Xiang Y, Mao-Zhu S. Nonlinear acoustic waves in a

collisional self-gravitating dusty plasma. Chinese Phys. 2010;B19:115203.
35. Jana MR, Sen A, Kaw PK. Collective effects due to charge-fluctuation dynamics in a dusty plasma. Phys. Rev. 1993;E48:3930.
36. Cramer NF, Vladimirov SV. The Alfvén resonance in a magnetized dusty plasma. Phys. Scipta. 1996;53:586-590.
37. Duan WS, Liu KP, Zhao JB. Hot dust acoustic solitary waves in dust Plasma with variable dust charge. Chin. Phys. Lett. 2001;18:1088-1089.
38. Pakzad HR, Javidan VBK. Solitary waves in dusty plasmas with variable dust charge and two temperature ions. Chaos, Solitons and Fractals. 2009;42:2904-2913.
39. Amin MR, Paul SK, Mondal G, Mamun AA. Effects of vortex-like (trapped) electron distribution on non-linear dust-acoustic waves with positive dust charge fluctuation. J. Plasma Phys. 2010;76:477-485.
40. Asgari H, Muniandy SV, Wang CS. Dust acoustic dressed soliton with dust charge fluctuations. Phys. Plasmas. 2010;17:063704.
41. Maitra S. The effect of dust size distribution and dust charge fluctuation on dust acoustic solitary waves in a magnetized dusty plasma Phys. Plasmas. 2012;19:03701.
42. Paul SN, Roychowdhury K, Burman S, Roychowdhury A, Paul B. On the existence of ion-acoustic solitary waves in a gravitating dusty plasma having charge fluctuation. Czech. J. Phys. 2006;56:1453-1460.
43. Paul SN, Pakira G, Paul B, Ghosh B. Ion-acoustic solitary waves in a self-gravitating dusty plasma having two-temperature electrons. World Acad. Sci. Engng Technol. 2011;5:472-477.
44. Paul SN, Pakira G, Paul B, Ghosh B. Nonlinear ion-acoustic waves in gravitating dusty plasma with non-isothermal electrons and fluctuating dust charges. Acta Physica Polonica. 2012; A122:116-121.
45. Majumdar SR, Paul SN, Das KP. Third-order contributions to ion-acoustic solitons in a plasma with two types of cold positive ions and two-temperature non-isothermal electrons. J.Plasma Phys. (U.K). 2000;64(3):297-308.
46. Duan WS, Zhao ZB Korteweg-de Vries solitons in inhomogeneous plasma. Phys. Plasmas. 1999;6:3484-3488.
47. Washimi H, Taniuti T. Propagation of ion-acoustic solitary waves of small amplitude. Phys. Rev. Letts. 1966;17:966 .
48. Davidson RC, Methods in nonlinear plasma theory. New York, Academic; 1972.
49. Das GC, Karmakar B, Paul SN. Ion-acoustic solitary waves in two-temperature electron plasmas. Phys. Fluids. 1986;29:2192.
50. Goertz CK. Dusty plasmas in the solar system. Rev. Geophys. 1989;27:271-292.
51. Mendis DA, Rosenberg M. Cosmic dusty plasma - annual review of astronomy and astrophysics. Ann. Rev. Astron. Astrophys. 1994;32:419-463.
52. Verheest F. Waves in dusty plasma. Kluwer Academic, Dordreecht; 2000.
53. Shukla PK, Mamun AA. Introduction to dusty plasma physics. Institute of Physics, Bristol, U.K; 2002.
54. Kim YW, Jung YD. Nonlinear effects on bremsstrahlung emission in dusty plasmas. Phys. Letts. 2004;A333:426-433.
55. Maharaj SK, Pilley SR, Bharuthram R, Singh SV, Lakhina GS. A parametric study of the influence of non-thermal ions on linear dust-acoustic waves in an unmagnetized dusty plasma. J. Plasma Phys. 2005;71:345-358.
56. Tsytovich VN, Morfill GE, Vlidimirov SV, Thomas HM. Elementary Physics of Complex Plasmas. Lecture Notes in Physics 731; 2008. Springer Berlin Heidelberg.
57. Shah MG, Rahman MM, Hossen MR, Mamun AA. Roles of superthermal elctrons and adiabatic heavy ions on heavy-ion-acoustic solitary and shock waves in a multi-component plasma. Communications Theo. Phys. 2015; 64(2):208-211.
58. Shah MG ,Rahman MM, Hossen MR, Mamun AA . Properties of cylindrical and spherical heavy ion-acoustic solitary and shock structures in a multispecies plasma with superthermal electrons. Plasma Physics Reports . 2016; 42 (2): 168-176.
59. Ema SA, Hossen MA, Mamun AA. Linear and nonlinear heavy ion-acoustic waves in a strongly coupled plasma .Phys. Plasmas. 2015;22:092108.
60. Ema SA, Hossen MR, Mamun AA , Nonplanar shocks and solitons in a strongly coupled adiabatic plasma: The roles of heavy ion dynamics and nonextensitivity. Contrib. Plasma Phys. 2015;55(8):596-605.
61. Ali S, Shukla PK. Dust acoustic solitary waves in a quantum plasma. Phys. Plasmas. 2006;13: 022313.
62. Misra AP, Shukla PK, Bhowmik C. Electron-acoustic solitary waves in dense quantum electron-ion plasmas. Phys. Plasmas. 2007; 14:082309.

63. Misra AP, Samanta S, Roychowdhury A. Nolinear propagation of dust ion-acoustic waves in a dusty quantum magnetoplasma. J. Plasma Phys. 2008;74:197-205.
64. Ghosh B, Chandra S, Paul SN. Amplitude modulation of electron plasma waves in a quantum plasma. Phys. Plasmas. 2011;18: 012106.
65. Chandra S, Paul SN, Ghosh B. Solitary wave structure in weakly relativistic degenerate electron plasma, Astrophys. and Space Sci. 2012;342:417-424.
66. Chandra S, Paul SN, Ghosh B. Linear and Nonlinear Propagation of electron plasma wave in quantum Plasma. Ind. J. Pure & Appl. Phys. 2012;50:314-319.
67. Ghosh B, Paul SN, Chandra S. Relativistic effects on the modulational instability of electron plasma waves in quantum plasma. Pramana. 2012;78:779-790.
68. Shahmansouri M. Dust-acoustic solitons in quantum plasma with kappa-distributed ions. Pramana. 2013;80:295-306.
69. Ghosh B, Paul SN, Banerjee S. Instability of electron plasma waves in an electron-hole bounded quantum dusty plasma. Internat. J. Math. Comput., Phys. Quantum Engineering. 2014;8(6):976.
70. Bhat JR, Pandey BP. Self-consistent charge dynamics and collective modes in a dusty plasma. Phys. Rev. E. 1994;50:3980.

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ORIGINAL PAPER

Dressed solitons and double layers of ion-acoustic waves in a dusty plasma having ultra-relativistic degenerate two-temperature electrons

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Abstract: Using the integral form of governing equation in terms of pseudopotential, the effect of higher-order nonlinear and dispersive effects on the dressed solitons (DS) and double layers (DL) of ion-acoustic waves has been theoretically studied in an ultra-relativistic degenerate (URD) plasma consisting of cold inertial ions, degenerate two-temperature electrons and negatively charged dust particles. The solution of DS in such plasma is obtained up to second-order approximation. The profiles of DS are drawn and discussed for different values of density and temperature of this plasma. The DS is compressive in the plasma for different values of density and temperature of degenerate electrons, and its amplitude is much larger than that of the first-order (KdV) solitons. The DS is compressive with wave-like structure for some value of the temperature of degenerate electrons, and its amplitude is lower than that of the KdV solitons. Moreover, the solution for the DS near-critical state is obtained and discussed graphically. Near-critical state, the temperatures of URD electrons have important roles in the DS. The solution of DL in the plasma has also been obtained and graphically discussed. Both compressive and rarefactive DL will be excited in URD two-temperature-electron plasma for different values of the electron density. But, only rarefactive DL would be excited in this plasma for different values of the electron temperature. The results are new and these would be applicable in space plasma.

Keywords: Dressed solitons; Double layers; Ion-acoustic wave; Ultra-relativistic degenerate plasma; Two-temperature electrons

1. Introduction

The solitary waves (SW) and double layers (DL) of ion-acoustic waves (IAW) have been studied by various authors considering different types of plasma (negative ion plasma, positron plasma, quantum plasma, dusty plasma, etc.) having different parameters (ion temperature, non-isothermal electrons, two-temperature electrons, nonthermal electrons, superthermal electrons, Tsallis distributed electrons, etc.) to fit their results in various physical situations in the laboratory and astrophysical plasma. However, two-temperature electrons in the plasma have special kind of effects in exciting the SW and DL of IAW. It has been found that a small percentage of the cooler component of electrons leads to effect qualitatively different from those obtained for a plasma having only one-temperature electron species. Goswami and Buti [1] and some other authors [2–8] have investigated the SWs and DLs and other aspects of ion-acoustic waves in plasma having two-temperature electrons.

However, from the experimental observations of various authors [9–11], it is found that theoretically predicted values of the amplitude, width and velocity of SW do not obey the experimental results. To remove the discrepancy between the theoretical and experimental results, a large number of authors have studied and analyzed the inclusion of higher-order perturbation corrections into the Korteweg–deVries (KdV) solitons, and the resulting solution has been termed as dressed solitons (DS). Initially, Ichikawa et al. [12] have first examined such higher-order effects by the reductive perturbation method (RPM) for SW in a plasma consisting of cold ions. Later, the temperature of ions in the plasma has been considered by Lai [13] and has investigated the higher-order effect on the SW of IAW. Subsequently, Kodama and Taniuti [14] have reconsidered this

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problem and have shown by the method of renormalization how to eliminate the secular terms appearing in higher-order terms of the expansion. Later, several authors have considered the effect of higher-order nonlinearity for investigating the DS of IAW in nondegenerate plasma and obtained interesting results [15–20].

Considering quantum (degenerate) plasma, Chatterjee et al. [21] have used the RPM for the study of DS of IAW in a dense quantum plasma whose constituents are electrons, positrons and positive ions. Later, Roy and Chatterjee [22] have investigated the DSs in an unmagnetized two-species electron–ion quantum plasma by using the RPM. They have derived a higher-order inhomogeneous (KdV-type) differential equation for the second-order correction and have obtained the nonsecular solution by the use of renormalization procedure. Moreover, they have applied a new technique to obtain the particular solution of the higher-order inhomogeneous equation which is found to be simpler compared to the technique used by previous investigators. Subsequently, Wang et al. [23] have presented a theory for the DS of quantum ion-acoustic waves (QIAW) in unmagnetized plasmas composed of positive and negative ions and weakly relativistic beams of electron considering higher-order corrections. They have investigated the properties of the QIAW using a quantum hydrodynamic (QHD) model from which a KdV equation has been derived using the RPM. They have also derived an equation including higher-order dispersion and nonlinearity corrections, and the effect of the physical parameter has been discussed for the importance of these corrections.

On the other hand, propagation of waves in relativistically degenerate (RD) and ultra-relativistic dense (URD) plasma has been studied by various authors following the works of Chandrasekhar [24, 25]. Using Sagdeev potential approach, Esfandyari-Kalejahi et al. [26] have investigated arbitrary amplitude SWs in electron–positron-ion plasma having ultra-relativistic or nonrelativistic degenerate electrons and positrons and have numerically investigated the matching criteria of existence of such solitary waves. Later, Shah et al. [27] have investigated the effect of trapping for the formation of solitary structures in RD plasma. Masood and Eliasson [28] have studied electrostatic solitary waves in quantum plasma having RD electrons and cold ions. Subsequently, Roy et al. [29] have investigated the DSs in a dense plasma composed of URD electrons and positrons, and inertial cold mobile ions, and negatively charged static dust particles by using the RPM. Later, using the QHD model and degenerate electron pressure Chandra et al. [30] have studied electro-acoustic solitary waves in RD quantum plasma with two-temperature electrons. It has been shown that the degeneracy parameter significantly influences the conditions of formation and properties of solitary structures. However, in a dense plasma consisting of URD electrons and positrons, cold and mobile inertial ions, and negatively charged static dust particles, Paul et al. [31] have investigated the modulation instability, envelope solitons and rogue waves of ion-acoustic waves using Fried and Ichikawa method [32].

From the discussions, it is seen that two-temperature electrons have important roles on the excitation of K-dV solitons, DL and DS of IAW in the plasma. Moreover, URD electrons have a significant effect on the nonlinear propagation of waves in plasma. But, no investigation has yet been reported on the DS in a dense dusty plasma with URD two-temperature electrons obeying the equation of state. So, in this paper, we are interested to study the DS and the DL of IAW in a dense dusty plasma with cold ions and URD two-temperature electrons with static negatively charged dust particles. To solve the problem we have used the integral form of governing equations in terms of pseudopotential. The effects of higher-order nonlinear and dispersive effects have been considered for obtaining the DS in URD two-temperature-electron plasma. It is to be mentioned that our mathematical technique has been used by several authors to investigate the effects of higher-order nonlinear and dispersive terms on the propagation of ion-acoustic solitary waves in nondegenerate plasma [33–35].

2. Formulation

We consider four-component plasma consisting of inertialess URD two-temperature electrons, cold inertial ion fluid and negatively charged static dust particles. The degenerate pressure of electrons has been expressed in terms of density by using the ultra-relativistic limit of Chandrasekhar [24, 25]. The nonlinear dynamics of the electrostatic wave in such URD dense plasma is described by the following equations:

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s u_s)=0 \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial \varphi}{\partial x} \tag{2}$$

$$n_{ec}\frac{\partial \varphi}{\partial x}-\frac{3}{4}\beta_{ec}\frac{\partial n_{ec}^{4/3}}{\partial x}=0 \tag{3}$$

$$n_{eh}\frac{\partial \varphi}{\partial x}-\frac{3}{4}\beta_{eh}\frac{\partial n_{eh}^{4/3}}{\partial x}=0 \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=\alpha_{ec}n_{ec}+\alpha_{eh}n_{eh}-n_i+\alpha_d \tag{5}$$

where the subscript ‘s’ (= e, i, p) denotes for electron, ion and positron; velocity u_s is normalized by ion-acoustic speed $C_i=(m_e c^2/e)^{1/2}$, density n_s is normalized by the equilibrium ion density, the electrostatic potential ϕ is normalized by m_ec^2/e, space variable x and time variable t are normalized by $(m_ec^2/4\pi n_{i0}e^2)^{1/2}$ and ion plasma period $(\omega_{pi})^{-1}$; $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$;

$$\begin{aligned} &\alpha_{ec} = \lambda_c n_{ec0}^{1/3}, \alpha_{eh} = \lambda_c n_{eh0}^{1/3}, \lambda_c = h/m_e c, \\ &\beta_{ec} = K_0\alpha_{ec}, \beta_{eh} = K_0\alpha_{eh}, K_0 = (\hbar/m_e c), \\ &\alpha_{ec} = (n_{ec0}/n_{i0}), \alpha_{eh} = (n_{eh0}/n_{i0}), \alpha_d = (Z_d n_{d0}/n_{i0}) \end{aligned} \tag{6}$$

n_{ec0}, n_{eh0} are the equilibrium densities of electrons at low and high temperature; n_{i0} is the equilibrium density of positive ions; Z_d is the number of negative charges of dust particles. Other parameters have their usual meanings.

The charge neutrality condition is

$$\alpha_{ec} + \alpha_{ec} + \alpha_d = 1, \; i.e.\, \alpha_{ec} + \alpha_{eh} = 1 - \alpha_d \tag{7}$$

In absence of dust particles, $\alpha_d = 0$, i.e.,$\alpha_{ec} + \alpha_{eh} = 1$.

For a solitary wave solution, we assume that the dependent variables depend on a single independent variable $\xi = x - Vt$, where V is the phase velocity of a solitary wave. Then from Eqs. (1)–(3), we obtain after using the boundary conditions $n_i \to 1, \varphi \to 0$ as $\xi \to \pm\infty$

$$n_i = \frac{1}{\sqrt{1-\frac{2\varphi}{V^2}}}, \; n_{ec} = \left(1+\frac{\varphi}{3\beta_{ec}}\right)^3, \; n_{eh} = \left(1+\frac{\varphi}{3\beta_{eh}}\right)^3 \tag{8}$$

where β_{ec} and β_{eh} are the temperature of degenerate electrons at low and high temperatures.

Now using (8) in (4) we get

$$\frac{\partial^2\varphi}{\partial\xi^2} = \Delta_1\varphi + \Delta_2\varphi^2 + \Delta_3\varphi^3 \tag{9}$$

where

$$\Delta_1 = \left[\frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}} - \frac{1}{V^2}\right] \tag{10a}$$

$$\Delta_2 = \left[\frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2} - \frac{3}{2V^4}\right] \tag{10b}$$

$$\Delta_3 = \left[\frac{\alpha_{ec}}{27\beta_{ec}^3} + \frac{\alpha_{eh}}{27\beta_{eh}^3} - \frac{5}{2V^6}\right] \tag{10c}$$

An integral of (9) satisfying the conditions $d\varphi/dX \to 0, \; \varphi \to 0$ as $\xi \to \pm\infty$ is

$$\frac{\varepsilon}{2}\left(\frac{d\varphi}{dX}\right)^2 = \frac{1}{2}\Delta_1\varphi^2 + \frac{1}{3}\Delta_2\varphi^3 + \frac{1}{4}\Delta_3\varphi^4 \tag{11}$$

where we have stretched the ξ-coordinate according to the relation

$$X = \varepsilon^{1/2}\xi. \tag{12}$$

In (11) and (12), ε is a smallness parameter. ε is positive and $\varepsilon < 1$ (i.e., $0<\varepsilon<1$).

For obtaining the solutions of higher-order nonlinear equations, we now follow the works of Refs. [33–35] and make the following perturbation expansions for electrostatic potential ϕ. and phase velocity V of the wave as

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + \varepsilon^3\varphi^{(3)} + \cdots\cdots, \tag{13a}$$

$$V = V_0 + \varepsilon V^{(1)} + \varepsilon^2 V^{(2)} + \varepsilon^3 V^{(3)} + \cdots\cdots \tag{13b}$$

where V_0 is the linear phase velocity of ion-acoustic wave.

In (13a) and (13b), the terms with $\varepsilon, \varepsilon^2, \varepsilon^3$, etc., represent the perturbed values of respective parameters.

Moreover, using the expansion of V given by (13b), we have expanded Δ_1, Δ_2 and Δ_3 of Eq. (9) in power series of ε and get the following equations:

$$\begin{aligned} &\Delta_1 = \delta_1^{(0)} + \varepsilon\delta_1^{(1)} + \varepsilon^2\delta_1^{(2)} + \varepsilon^3\delta_1^{(3)} + \cdots\cdots\cdots \\ &\Delta_2 = \delta_2^{(0)} + \varepsilon\delta_2^{(1)} + \varepsilon^2\delta_2^{(2)} + \cdots\cdots\cdots \\ &\Delta_3 = \delta_3^{(0)} + \varepsilon\delta_3^{(1)} + \cdots\cdots\cdots \end{aligned} \tag{14}$$

where

$$\begin{aligned} &\delta_1^{(0)} = 0, \\ &\delta_1^{(1)} = 2\gamma_1^2 V_0 V^{(1)}, \\ &\delta_1^{(2)} = 2\gamma_1^2 V_0 V^{(2)} + \gamma_1^2(1-4\gamma_1 V_0^2)(V^{(1)})^2, \\ &\delta_1^{(3)} = 2\gamma_1^2 V_0 V^{(3)} + 2\gamma_1^3(1-4\gamma_1 V_0^2)V^{(1)}V^{(2)} \\ &\qquad + 4\gamma_1^3 V_0(2\gamma_1^2 V_0^2 - 1)(V^{(1)})^3, \\ &\delta_2^{(0)} = \gamma_2 - \frac{3}{2}\gamma_1^2, \\ &\delta_2^{(1)} = 6\,\gamma_1^3 V_0 V^{(1)}, \\ &\delta_2^{(2)} = 6\,\gamma_1^3 V_0 V^{(2)} + 3\gamma_1^3(1-6\gamma_1 V_0^2)(V^{(1)})^2, \\ &\delta_3^{(0)} = \gamma_3 - \frac{5}{2}\gamma_1^3, \\ &\delta_3^{(1)} = 15\,\gamma_1^4\, V_0(V^{(1)})^2, \\ &\gamma_1 = \frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}}, \; \gamma_2 = \frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2}, \; \gamma_3 = \frac{\alpha_{ec}}{27\beta_{ec}^3} + \frac{\alpha_{eh}}{27\beta_{eh}^3}. \end{aligned} \tag{15}$$

Substituting expansions (15) in (11), we get the sequence of equations for the $\varphi^{(i)\prime}s$ and the equation for $\varphi^{(i)}$ at each order becomes a first-order inhomogeneous differential equation for $i > 1$.

For linear dispersion relation, $(\Delta_1)_0 = \partial_1^{(0)} = 0$. Therefore,

$$\frac{\alpha_{ec}}{\beta_{ec}}+\frac{\alpha_{eh}}{\beta_{eh}}=\frac{1}{V_0^2}=\gamma_1, \tag{16a}$$

$$i.e.\, V_0 == \sqrt{\frac{\beta_{ec}\beta_{eh}}{(\alpha_{ec}\beta_{eh}+\alpha_{eh}\beta_{ec})}},\ \gamma_1=V_0^{-2} \tag{16b}$$

3. Solitary wave solutions

3.1. First-order equation and its solution

In the first order, i.e., in the lowest order which is at the order of ε^3, we obtain the following equation for $\varphi^{(1)}$.

$$\frac{1}{2}\left(\frac{\mathrm{d}\varphi^{(1)}}{\mathrm{d}X}\right)^2=\frac{1}{2}\delta_1^{(1)}\varphi^{(1)^2}+\frac{1}{3}\delta_2^{(0)}\varphi^{(1)^3} \tag{17}$$

The solution can be written as

$$\frac{\mathrm{d}\varphi^{(1)}}{\mathrm{d}X}=\pm\sqrt{\frac{2}{3}}\varphi^{(1)}[3V_0-\varphi^{(1)}]^{\frac{1}{2}} \tag{18}$$

The solution of which is the first-order KdV solitons

$$\varphi^{(1)}=f_1 \sec h^2\eta \tag{19}$$

where

$$f_1=-\frac{6V_0^{-5}}{(2\gamma_2-3V_0^{-4})}V^{(1)}, \tag{20a}$$

$$\eta=\left(\frac{V_0^{-3}V^{(1)}}{2}\right)^{\frac{1}{2}}X, \tag{20b}$$

f_1 is the first-order amplitude. $\varphi^{(1)}$ vanishes at infinity, and it attains a maximum value at $\eta = 0$ giving the amplitude of first-order solitary wave $f_1, f_1=\alpha_{01}(say)$.

$$V^{(1)}=\frac{(2\gamma_2-3V_0^{-4})V_0^5 f_1}{6} \tag{21}$$

The first-order width D_1 of the solitary wave is

$$D_1=\left(\frac{2}{V_0^{-3}V^{(1)}}\right)^{\frac{1}{2}}\eta_1 \tag{22}$$

where η_1 is the positive root of the equation $\tanh^2\eta_1=0.58$.

The Mach number correct up to first order is given by

$$M_1=1+\frac{V^{(1)}}{V_0}=1+\frac{(2\gamma_2-3V_0^{-4})V_0^4 f_1}{6} \tag{23}$$

It is important to note that the perturbed part $V^{(1)}$ of the expansion for V in (13b) remains unspecified and hence the amplitude of the solitons, f_1, can be changed by varying $V^{(1)}$.

3.2. Second-order equation and its solution

At the second order, we get the following equation for $\varphi^{(2)}$ using (16) and (22)

$$\frac{\mathrm{d}\varphi^{(1)}}{\mathrm{d}X}\frac{\mathrm{d}\varphi^{(2)}}{\mathrm{d}X}-\left[\partial_1^{(1)}\varphi^{(1)}+\partial_2^{(1)}\varphi^{(1)^2}\right]\varphi^{(2)} =\frac{1}{2}\delta_1^{(2)}\varphi^{(1)^2}+\frac{1}{3}\delta_2^{(1)}\varphi^{(1)^3}+\frac{1}{4}\delta_3^{(0)}\varphi^{(1)^4} \tag{24}$$

Differentiating (24) with respect to $\varphi^{(1)}$ we get

$$\frac{d^2\varphi^{(1)}}{\mathrm{d}X^2}=\delta_1^{(1)}\varphi^{(1)}+\delta_2^{(0)}\varphi^{(1)^2} \tag{25}$$

By the use of this relation and by introducing the independent variables given by (22) Eq. (23) can be written as

$$\frac{\mathrm{d}\varphi^{(2)}}{d\eta}\frac{\varphi_{\eta\eta}^{(1)}}{\varphi_\eta^{(1)}}\varphi^{(2)}=\frac{\partial_1^{(2)}}{V_0V^{(1)}}\frac{\varphi_1^{(1)^2}}{\varphi_\eta^{(1)}}+\frac{2\delta_2^{(1)}}{3V_0V^{(1)}}\frac{\varphi_1^{(1)^3}}{\varphi_\eta^{(1)}} +\frac{\delta_3^{(0)}}{2V_0V^{(1)}}\frac{\varphi_1^{(1)^4}}{\varphi_\eta^{(1)}} \tag{26}$$

Multiplying both sides of (26) by the integrating factor $1\big/\varphi_\eta^{(1)}$ and then integrating we obtain the following solution $\varphi^{(2)}$ that vanishes at infinity and that attains a maximum at $\eta=0$.

$$\varphi^{(2)}=\frac{6V_0V^{(1)}\,\delta_1^{(2)}}{4V_0\ V^{(1)}}\,\eta\sec h^2\eta\tan h\eta +6V_0V^{(1)}\left(\frac{\delta_1^{(2)}}{4V_0V^{(1)}}+\frac{\delta_2^{(1)}}{2}+\frac{9\delta_3^{(0)}\ V_0V^{(1)}}{4}\right)\sec h^2\eta -\frac{27\delta_3^{(0)}\ V_0^2V^{(1)^2}}{4}\sec h^4\eta \tag{27}$$

Owing to the presence of the first term in the above expression, the $\varphi^{(2)}\big/\varphi^{(1)}$ does not remain finite at $\eta\to\pm\infty$.. So for a uniformly valid expansion of $\varphi^{(2)}$ the coefficient of this term must be equal to zero. Therefore we get $\delta_1^{(2)}=0$ which gives

$$V^{(2)}=-\frac{3V_0V^{(1)^2}}{2} \tag{28}$$

Removing the secular term, the solution of (27) is obtained as

$$\varphi^{(2)}=\left[f_2+f_3\ \tan h^2\eta\right]\sec h^2\eta \tag{29}$$

where

$$f_1 = -\frac{3}{2}\frac{\delta_1^{(1)}}{\delta_2^{(0)}} = -\frac{6V_0^{-5}}{(2\gamma_2 - 3V_0^{-4})}V^{(1)}$$
$$f_2 = \frac{2f_1^2}{\delta_1^{(1)}}\left[\frac{1}{3}\delta_2^{(1)} + \frac{1}{2}f_1\delta_3^{(0)}\right]$$
$$= \frac{36}{V_0^6(2\gamma_2 - 3V_0^{-4})^2}$$
$$\left[2 + \frac{3}{2(2\gamma_2 - 3V_0^{-4})}(2\gamma_3 - 5V_0^{-6}\right]V_0^2(V^{(1)})^2$$
$$f_3 = -\frac{f_1^3}{2}\frac{\delta_3^{(0)}}{\delta_1^{(1)}} = -\frac{f_1^3}{2}\frac{(\gamma_3 - \frac{5}{2}V_0^{-6})}{2V_0^{-3}V^{(1)}} = -\frac{f_1^3}{8}\frac{V_0^3(2\gamma_3 - 5V_0^{-6})}{V^{(1)}} \tag{30}$$

The amplitude of second-order correction term of (29) is

$$f_2 = \frac{36}{V_0^6(2\gamma_2 - 3V_0^{-4})^2}\left[2 + \frac{3}{2(2\gamma_2 - 3V_0^{-4})}(2\gamma_3 - 5V_0^{-6}\right]V_0^2(V^{(1)})^2 \tag{30a}$$

The second-order width D_2 of the SW is

$$D_2 = \left(\sqrt{\frac{2V_0^3}{V^{(1)}}}\right)\eta_2 \tag{31}$$

where η_2 is the positive root of the following equation,

$$(f_3/f_2)\tanh^4\eta_2 - [1 + 2(f_3/f_2)]\tanh^2\eta_2 + [(f_3/f_2) + 0.58] = 0 \tag{32}$$

3.3. Corrected higher-order solution up to second order (dressed solitons)

Using (19) and (29) the solitary waves correct up to second order become

$$\varphi_2 = \varphi^{(1)} + \varphi^{(2)} = f_4(1 + f_5\tanh^2\eta)\sec h^2\eta$$

where, $f_4 = f_1 + f_2, f_5 = f_3/f_4$

The amplitude of corrected second-order SW (or DS) is f_4,

$$f_4 = \frac{6V^{(1)}}{V_0^7(2\gamma_2 - 3V_0^{-4})}\left[1 + \frac{6V_0^{-2}}{(2\gamma_2 - 3V_0^{-4})}\left\{2 + \frac{3}{(2\gamma_2 - 3V_0^{-4})}(\gamma_3 - \frac{5}{2}V_0^{-6})\right\}V_0V^{(1)}\right] \tag{34b}$$

The Mach number of second-order correction term is

$$M_2 = \frac{{V^{(1)}}^2}{2V_0^2} \tag{35}$$

The Mach number of DS is

$$M = 1 + \frac{V^{(1)}}{V_0} + \frac{{V^{(1)}}^2}{2V_0^2}, \tag{36}$$

where $V^{(1)}$ is given by (21).

4. Special case: dressed solitons near-critical state

In the critical state, the nonlinear coefficient vanishes, i.e., $\delta_2^{(0)} = 0$. Under this condition, the SW will not be excited in plasma. But, the near- critical value of phase velocity V_0 (i.e., when $\delta_2^{(0)} \approx 0$), the SW will be excited with *sechη* profile instead of usual *sech*^{2}*η* profile. To study the SW near-critical state when $\delta_2^{(0)} \approx 0$, the following perturbation expansion for ϕ should be used,

$$\varphi = \varepsilon^{1/2}\varphi^{(1)} + \varepsilon\varphi^{(2)} + \varepsilon^{3/2}\varphi^{(3)} + \cdots\cdots \tag{37}$$

where ε is a smallness parameter, $0 < \varepsilon < 1$.

Substituting (12), (13b) and (37) in (9) and equating the coefficients of ε^2 from l.h.s. and r.h.s. we get

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\delta_1^{(1)}{\varphi^{(1)}}^2 + \delta_2^{(0)}{\varphi^{(1)}}^2\varphi^{(2)} + \frac{1}{4}\delta_3^{(0)}{\varphi^{(1)}}^4 \tag{38}$$

Near-critical state when $\delta_2^{(0)} \approx 0$, Eq. (38) is reduced to

$$\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \delta_1^{(1)}{\varphi^{(1)}}^2 + \frac{1}{2}\delta_3^{(0)}{\varphi^{(1)}}^4 \tag{39}$$

Equation (39) has the solution for SW in the form

$$\varphi_c^{(1)} = A_1\sec h\eta \tag{40}$$

where A_1 is amplitude of first-order SW near-critical state,

$$A_1^2 = -\frac{2\delta_1^{(1)}}{\delta_3^{(0)}} = -\frac{4V_0^{-3}}{(\gamma_3 - 2.5V_0^{-6})}V^{(1)} \tag{41}$$

$$\eta = \frac{1}{2}\left(\sqrt{\delta_1^{(1)}}\right)X,\ \delta_1^{(1)} = 2V_0^{-3}V^{(1)}$$

In near-critical state, $\delta_2^{(0)} \approx 0$; so, from (15) we obtain

$$\gamma_2 - \frac{3}{2}\gamma_1^2 = 0 \tag{42}$$

where

$$\gamma_1 = \frac{\alpha_{ec}}{\beta_{ec}} + \frac{\alpha_{eh}}{\beta_{eh}},\ \gamma_2 = \frac{\alpha_{ec}}{3\beta_{ec}^2} + \frac{\alpha_{eh}}{3\beta_{eh}^2}$$

Using the values of γ_1 and γ_2 in (42), we obtain the following quadratic equation for the near-critical value of density of URD electrons at $\delta_2^{(0)} \approx 0$,

$$\alpha_{ec}^2 - \left[\frac{(\beta_{eh} + 9\alpha_{eh}\beta_{ec})}{3\beta_{eh}}\right]\alpha_{ec} - \left[\frac{(2 - 3\alpha_{eh})\alpha_{eh}\beta_{ec}^2}{2\beta_{eh}^2}\right] = 0 \tag{43}$$

From (43) we obtain two values of the density of URD electrons at the low temperature given by

$$\alpha_{ec1} = B + (B^2 + 4C)^{1/2} \tag{44a}$$

$$\alpha_{ec2} = B - (B^2 + 4C)^{1/2} \tag{44b}$$

where

$$B = \left[\frac{(\beta_{eh} + 9\alpha_{eh}\beta_{ec})}{3\beta_{eh}}\right],\ C = \frac{(2 - 3\alpha_{eh})\alpha_{eh}\beta_{ec}^2}{2\beta_{eh}^2}$$

From (44a) and (44b) the critical values of density α_{ec1} and α_{ec2} of electrons at the low temperature may be obtained, if other plasma parameters, e.g., $\alpha_{eh}, \beta_{ec}, \beta_{eh}$, are known.

Similarly, at near-critical state, values of the temperatures β_{ec} of URD electrons at low temperature for the excitation of SW of IAW may be obtained from the quadratic equation of β_{ec} using Eq. (42).

Now, to find the higher-order correction in solitons at near-critical state of second-order Eq. (24) we equate the coefficient €5/2 and obtain the following equation,

$$\frac{d\varphi^{(1)}}{dX}\frac{d\varphi^{(2)}}{dX} = \partial_1^{(1)}\varphi^{(1)}\varphi^{(2)} + \partial_2^{(0)}\varphi^{(1)}\varphi^{(2)^2} + \partial_2^{(0)}\varphi^{(1)^2}\varphi^{(3)} + \frac{1}{3}\delta_2^{(1)}\varphi^{(1)^3} + \delta_3^{(0)}\varphi^{(1)^3}\varphi^{(2)} \tag{45}$$

Assuming $\delta_2^{(0)} \approx 0$ for near-critical state, we solve Eq. (45) for obtaining the second-order correction term $\varphi^{(2)}$ given by

$$\varphi_c^{(2)} = [A_2 \sec h\eta + \tanh \eta] \sec h\eta \tag{46}$$

where A_2 is the amplitude of second-order correction term at near-critical state,

$$A_2 = -\frac{2}{3}\frac{\delta_2^{(1)}}{\delta_3^{(0)}} = -\frac{4\gamma_1^3 V_0 V^{(1)}}{(\gamma_3 - 5\gamma_1^3/2)} = -\frac{4\, V_0^{-5}V^{(1)}}{(\gamma_3 - 2.5V_0^{-6})} \tag{47}$$

Using (40) and (46) the corrected solution of DS of IAW at near-critical state is obtained as

$$\varphi_{2C} = \varphi_c^{(1)} + \varphi_c^{(2} = [A_1 + A_2 \sec h\eta + \tanh \eta] \sec h\eta$$

5. Double layers

The solution of DL of IAW in URD two-temperature-electron plasma can also be obtained from Eq. (9). To obtain the solution of DL, we make an approximation taking the coefficients of φ^3 of Eq. (9) as $\varepsilon\Delta_3$ where ε is smallness parameter, $0 < \varepsilon < 1$.

Moreover, we substitute a new scaling with different perturbation expansions for V and ξ given by

$$V = V_0 + \varepsilon^2 V^{(1)} + \varepsilon^3 V^{(2)} \tag{49a}$$

and

$$X = \varepsilon\xi \tag{49b}$$

where ε is a smallness parameter, $0 < \varepsilon < 1$.

Using (49a) and (49b) in Eq. (9), we get after integration

$$\frac{\varepsilon^2}{2}\left(\frac{d\varphi}{dX}\right)^2 = \frac{1}{2}\Delta_1\,\varphi^2 + \frac{1}{3}\Delta_2\varphi^3 + \frac{1}{4}\varepsilon\Delta_3\varphi^4 \tag{50}$$

Now using Δ_1, Δ_2 and Δ_3 of (9) in (50) and equating the coefficient of ε^4 on both l.h.s. and r.h.s. we obtain

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 = \frac{1}{2}\partial_1^{(1)}\,(\varphi^{(1)})^2 + \frac{1}{3}\partial_2^{(0)}(\varphi^{(1)})^3 + \frac{1}{4}\partial_3^{(0)}(\varphi^{(1)})^4 \tag{51}$$

Now, Eq. (51) is written in the form

$$\frac{1}{2}\left(\frac{d\varphi^{(1)}}{dX}\right)^2 + \psi(\varphi^{(1)}) = 0 \tag{52}$$

where

$$\psi(\varphi^{(1)}) = -\frac{1}{2}\partial_1^{(1)}\,(\varphi^{(1)})^2 - \frac{1}{3}\partial_2^{(0)}(\varphi^{(1)})^3 - \frac{1}{4}\partial_3^{(0)}(\varphi^{(1)})^4 \tag{53}$$

Equation (52) describes the motion of a pseudoparticle of unit mass with velocity $d\varphi^{(1)}/dX$ and position $\varphi^{(1)}$ in a potential well $\psi(\varphi^{(1)})$. The first term of Eq. (53) can be regarded as the kinetic energy of the pseudoparticle, whereas the second term is the potential energy of the same particle at that instant.

For the solutions of DL, $\psi(\varphi^{(1)})$ must satisfy the following boundary conditions:

$$\text{(i)}\ \psi(\varphi^{(1)}) = 0, d\psi/d\varphi^{(1)} = 0 \text{ and } d^2\psi/d\varphi^2 < 0 \text{ at } \varphi = 0, \tag{54a}$$

$$\text{(ii)}\ \psi(\varphi^{(1)}) = 0, d\psi/d\varphi^{(1)} = 0 \text{ and } d^2\psi/d\varphi^{(1)^2} < 0 \text{ at } \varphi^{(1)} = \varphi_m^{(1)}(\neq 0), \tag{54b}$$

$$\text{(iii) for } 0 < \left|\varphi^{(1)}\right| < \left|\varphi_m^{(1)}\right| \tag{54c}$$

When the above conditions are satisfied, $\varphi^{(1)} = 0$ is an unstable equilibrium position. The pseudoparticle is not reflected at $\varphi^{(1)} = \varphi_m^{(1)}$ because the pseudoforce and the pseudovelocity have vanished. Instead, it goes to another state producing an asymmetrical DL with a net potential drop $\varphi_m^{(1)}$.

Applying boundary conditions (54a) and (54b) in (53) we obtain

$$\varphi_m = -\frac{2}{3}\frac{\partial_2^{(0)}}{\partial_3^{(0)}} = -\frac{2}{3}\frac{(\gamma_2 - 1.5V_0^{-4})}{(\gamma_3 - 2.5V_0^{-6})} \tag{55}$$

and

$$\psi(\varphi^{(1)}) = -\frac{\partial_3^{(0)}}{4}\varphi^{(1)^2}(\varphi_m^{(1)} - \varphi^{(1)})^2 \tag{56a}$$

$$\text{i.e. } \psi(\varphi^{(1)}) = -\frac{(\gamma_3 - 2.5V_0^{-6})}{4}\varphi^{(1)^2}(\varphi_m^{(1)} - \varphi^{(1)})^2 \tag{56b}$$

The solution of (52) for DL satisfying (55) is obtained as

$$\varphi_D = \frac{\varphi_m^{(1)}}{2}\left(1 - \tanh[\sqrt{(\partial_3^{(0)}/8\varphi_m^{(1)})}X]\right) \tag{57a}$$

$$\text{i.e. } \varphi_D = -\frac{1}{3}\frac{(\gamma_2 - 1.5V_0^{-4})}{(\gamma_3 - 2.5V_0^{-6})}\left(1 - \tanh[\sqrt{\{(\gamma_3 - 2.5V_0^{-6})/8\varphi_m^{(1)}\}}X]\right) \tag{57b}$$

The width of DL is given by

$$W_{DL} = 2\left(\frac{\partial_3^{(0)}}{8\left|\varphi_m^{(1)}\right|}\right)^{1/2} \tag{58a}$$

$$\text{i.e. } W_{DL} = 2\left(\frac{[\gamma_3 - 2.5V_0^{-6}]}{8\left|\varphi_m^{(1)}\right|}\right)^{1/2} \tag{58b}$$

It is seen that the width of DL depends on the parameters of URD two-temperature-electron plasma. It is important to note that the DL in plasma can only exist for positive value of $\delta_3^{(0)}$, i.e., $\partial_3^{(0)} > 0$. To satisfy this condition, the plasma parameters should have some critical values. Using (15) we get the critical value of the density of URD electrons at low temperature,

$$\alpha_{ec} > [27\beta_{ec}^3(2.5V_0^{-6} - \frac{\alpha_{eh}}{27\beta_{eh}^3})] \tag{59}$$

From Eqs. (55) and (57b) it is observed that the nature of DL (compressive or rarefactive) depends on the sign of the coefficient of the quadratic nonlinear term. Moreover, from (55) it is evident that for positive values of $\delta_2^{(0)}$ the rarefactive DL is formed since $\delta_3^{(0)}$ is positive. On the other hand, if $\delta_2^{(0)}$ is negative, the compressive DL will be excited. The nature, amplitude and width of the DL depend on the plasma parameters such as the density and temperature of electrons of URD two-temperature-electron plasma.

6. Results and discussion

We have studied the DS and the DL of IAW in dense dusty plasma having URD two-temperature electrons considering higher-order nonlinear and dispersive effects by carrying out calculations up to second order. The secularity conditions at each order give corrections to the velocity of the SW at the corresponding higher order. It is shown that higher-order nonlinear and dispersion terms have significant contributions to the formation of DS in the plasma under consideration. To find the DS we have first obtained the first-order SW (KdV solitons), then second-order correction to the KdV solitons and finally the corrected second-order SW (i.e., the DS). From (19), (29) and (33) it is seen that the density and the temperature of URD two-temperature electrons have significant contribution to excite the DS. Considering a model URD plasma, we have numerically estimated the respective amplitudes and have drawn the profiles of KdV solitons, second-order correction to the KdV soliton and finally the DS in the plasma. The effects of plasma parameters on the DS are graphically shown in Figs.(1)–(4).

To understand the nature of DS in URD two-temperature-electron plasma, the profiles are drawn taking different values of density and temperature of degenerate electrons. In Fig. 1 to Fig. 4 the structures of KdV solitons and DS are shown with the variation of density and temperature of ultra-relativistic degenerate two-temperature electrons (dimensionless) for fixed values of other plasma parameters.

i) **Effect of** α_{ec}.

In Fig. 1(a), the profile of KdV solitons ($\varphi^{(1)}$) is shown for different values of degenerate electron density (α_{ec}) at low temperature for fixed values of the plasma parameters $\beta_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$. It is observed that rarefactive KdV solitons would be excited in the plasma and the amplitude will increase with the increase of α_{ec}. In Fig. 1(b), the structure of second-order correction to KdV solitons ($\varphi^{(2)}$) is shown for the same plasma. It is observed that second-order correction term $\varphi^{(2)}$ is compressive in nature and the amplitude is large for the large values of density of degenerate electrons at low temperature. Figure 1(c) shows the profiles of the DS (dressed solitons) (φ_2) for degenerate electron density $\alpha_{ec} = 0.1$, for fixed values of the plasma parameters $\beta_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$. It is seen that the DS is compressive in nature and the amplitude of DS is much larger than that of the KdV solitons. The

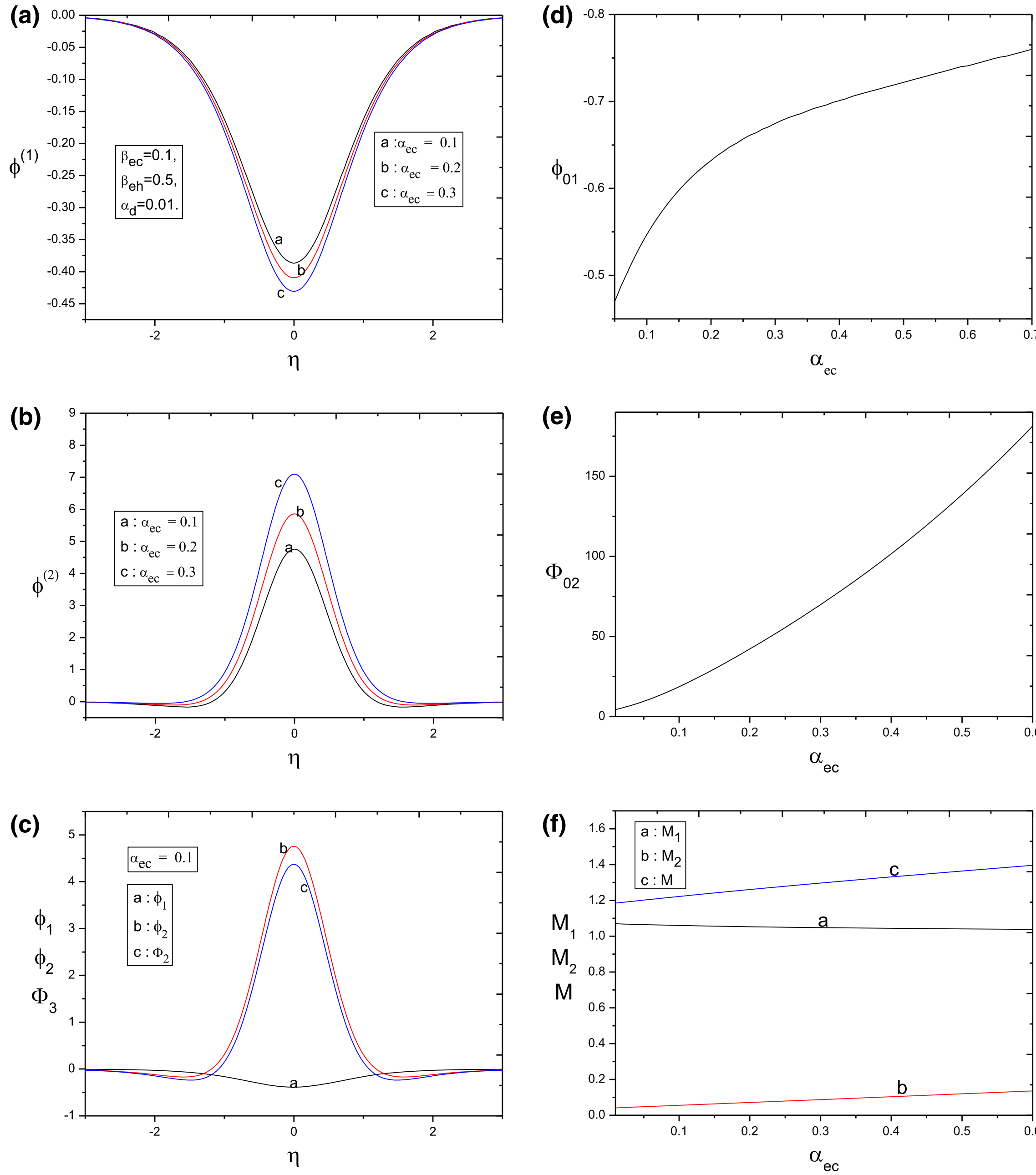
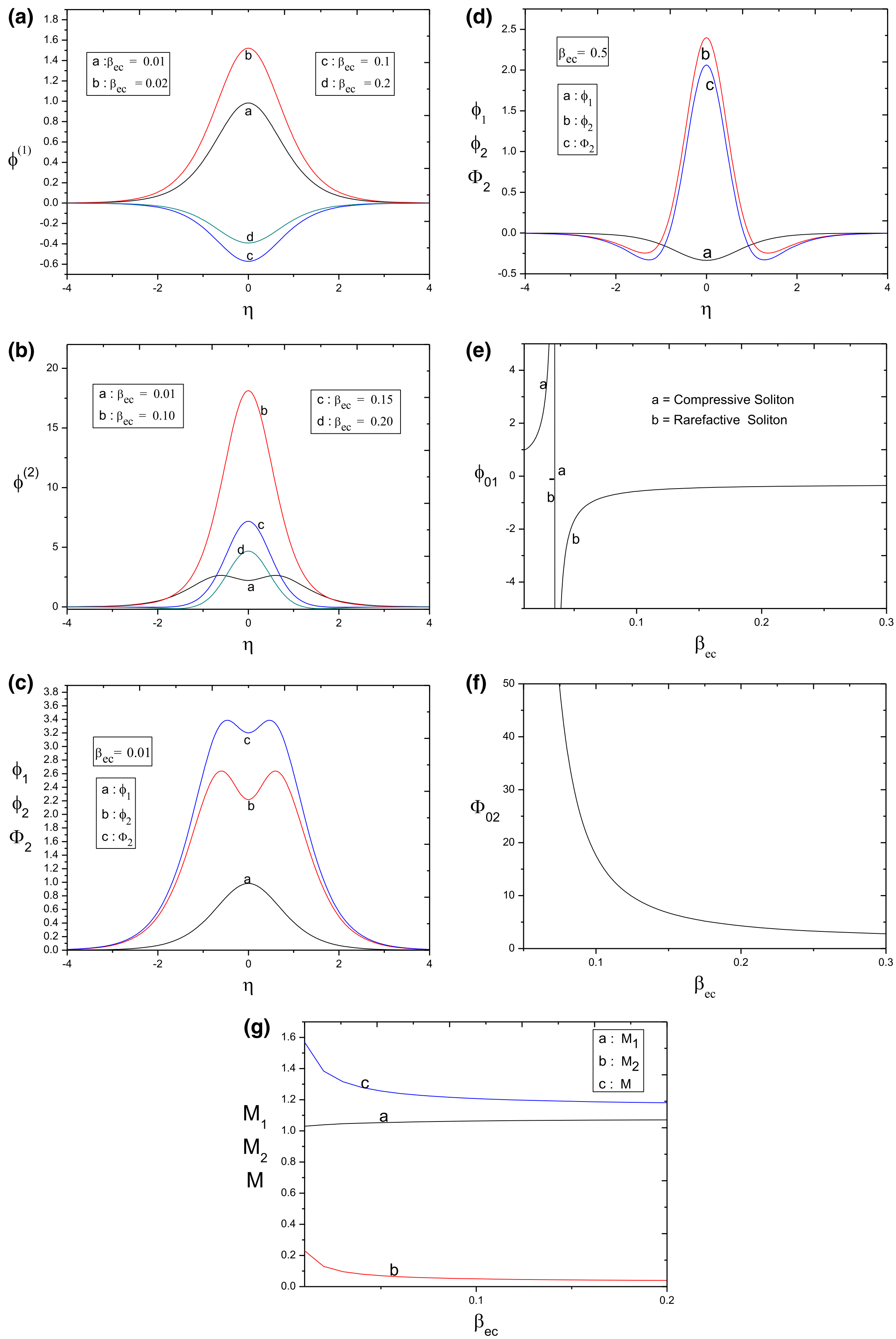


Fig. 1 (a) (colour online) The profiles of KdV solitons ($\varphi^{(1)}$) for different values of density (α_{ec}) of degenerate electrons for fixed values of the plasma parameters β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01; the graphs a and c correspond to α_{ec} = 0.1, 0.2 and 0.3; (b) (colour online) the profiles of second-order correction ($\varphi^{(2)}$) to the KdV solitons for different values of density (α_{ec}) of degenerate electrons for fixed values of the plasma parameters β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01; the graphs a, b and c correspond to α_{ec} = 0.1, 0.2 and 0. (c) (colour online) The profiles of the dressed solitons (φ_2) for density α_{ec} = 0.1 of degenerate electrons for fixed values of the plasma parameters β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01; the graphs a, b and c represent the KdV solitons ($\varphi^{(1)}$), second-order correction ($\varphi^{(2)}$) and dressed solitons ($\varphi_2 = \varphi^{(1)} + \varphi^{(2)}$(d) (colour online) Variation of the amplitude of KdV solitons for different values of density (α_{ec}) of degenerate electron for fixed values of β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01. (e) (colour online) Variation of the amplitude of dressed solitons for different values of density (α_{ec}) of degenerate electrons for fixed values of β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01. (f) (colour online) Variation of the Mach number with density (α_{ec}) of degenerate electrons for fixed values of β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.0; curves (a), (b) and (c) represent the Mach number of the KdV solitons (M_1), second-order correction of solitons (M_2) and dressed solitons ($M = M_1 + M_2$), respectively


(a)
a : βec = 0.01
b : βec = 0.02
c : βec = 0.1
d : βec = 0.2
φ(1)
η
(b)
a : βec = 0.01
b : βec = 0.10
c : βec = 0.15
d : βec = 0.20
φ(2)
η
(c)
βec = 0.01
a : φ1
b : φ2
c : Φ2
φ1
φ2
Φ2
η
(d)
βec = 0.5
a : φ1
b : φ2
c : Φ2
φ1
φ2
Φ2
η
(e)
a = Compressive Soliton
b = Rarefactive Soliton
φ01
βec
(f)
Φ02
βec
(g)
a : M1
b : M2
c : M
M1
M2
M
βec

◀**Fig. 2** (a) (colour online) The profiles of KdV solitons ($\varphi^{(1)}$) for different values of temperature (β_{ec}) of degenerate electron for fixed values of the plasma parameters α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01; the red, blue, green and magenta graphs represent β_{ec} = 0.01, 0.02, 0.1 and 0.2. (b) (colour online) The profiles of the second-order correction ($\varphi^{(2)}$) to the KdV solitons for different values of temperature (β_{ec}) of degenerate electrons for fixed values of the plasma parameters α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01. The graphs a, b, c and d represent β_{ec} = 0.01, 0.1, 0.15 and 0.2; (c) (colour online) the profiles of dressed solitons (φ_2) for the temperature β_{ec} = 0.01 of degenerate electrons for fixed values of the plasma parameters α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01; the graphs a, b and c represent the KdV solitons ($\varphi^{(1)}$), second-order correction ($\varphi^{(2)}$) and dressed solitons ($\varphi_2 = \varphi^{(1)} + \varphi^{(2)}$); (d) (colour online) the profiles of dressed solitons for the temperature β_{ec} = 0.5 of degenerate electron with fixed values of α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01; curves (a), (b) and (c) represent KdV solitons ($\varphi^{(1)}$), second-order correction ($\varphi^{(2)}$) and dressed solitons ($\varphi_2 = \varphi^{(1)} + \varphi^{(2)}$); (e) (colour online) variation of the amplitude of KdV solitons for different values of temperature (β_{ec}) of degenerate electrons with fixed values of α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01. (f) (colour online) Variation of amplitude of dressed solitons for different values of temperature (β_{ec}) of degenerate electrons with fixed values of α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01. g (colour online) Variation of the Mach number with temperature (β_{ec}) of degenerate electrons for fixed values of α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01; curves (a), (b) and (c) represent the Mach number of the KdV solitons (M_1), second-order correction of solitons (M_2) and dressed solitons ($M = M_1 + M_2$), respectively

variation of amplitude of the KdV solitons and the DS in URD two-temperature electrons are shown in Fig. 1(d) and Fig. 1(e). It is observed that for the increase of α_{ec} the amplitudes of rarefactive KdV solitons and the compressive DS are decreasing. Figure 1(f) shows the variation of Mach numbers of the KdV solitons (M_1), second-order correction to KdV solitons (M_2) and DS ($M = M_1 + M_2$) is described for different values of α_{ec} in URD plasma for fixed values of the plasma parameters β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01. It is observed that the Mach number of dressed solitons (M) is greater than that of KdV solitons (M_1).

ii) **Effect of** β_{ec}

Similarly, the effect of temperature (β_{ec}) of URD electrons at low temperature on the KdV solitons is shown in Fig. 2a in the plasma for fixed values of other parameters α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01. It is observed that both compressive and rarefactive SW may be excited in the plasma depending upon the values of β_{ec}. For β_{ec} = 0.01 and 0.02, the SW will be compressive and the amplitude is large for the large value of β_{ec}, but when β_{ec} = 0.1 and 0.2 the rarefactive SW will be excited and amplitude is increased with the increase of β_{ec}. The profiles of second-order correction $\varphi^{(2)}$ are shown in Fig. 2b for different values temperature of degenerate electrons β_{ec} for fixed values of the plasma parameters α_{ec} = 0.1, β_{eh} = 0.6 are α_d = 0.01. It is observed that second-order corrections $\varphi^{(2)}$ to KdV solitons are compressive in nature and amplitude is large for the large values of β_{ec}.. At low value of β_{ec}, it is compressive with wave-like behavior.

The profiles of DS are shown in Fig. 2c, d in URD two-temperature-electron plasma for the values of β_{ec} = 0.01 and β_{ec} = 0.5 for fixed values of α_{ec} = 0.1, β_{eh} = 0.5 and α_d = 0.01. It is observed that structure of DS is more interesting for low values of ($\beta_{ec} = 0.01$)

than large values of β_{ec} = 0.5.

The variation of the amplitude of the KdV solitons and the DS in URD plasma having two-temperature electrons is shown in Fig. 2e, f. It is observed that for the increase of β_{ec} the amplitudes of compressive DS are increasing for the low value β_{ec} and the amplitudes are decreasing for the large values β_{ec}. Figure 2g shows the variation of Mach numbers of the KdV solitons (M_1), the second-order correction to KdV solitons (M_2) and the DS ($M = M_1 + M_2$) with the temperature of degenerate electrons (β_{ec}) for fixed values of α_{ec} = 0.1, β_{eh} = 0.6, α_d = 0.01. It is observed that the Mach number (M) of DS is greater than that of the KdV solitons (M_1).

Special Cases:

Dressed solitons near-critical state

Effect of α_{ec} In near-critical state, the nonlinear coefficient vanishes, i.e., $\delta_2^{(0)} \approx 0$, the profiles of KdV solitons (φ_{1C}) are shown in Fig. 3a for different values of α_{ec} in URD two-temperature electrons for fixed values of β_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01. It is observed that in near-critical state the KdV solitons are compressive and the amplitude increases with the increase of α_{ec}. The second-order correction to the KdV solitons $\varphi^{(2)}$ is shown in Fig. 3b for different values of α_{ec} for fixed values of the parameters of same plasma. It is observed that the nature of second-order corrections to the KdV solitons ($\varphi^{(2)}$) may be compressive and rarefactive and the amplitude is large for the large values of α_{ec}. At near- critical state, the DS in URD two-temperature electrons is shown in Fig. 3c for the electron density α_{ec} = 0.1 with fixed values of β_{ec} = 0.1, β_{eh} = 0.5 and α_d = 0.01. It is seen from Fig. 3d that amplitude of the DS is lower than the KdV solitons.

Effect of β_{ec} The temperatures of URD two-temperature electrons also have important roles of the nature of DS at near-critical state. Let us first see the effect of β_{ec} on the KdV solitons. In Fig. 4a, the structures of KdV solitons are shown for different values of β_{ec} in plasma for fixed values of α_{ec} = 0.1, β_{eh} = 0.5 and α_d = 0.01. It is seen that the KdV solitons are compressive in nature and the amplitude of it decreases with the increase of β_{ec}. At near-critical

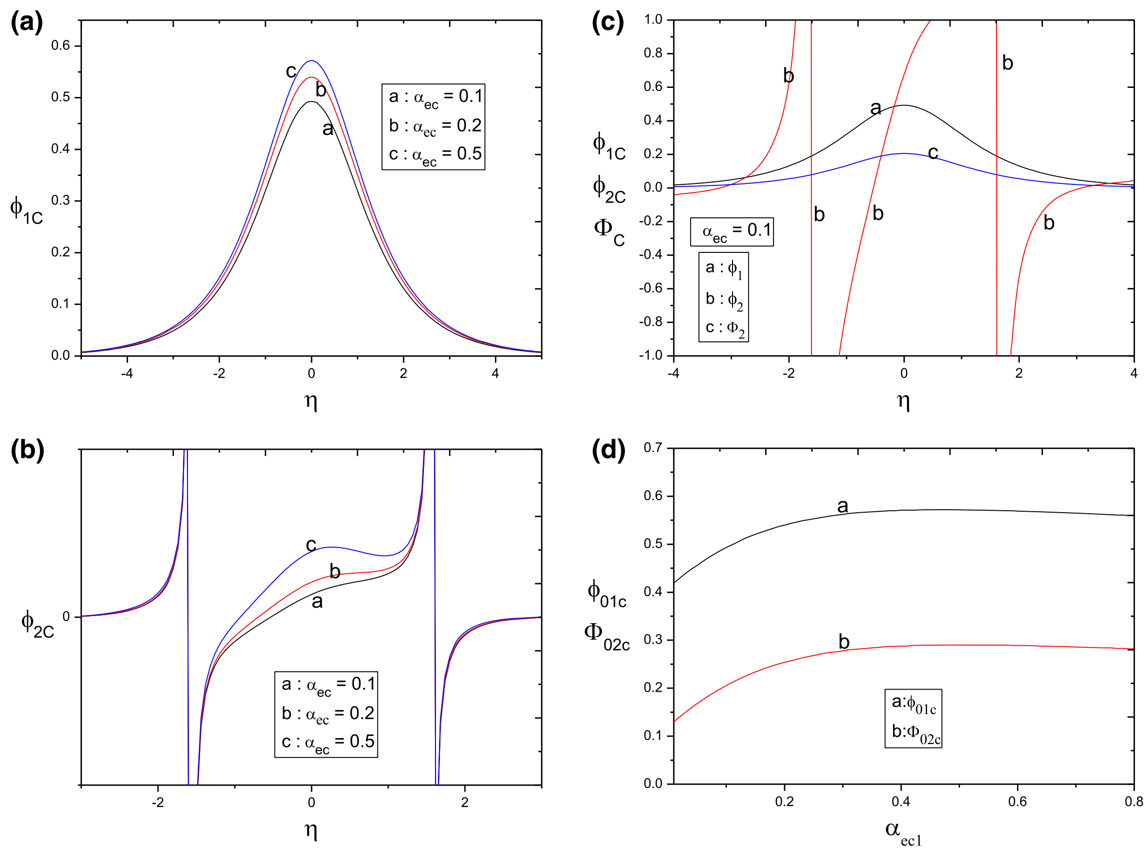


Fig. 3 (a) (colour online) The profiles of first-order KdV solitons (φ_{1C}) near-critical state for different values of density (α_{ec}) of degenerate electron with fixed values of $\beta_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$; curves (a), (b) and (c) represent three values of $\alpha_{ec} = 0.1$, 0.2 and 0.5, respectively. (b). (colour online) The profiles of second-order correction to KdV solitons (φ_{2C}) near-critical state of degenerate electron density (α_{ec}) for fixed values of $\beta_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$; curves (a), (b) and (c) represent three different values of $\alpha_{ec} = 0.1$, 0.2 and 0.5, respectively. (c) (colour online) The profiles of the dressed solitons near-critical state for degenerate electron density $\alpha_{ec} = 0.1$ with fixed values of $\beta_{ec} = 0.1$, $\beta_{eh} = 0.6$, $\alpha_d = 0.01$; curves (a), (b) and (c) represent the KdV solitons (φ_{1C}), second-order correction (φ_{2C}) and dressed solitons ($\varphi_{2C} = \varphi_{1C} + \varphi_{2C}$). (d) (colour online) Variation of amplitude of first-order KdV solitons and dressed solitons near-critical state for different values of degenerate electron density (α_{ec1}) for fixed values of $\beta_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$. Curves a and b represent φ_{01c} and φ_{02c}, respectively

state, the higher-order corrections (φ_{2C}) to the KdV solitons for different values of β_{ec} are shown in Fig. 4b in same plasma. It is observed that the correction term φ_{2C} may be compressive and rarefactive in nature. Moreover, the amplitude of φ_{2C} is small for the large values of β_{ec}. The DS at near-critical state in URD two-temperature electrons is shown in Fig. 4c for $\beta_{ec} = 0.1$ with fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.6$ and $\alpha_d = 0.01$. It is seen that the amplitude of DS is lower than the first-order SW. In this regard, it is necessary to state that the DS will be excited in the URD plasma at near-critical state ($\delta_2^{(0)} \approx 0$), when the density α_{ec} will have two critical values α_{ec1} and α_{ec2} given by (48a) and (48b). The variations of α_{ec1} and α_{ec2} with β_{ec} for fixed values of $\beta_{eh} = 0.6$ and $\alpha_d = 0.01$ are shown in Fig. 4(d)-1,-2. It is seen that with the increase of β_{ec} both α_{ec1} and α_{ec2} are increased.

Double Layers in URD plasma

The solution of DL of IAW in URD two-temperature-electron plasma is obtained from Eq. (9) which is given by (57b), and the amplitude and width are given by (55) and (58b), respectively. It is to be noted that there are some critical values of α_{ec} and β_{ec} for the excitation of the DL in URD plasma. The variation of critical value of α_{ec} with β_{ec} is shown in Fig. 5a for excitation of DL for the fixed values of $\beta_{eh} = 0.6$, $\alpha_d = 0.01$ in URD plasma.

Effect of α_{ec} Using the limiting values of critical α_{ec} and β_{ec} the profiles of DL in URD two-temperature-electron plasma are drawn for different values of α_{ec} and β_{ec}. In Fig. 5b it is shown that the formation of both compressive and rarefactive types of DL is possible to be excited in the URD plasma understudy for different values of α_{ec} with the fixed values of $\beta_{ec} = 0.01$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$. It is found that the amplitude of rarefactive DL is increasing with the increase of α_{ec}. The variation of width of DL with α_{ec} for different values of β_{ec} of URD electrons for the fixed

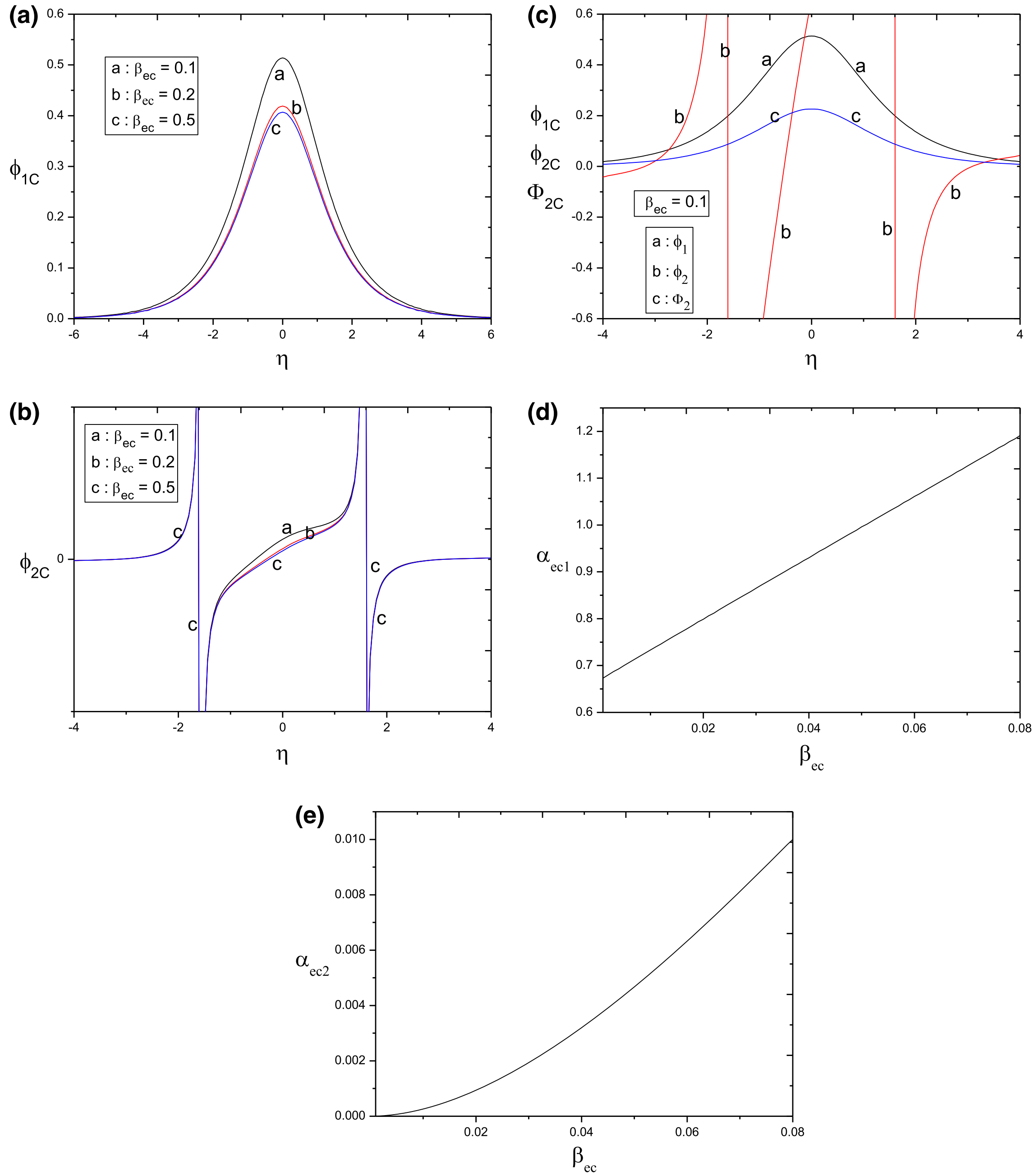


Fig. 4 (a) (colour online) The profiles of first-order KdV solitons (φ_{1C}) near-critical state for different values of temperature (β_{ec}) of degenerate electrons with fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$; curves (a), (b) and (c) represent three different values of $\beta_{ec} = 0.1$, 0.2 and 0.5, respectively. (b) (colour online) The profiles of second- order of solitons (φ_{2C}) near-critical state for different values of temperature (β_{ec}) of degenerate electrons with fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$; curves (a), (b) and (c) represent three different values of $\beta_{ec} = 0.1$, 0.2 and 0.5, respectively. (c) (colour online) The profiles of dressed solitons near-critical state for the temperature $\beta_{ec} = 0.1$ of degenerate electrons with fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.6$, $\alpha_d = 0.01$; curves a, b and c represent the KdV solitons (φ_{1C}), second-order correction (φ_{2C}) and dressed solitons ($\varphi_{2C} = \varphi_{1C} + \varphi_{2C}$), respectively. (d) (colour online) Variation of critical value of electron density α_{ec1} with electron temperature β_{ec} in ultra-relativistic degenerate plasma for fixed values of $\beta_{eh} = 0.6$, $\alpha_d = 0.01$. (e) (colour online) Variation of critical value of electron density α_{ec2} with electron temperature β_{ec} in ultra-relativistic degenerate plasma for fixed values of $\beta_{eh} = 0.6$, $\alpha_d = 0.01$

values of $\beta_{eh} = 0.5$ and $\alpha_d = 0.01$ is depicted in Fig. 5c. It is seen that the width of DL increases with the increase of α_{ec}, but it decreases with the increase of β_{ec}.

Effect of β_{ec} In Fig. 6a the profiles of DL of IAW are shown for different values of β_{ec} with the fixed values of $\alpha_{ec} = 0.1$, $\beta_{eh} = 0.5$ and $\alpha_d = 0.01$ in URD two-temperature-electron plasma. Figure 6a shows that only rarefactive

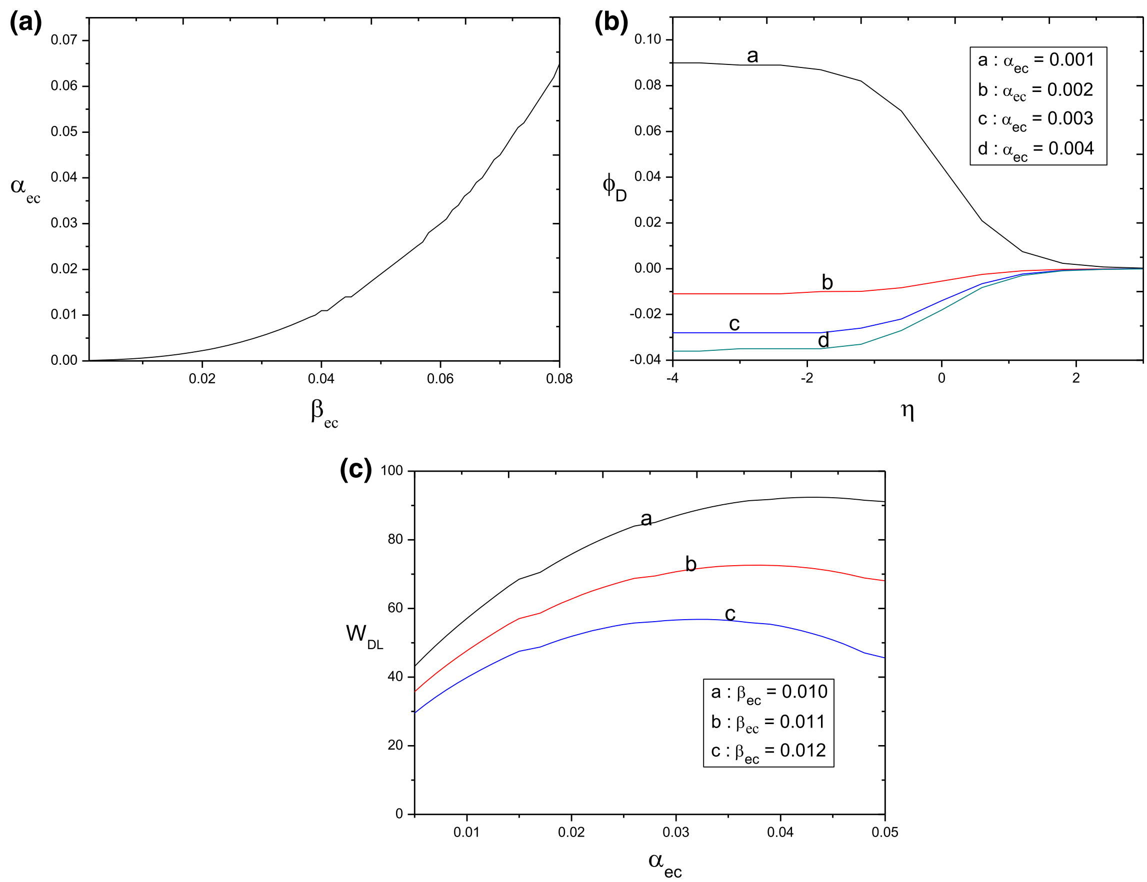


Fig. 5 (a) (colour online) Variation of critical value of degenerate electrons density (α_{ec}) with degenerate electron temperature (β_{ec}) for fixed values of $\beta_{eh} = 0.6$, $\alpha_d = 0.01$ for excitation of double layers. (b) (colour online) The profiles of double layers for different values of degenerate electron density (α_{ec}) with fixed values of $\beta_{ec} = 0.01$, $\beta_{eh} = 0.5$, $\alpha_d = 0.01$; curves a, b, c and d represent four different values of $\alpha_{ec} = 0.0011$, 0.0022, 0.0033 and 0.0044, respectively. (c) (colour online) Variation of the width (W_{DL}) of double layers with α_{ec} for different β_{ec} of degenerate electrons with fixed values of $\beta_{eh} = 0.5$, $\alpha_d = 0.001$; curves (a), (b) and (c) represent three different values of $\beta_{ec} = 0.010$, 0.011 and 0.012, respectively

DL is excited in the plasma and the amplitude of rarefactive DL is increased with the increase of β_{ec}. The variation of width of DL with β_{ec} is shown in Fig. 6b for different values of α_{ec} with the fixed values of $\beta_{eh} = 0.8$, $\alpha_d = 0.1$. It is observed that width of DL decreases with the increase of β_{ec}, but it increases with the increase of α_{ec}.

7. Conclusions

In this paper, we have theoretically studied the DS and the DL of IAW in a dusty plasma composed of URD two-temperature electrons, cold ions and negatively charged dust particles. Starting from an integrated form of the system of governing equations in terms of the pseudopotential considering higher-order nonlinear and dispersive effects, the DS and the DL have been theoretically studied using the approach based on the expansion of the Sagdeev potential up to second order and the RPM. Most important and significant results in this paper on the DS and the DL in cold ion and dense dusty plasma having URD two-temperature electrons are:

- The rarefactive first-order SW (KdV solitons) would be excited for different values of α_{ec} in URD two-temperature-electron plasma, and the amplitude of KdV solitons will increase with the increase of α_{ec}. In URD two-temperature-electron plasma, the second-order correction to the KdV solitons is compressive in nature for different values of α_{ec} and the amplitude of KdV solitons is large for the large values of α_{ec}. The DS is compressive in URD two-temperature-electron plasma for different values of α_{ec}. The amplitude of DS is much larger than that of the KdV solitons.
- Both compressive and rarefactive KdV solitons would be excited in URD two-temperature-electron plasma depending upon the values of β_{ec}. For the values of $\beta_{ec} = 0.01$ and 0.02, the KdV solitons will be compressive and the amplitude is large for the large value

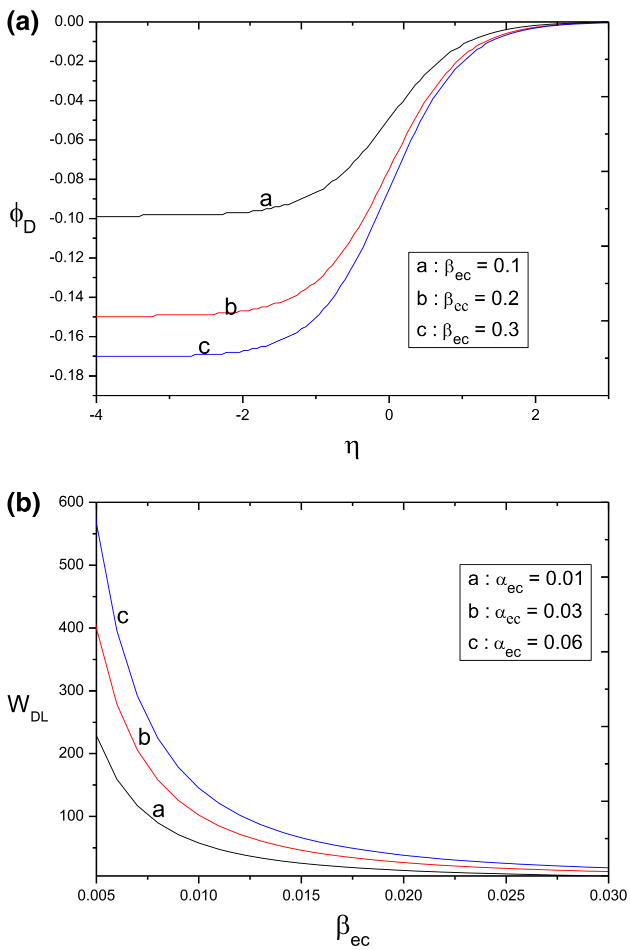


Fig. 6 (a) (colour online) The profiles of double layers for different degenerate electron temperature (β_{ec}) for fixed values of α_{ec} = 0.1, β_{eh} = 0.5, α_d = 0.01; curves (a), (b) and (c) represent three different values of β_{ec} = 0.1, 0.2 and 0.3, respectively. (b) (colour online) Variation of the width (W_{DL}) of double layer with β_{ec} for different α_{ec} of degenerate electrons with fixed values of β_{eh} = 0.8, α_d = 0.1; curves (a), (b) and (c) represent three different values of α_{ec} = 0.01, 0.03 and 0.06, respectively

of β_{ec}, but for β_{ec} = 0.1 and 0.2 the rarefactive KdV solitons will be excited and the amplitude is increased with increase of β_{ec}.

The structures of DS are found interesting with the increase of temperature of electrons. The DS is compressive with wave-like structure, and its amplitude is lower than that of the KdV solitons.

The second-order corrections to KdV solitons are compressive having wave-like structure for low value of β_{ec}.

- In near-critical state, the KdV solitons are compressive and the amplitude increases with the increase of α_{ec}. The DS is also compressive, and its amplitude is lower than that of the KdV solitons.

  In near-critical state, the temperatures of URD electrons have important roles on the nature of the SW. It is seen that the KdV solitons are compressive and its amplitude decreases with the increase of β_{ec}. The DS is also compressive, and the amplitude of DS is lower than that of the KdV solitons.
- It is possible to excite both compressive and rarefactive DL of IAW in URD two-temperature-electron plasma for different values of α_{ec}. The amplitude of the rarefactive DL will be increased with the increase of α_{ec}. Moreover, the width of DL increases with the increase of α_{ec}, but it decreases with the increase of β_{ec}.

Only rarefactive DL would be excited in URD two-temperature-electron plasma for different values of β_{ec}, and the amplitude of DL will be increased with the increase of β_{ec}. The width of DL decreases with the increases of β_{ec}, but it increases with the increase of α_{ec}.

It is important to be mentioned that most of the authors have studied the DS by the use of renormalization procedure in RPM. But, in the present paper, we have used the integral form of governing equations in terms of pseudopotential. The advantage of this method over the RPM is that instead of solving a second-order inhomogeneous differential equation at each order one has to solve a first-order inhomogeneous equation at each order except at the first. This technique has been successfully used by some authors [33–35] for investigating the effects of higher-order nonlinear and dispersive terms on the propagation of ion-acoustic SW in nondegenerate plasma systems. In this paper, we have also investigated the DS in URD two-temperature electrons near-critical state. It is necessary to be mentioned that using the renormalization procedure in RPM previous authors have studied the DS in plasma but not considered the situation in near-critical state. In our present investigation, we have considered the density of dust particles is low and is much smaller than the density of URD electrons. In dense dusty plasma, the dust particles would play significant roles on the nature of KdV solitons and the DS. The space plasmas are known to have multi-species composition with two-temperature electrons. So, our results on the DS and the DL may be useful for the study of nonlinear propagation of waves in understanding many phenomena in space plasma environments where the effects of URD electrons are important.

References

[1] B N Goswami and B Buti *Phys. Letts.* **57A** 149 (1976)
[2] K Nishihara and M Tajiri *J. Phys. Soc. Japan* **50** 4047 (1981)
[3] K K Ghosh and B Paul Chandra Das and S N Paul *J. Phys. A: Math. Theor.* **41** 335501 (2008)
[4] T K Baluku, M A Hellberg and F Verheest *Euro Phys. Letts.* **91** 15001 (2010)

[5] O R Rufai, R Bharuthram, S V Singh and G S Lakhina *Phys. Plasmas* **19** 122308 (2012)
[6] T Akhter, M M Hossain and A A Mamun *Chin. Phys. B* **22** 075201 (2013)
[7] Shalini and N. S. Saini, *Phys. Plasmas* **21** 102901 (2014)
[8] F Verheest and M A Hellberg *Phys. Plasmas* **22** 072303 (2015)
[9] H Ikezi, R J Tayler and D R Baker *Phys. Rev. Letts.* **25** 11 (1970)
[10] M Q Tran *Physica Scripta* **20** 317 (1979)
[11] K E Lonngren *Plasma Phys.* **25** 943 (1983)
[12] Y H Ichikawa, T Matsuhasi and K Konno *J. Phys. Soc. Japan* **41** 1382 (1976)
[13] C S Lai Canadian, *J. Phys.* **57** 490 (1979)
[14] Y Kodama and T Taniuti *J. Phys. Soc. Japan* **45** 298 (1978)
[15] R S Tiwari and M K Mishra *Phys. Plasmas* **13** 062112 (2006)
[16] S K El-Labany, W F El-Taibany and O M El-Abbasy *Chaos Solitons Fractals* **33** 813 (2007)
[17] T S Gill, P Bala and H Kaur *Phys. Plasmas* **15** 122309 (2008)
[18] K Roy, P Chatterjee and S Kundu *Advances in Space Research* **50** 1288 (2012)
[19] R Amour, L A Gougam and M Tribeche *Physica A* **436** 856 (2015)
[20] E K El-Shewy, N F Abdo and M S Yousef *Indian J. Phys* **90** 959 (2016)
[21] P Chatterjee, K Roy, S V Muniandy and C S Wong *Phys. Plasmas* **16** 112106 (2009)
[22] K Roy and P Chatterjee *Indian J. Phys.* **85** 1653 (2011)
[23] Y Wang, Y Dong and B Eliasson *Physics Letters A* **377** 2604 (2013)
[24] S Chandrasekhar *Philosophical Magazine* **11** 592 (1931)
[25] S Chandrasekhar *Mon. Not. R. Astron. Soc.* **170** 405 (1935)
[26] A Esfandyari-Kalejahi, M Akbari-Moghanjoughi and E Saberian *Plasma and Fusion Research: Regular Articles* **5** 045 (2010)
[27] H A Shah, W Masood, M N S Qureshi and N L Tsintsadze *Phys. Plasmas* **18** 102306 (2011)
[28] W Masood and B Eliasson *Phys. Plasmas* **18** 034503 (2011)
[29] N Roy, M S Zobaer and A A Mamun *Journal of Modern Physics* **3** 850 (2012)
[30] S Chandra, B Ghosh and S N Paul *Astrophys. Space Sci.* **343** 213 (2013)
[31] S N Paul and A Roychowdhury Indrani Paul *Plasma Physics Reports* **45** 1011 (2019)
[32] B D Fried and Y H Ichikawa *J. Phys. Soc. Japan* **34** 1073 (1973)
[33] K P Das and S R Majumdar Canadian *J Phys.* **69** 822 (1991)
[34] K P Das, S R Majumdar and S N Paul *Phys Rev E* **51** 4796 (1995)
[35] S R Majumdar S N Paul and A Roy Chowdhury *Physica Scripta* **69** 335 (2004)

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Nonlinear propagation of ion acoustic waves in quantum plasma in the presence of an ion beam

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Abstract

Nonlinear propagation of ion acoustic waves has been studied in unmagnetized quantum (degenerate) plasma in the presence of an ion beam using the one-dimensional quantum hydrodynamic model. The Korteweg–de Vries (K–dV) equation has been derived by using the reductive perturbation technique. The solution of ion acoustic solitary waves is obtained from the K–dV equation. The theoretical results have been analyzed numerically for different values of plasma parameters and the results are presented graphically. It is seen that the formation and structure of solitary waves are significantly affected by the ion beam in quantum plasma. The solitary waves will be compressive or rarefactive depending upon the values of velocity, concentration, and temperature of the ion beam. The critical value of ion beam density for the nonexistence of solitary wave has been numerically estimated, and its variation with velocity and temperature of ion beam has been discussed graphically. The results are new and would be very useful for understanding the beam–plasma interactions and the formation of nonlinear wave structures in dense quantum plasma.

Introduction

The most general class of nonlinear wave structures is solitary waves or soliton having approximately self-consistent wave amplitudes and phases. Solitons are important and extensively studied because a whole class of nonlinear differential equations encountered in plasma physics, solid-state physics, particle physics, hydrodynamics, nonlinear optics, and biology are seen to support the solutions of solitary waves. In the last few years, solitary waves in plasma have been studied theoretically and experimentally by various authors. Washimi and Taniuti (1966) first theoretically studied ion acoustic solitary waves (IASWs) in cold collisionless unmagnetized plasma deriving the K–dV equation using the reductive perturbation technique. The solitary wave in plasma was observed experimentally by Ikezi (1973), Lonngren (1983) and others. A lot of theoretical works on the nonlinear propagation of ion acoustic (IA) waves have been done by various authors incorporating different parameters in the plasma example, ion temperature (Tagare, 1973), two-temperature electrons (Ghosh *et al*., 2008), resonant electrons (Schamel, 1973), gravitation (Paul *et al*., 2017), negative ion (Chattopadhyaya and Paul, 2012), positrons (Paul *et al*., 2012), nonthermal electrons (Gill *et al*., 2004), kappa-distributed electron (Baluku and Hellberg, 2012), Tsallis-distributed electrons (Bala *et al*., 2017), etc. and have shown that these plasma parameters have considerable impact on the excitation of solitary waves. However, in recent years, there is considerable interest in the nonlinear propagation of waves in a plasma consisting of electron/ion beams. The studies of the propagation of waves in beam plasma are important in magnetospheric and solar physics (Goldman, 1983; Hoffmann and Evans, 1968). The nonlinear structure of a plasma may change considerably in the presence of electron/ion beams. An important property of an electron beam plasma is that it can change the propagation characteristic of the Trivelpiece–Gould (TG) solitons (Krivoruchko *et al*., 1975). The effects of an ion beam on the motion of solitons in an ion beam-plasma system has been studied by Gell and Roth (1977). The effects of ion beams are more important on the excitation of ion acoustic solitary waves in plasma. Abrol and Tagare (1979) obtained a modified K–dV equation for an IASW in an ion–beam–plasma system with cold ions, beam ions, and nonisothermal electrons. They investigated the effect of the resonant electrons, both trapped and free electrons, of the plasma–ion to beam–ion mass ratio and of the beam–ion concentration on the amplitude and width of the solitary wave. Later, many authors (Karmakar *et al*., 1988; Das and Singh, 1991; El-Labany, 1995; Huibin and Kelin, 2009; Zank and McKenzie, 1998; Das *et al*. 2011; Misra and Adhikary, 2011; Das, 2012; Das and Deka, 2015) have studied solitary waves in ion beam plasma and have obtained some fascinating results which are important in different situations in laboratory and space plasma. Recently, Kaur *et al*. (2017) have investigated the nonlinear propagation of ion acoustic solitary waves (IASWs) in an unmagnetized plasma composed of a positive warm ion fluid, two-temperature electrons obeying kappa-type distribution, and penetrated by a positive ion beam. They have used the reductive perturbation method to derive the nonlinear equations, namely K–dV, modified K–dV (mK–dV), and Gardner equations. The characteristic features of both compressive and rarefactive nonlinear excitations from the solution of these equations are studied and compared in the context with the observation of the He^+ beam in the polar cap region near solar maximum by the Dynamics Explorer 1 satellite. It is observed that the superthermality and density of cold electrons, number density, and temperature of the positive ion beam crucially modify the basic properties of compressive and rarefactive IASWs in the K–dV and mK–dV regimes. It is further analyzed that the amplitude and width of Gardner solitons are appreciably affected by different plasma parameters. The characteristics of double layers (DLs) are also studied in detail below the critical density of cold electrons. The theoretical results may be useful for the observation of nonlinear excitations in laboratory and ion beam-driven plasmas in the polar cap region near solar maximum and polar ionosphere as well in Saturn's magnetosphere, solar wind, pulsar magnetosphere, etc., where the population of two-temperature superthermal electrons is present. More recently, Kaur *et al.* (2018) have investigated the propagation characteristics of dust acoustic solitary and rogue waves in an unmagnetized ion beam plasma with electrons and ions following the kappa-type distribution in nonplanar geometry. The reductive perturbation method is employed to derive the cylindrical/spherical K–dV equation, which is further transformed into a standard K–dV equation by neglecting the geometrical effects. Using new stretching coordinates, the nonlinear Schrödinger equation has also been derived from the standard K–dV equation to study the different order rational solutions of dust acoustic rogue waves. The impact of various physical parameters on the characteristics of dust acoustic solitary waves is elaborated specifically in nonplanar geometry. Further, the effects of ion beam and superthermality of electrons/ions on the characteristics of rogue waves are studied. It is predicted that the results obtained may be useful in comprehending a variety of phenomena in Earth's magnetosphere polar cap region where the presence of positive ion beam has been detected and also in other regions of space/astrophysical environments where dust along with superthermal electrons and ions exists.

However, in recent years, researchers are very much interested to study the propagation of waves in a quantum plasma because quantum effects become important in a variety of environments, such as in intense laser–solid density plasma interaction experiments, dense astrophysical and cosmological environments, ultrasmall electronic devices, and metal nanostructures. The recent developments in nanoscience (Manfredi *et al.*, 2009), biophotonics (Barnes *et al.*, 2003), fiber optics (Agrawal, 1989), quantum confinement (Ang *et al.*, 2003), ultracold plasmas (Killian, 2007), etc., the study has gained momentum in a totally new direction. The quantum effects are also important in laboratory-produced ultra-dense plasmas (Kremp *et al.*, 1999) and laser-based inertial fusion experiments (Azechi, 2006) as well as the interior of Jovian planets, neutron stars (Shapiro and Teukolsky, 1983), and white dwarfs (Madelung, 1927), etc., where the density is very high. Quantum plasmas show numerous nonlinearities which are absent in classical plasmas. This has led to the investigation of waves in quantum plasma more important than ever. There have been many modes of waves in plasmas like ion acoustic waves (IAWs), electron acoustic waves, dust acoustic waves, dust ion acoustic waves, electron plasma waves, and shock waves. Using the Quantum Hydrodynamic (QHD) model, Haas *et al.* (2003) has studied the importance of the role of quantum diffraction in a linear and nonlinear regime. They adopted an equation of state pertaining to a zero-temperature Fermi gas for the electrons by disregarding pressure effects for the ions. By an appropriate rescaling of the variables, they identified a nondimensional parameter H, proportional to quantum diffraction effects. The system is shown to support linear waves, which, in the limit of small H, resemble the classical IAWs. In the weakly nonlinear limit, the quantum plasma is shown to support waves described by a deformed K–dV equation which depends in a nontrivial way on the quantum diffraction parameter H. In the fully nonlinear regime, the system also admits traveling waves which can exhibit periodic patterns. The works of Haas *et al.* (2003) have encouraged a large number of authors to study the nonlinear propagation of acoustic waves (e.g., solitary waves and modulation instability) in quantum plasma. We have here mentioned some of the important works on solitary waves in quantum plasma. Ali and Shukla (2006) have considered a one-dimensional QHD model for a three-species quantum plasma to derive the K–dV equation incorporating quantum corrections and studied the nonlinear properties of dust acoustic solitary waves. They examined the quantum mechanical effects numerically both on the profiles of the amplitude and the width of dust acoustic solitary waves. It is found that the amplitude remains constant but the width shrinks for different values of a dimensionless electron quantum diffraction parameter H. Subsequently, Ali *et al.* (2007) have investigated the linear and nonlinear properties of the IAWs by using the QHD equations together with the Poisson equation in a three-component quantum electron–positron–ion plasma. They have derived the linear dispersion relation, the K–dV equation, and an energy equation containing quantum corrections. They have also performed the computational investigations to examine the quantum mechanical effects on the linear and nonlinear waves. It is found that both the linear and nonlinear properties of the IAWs are significantly affected by the inclusion of the quantum corrections. Later, Misra and Bhowmik (2007) have considered a nonplanar spherical geometry to study the nonlinear properties of ion acoustic (IA) waves in an electron–ion quantum plasma with the effects of quantum corrections. Using the QHD model and standard reductive perturbation method, they derived the K–P equation having a variable coefficient and examined numerically the importance of quantum mechanical effects on the compressive and rarefactive solitons. It is found that H plays a significant role in the formation of compressive and rarefactive solitons. A critical value of H is also found which depends on the phase velocity of the wave and the ion to electron Fermi temperature ratio, for which the soliton formation ceases to exist. Subsequently, Mushtaq and Khan (2007) have studied ion acoustic solitary wave with weakly transverse perturbations in quantum electron–positron–ion plasma using the QHD model. They have derived the linear dispersion relation in the linear regime and the K–P equation in the nonlinear regime. It is found that compressive solitary wave can propagate in this system. The quantum effects are also studied graphically for both the linear and nonlinear profiles of ion acoustic wave. Using the energy consideration method, conditions for the existence of stable solitary waves are obtained. It is found that stable solitary waves depend on quantum corrections, positron concentration, and direction cosine of the wave vector.

Later, Sah and Manta (2009) have investigated the nonlinear wave structure of electro-acoustic waves (EAWs) in a three-component unmagnetized dense quantum plasma consisting of two distinct groups of electrons (one inertial cold electron and other inertialess hot electrons) and immobile ions. Using one-dimensional QHD and the standard reductive perturbation technique, they derived the K–dV equation governing the dynamics of EAWs. Both compressive and rarefactive solitons along with periodical potential structures are found to exist for various ranges of dimensionless quantum parameter H. The quantum mechanical effects are also examined numerically on the profiles of the amplitude and the width of electro-acoustic solitary waves. It is observed that both the amplitude and the width of electro-acoustic solitary waves are significantly affected by the parameter H. Akbari-Moghanjoughi (2010) has investigated large-amplitude IASW in a degenerate dense electron–positron–ion plasma considering the ion temperature as well as electron/positron degeneracy effects. It is shown that the ion temperature effects play an important role in the existence criteria and allowed a Mach-number range in such plasmas. He has also pointed out the fundamental difference in the existence of supersonic IASW propagations between degenerate plasmas with nonrelativistic and ultra-relativistic electrons and positrons. Chandra *et al.* (2012) have theoretically studied the linear and nonlinear propagation of electron plasma wave in a two-component unmagnetized dense quantum plasma with streaming of ions both analytically and numerically using one-dimensional QHD. It has been shown that the quantum effect modifies the linear dispersion character of the electron plasma waves in the presence of streaming motion and makes the possibility of two distinct modes. They have also derived the K–dV equation and have shown that both compressive and rarefactive solitary waves would be excited in the model plasma on some critical values of the quantum diffraction parameter. Dip *et al.* (2017) have investigated higher-order nonlinearity of the EAWs, specifically IA waves in an unmagnetized, collisionless, quantum electron–positron–ion plasma. They derived the mK–dV equation. The plasma system is supposed to be formed of positively charged inertial heavy ions, inertialess electrons and positrons to analyze the solitary waves (SWs), and the standard Gardner (SG) equation to analyze the higher-order SWs as well as DLs. The basic features (namely, amplitude, width, phase speed, etc.) of the IA SWs and DLs are examined. The comparison between the mK–dV SWs and SG SWs is also made. It is found that the amplitude, width, and phase speed of the IA SWs and DLs are significantly modified by the effects of both Fermi temperatures as well as pressures and Bohm potentials of electrons and positrons.

However, the propagation of ion acoustic and EAWs in quantum (degenerate) plasma having electron beam or ion beam would give fascinating results which are not studied yet critically, though many authors studied the wave propagation in nondegenerate/classical plasma (Yajima and Wadati, 1990; Mondal *et al.*, 1998; El-Labany, 1995; Zank and McKenzie, 1998; Bailung *et al.*, 2010; Saberian *et al.*, 2013; Danehkar, 2018). So, in the present paper, we are interested to study the nonlinear propagation of ion acoustic waves in a quantum plasma consisting of warm ion beams. Using the reductive perturbation method, we have derived the K–dV equation and have obtained the solution of ion acoustic solitary in the quantum plasma having nonrelativistic ion beam. The effects of ion beam parameters, that is ion beam density, ion beam temperature, and ion beam velocity, on the solitary waves are critically discussed with graphical representation. The quantum diffraction parameter H has a significant role in the width of solitary waves in the quantum plasma. For $H < 2$ and $H > 2$, the width may be increased or decreased depending upon the values of ion beam parameters.

Basic equations

We consider that the quantum plasma is unmagnetized and collissionless and it consists of positive ions, beam ions, and inertialess electrons. All species of the plasma are assumed to follow the Fermi–Dirac statistics. The positive ions and positive beam ions are considered to be singly ionized. Ion acoustic wave is a low-frequency wave, in which the restoring force comes from the pressure of inertialess electrons and the ion masses provide the driving force to maintain the wave. The electrons are considered inertialess by assuming that the phase velocity of ion acoustic wave is much less than the Fermi velocity of electron and much greater than the Fermi velocities of positive and beam ions. We assume that the plasma particles behave as a one-dimensional Fermi gas at zero temperature and therefore the pressure law is:

$$p_j = \frac{m_j V_{Fj}^2}{3n_{j0}^2} n_j^3 \tag{1}$$

where j = e for electrons, j = i for ions, and j = b for beam ions; m_j is the mass; $V_{Fj} = \sqrt{2k_B T_{Fj}/m_j}$ is the Fermi speed, T_{Fj} is the Fermi temperature, and k_B is the Boltzmann constant; n_j is the number density with the equilibrium value n_{j0}. $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ is the electron plasma oscillation frequency and V_{Fe} is the Fermi thermal speed of electrons.

The normalized equations governing the plasma dynamics are:

$$0 = \frac{\partial\phi}{\partial x} - n_e\frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \tag{2}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x}\right)u_i = -\frac{\partial\phi}{\partial x} - \sigma_i n_i\frac{\partial n_i}{\partial x} \tag{4}$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial(n_b u_b)}{\partial x} = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial t} + u_b\frac{\partial}{\partial x}\right)u_b = -\mu\frac{\partial\phi}{\partial x} - \mu\sigma_b n_b\frac{\partial n_b}{\partial x} \tag{6}$$

$$\frac{\partial^2\phi}{\partial x^2} = \chi n_e - n_i - (1-\chi)n_b \tag{7}$$

It is to be mentioned that the following normalization has been used in the above equations:

$$x \to \frac{x\omega_{pi}}{V_{Fi}},\ t \to t\omega_{pi},\ \varphi \to \frac{e\varphi}{2k_B T_{Fj}},\ n_j \to \frac{n_j}{n_0} \text{ and } u_j \to \frac{u_j}{V_{Fe}}$$

and $H = \hbar\omega_{pe}/2k_B T_{Fe}$ is a nondimensional quantum parameter proportional to the quantum diffraction and is equal to the ratio between the plasma energy $\hbar\omega_{pe}$ (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy $k_B T_{Fe}$; $\chi = n_{e0}/n_{i0}$ is the ratio of unperturbed electron density and ion density; $\mu = m_i/m_b$ is the ratio of positive ion and beam ion masses and $\sigma_{i,b} = T_{Fi,b}/T_{Fe}$ is the ratio of ion (beam) Fermi temperature to electron Fermi temperature.

K–dV equation

In order to investigate the behavior of IA, we make the following perturbation expansions for the field quantities n_e, n_i, n_b, u_e, u_i, u_b, and φ about their equilibrium values:

$$\begin{bmatrix} n_e \\ u_e \\ n_i \\ u_i \\ n_b \\ u_b \\ \phi \end{bmatrix} = \begin{bmatrix} n_{e0} \\ 0 \\ n_{i0} \\ 0 \\ n_{b0} \\ u_{b0} \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{e1} \\ u_{e1} \\ n_{i1} \\ u_{i1} \\ n_{b1} \\ u_{b1} \\ \phi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{e2} \\ u_{e2} \\ n_{i2} \\ u_{i2} \\ n_{b2} \\ u_{b2} \\ \phi_2 \end{bmatrix} + \ldots \tag{8}$$

where $n_{e0} = \chi$, $n_{b0} = 1 - \chi$, and $n_{i0} = 1$ which are obtained from the equilibrium condition of densities after normalization.

To get the desired K–dV equation describing the nonlinear behavior of ion acoustic waves, we use the standard reductive perturbation technique (Washimi and Taniuti, 1966). We introduce the usual stretching of the space and time variables:

$$\xi = \varepsilon^{1/2}(x - V_p t) \quad \text{and} \quad \tau = \varepsilon^{3/2} t \tag{9}$$

where V_p is the phase speed normalized by electron Fermi speed V_{Fi} and ε is a smallness parameter measuring the dispersion and nonlinear effects.

Using Eq. (9), Eqs (2)–(7) are written in terms of the stretched coordinates ξ and τ and then the perturbation expansions (8) are substituted. Solving the lowest-order equations in ε (i.e., $\varepsilon^{3/2}$), we obtain the linear long-wave phase speed V_p in terms of plasma parameters after using the boundary conditions:

$$\chi - \frac{1}{V_p^2 - \sigma_i} - \frac{(1-\chi)\mu}{[(V_p - u_{b0})^2 - \sigma_b(1-\chi)^2]} = 0 \tag{10}$$

Simplifying Eq. (10), we obtain the following equation:

$$c_4 V_p^4 - 2c_3 V_p^3 + c_2 V_p^2 + 2c_1 V_p + c_0 = 0 \tag{11}$$

where

$$\begin{aligned} c_4 &= 1 \\ c_3 &= b_2/2 \\ c_2 &= -[b_1 - b_2 - b_3 + \mu(1-\chi)]/\chi \\ c_1 &= [b_2(b_1 - 1)]/2\chi \\ c_0 &= -[b_3(b_1 + 1)]/\chi \\ b_3 &= [u_{b0}^2 - \sigma_b(1-\chi)^2] \\ b_2 &= 2u_{b0} \\ b_1 &= \sigma_i \end{aligned}$$

Equation (11) is a bi-quadratic equation in V_p and it gives four values of phase speed of the ion acoustic wave in quantum plasma (real or imaginary). The four values of V_p from the above bi-quadratic equation are obtained as follows:

$$\begin{aligned} V_p &= \frac{1}{2}[(c_3 + d_1) + \sqrt{(d_1 + c_3) - 4(d_2 + d_3)} \\ V_p &= \frac{1}{2}[(c_3 + d_1) - \sqrt{(d_1 + c_3) - 4(d_2 + d_3)} \\ V_p &= \frac{1}{2}[(c_3 - d_1) + \sqrt{(d_1 - c_3) - 4(d_2 - d_3)} \\ V_p &= \frac{1}{2}[(c_3 - d_1) - \sqrt{(d_1 - c_3) - 4(d_2 - d_3)} \end{aligned} \tag{12}$$

where

$$\begin{aligned} d_1^2 &= c_3^2 - c_2 + 2d_2 \\ d_1 d_3 &= c_1 + c_3 d_2 \\ d_3^2 &= d_2^2 - c_0 \end{aligned}$$

$$d_2 = \left[\frac{1}{2}(d_4 + \sqrt{(d_4^2} - d_5)\right]^{1/3} + \left[\frac{1}{2}(d_4 - \sqrt{(d_4^2} - d_5)\right]^{1/3} + \frac{1}{6}c_2$$

$$\begin{aligned} d_4 &= \frac{1}{6}c_2(c_0 + c_1 c_3) + \frac{1}{108}c_2^3 - \frac{1}{2}(c_0 c_2 - c_0 c_3^2 - c_1^2) \\ d_5 &= \frac{4}{729}\left[3(c_0 + c_1 c_3) + \frac{1}{4}c_2^2\right]^2 \end{aligned}$$

From Eq. (12), the phase speed of IAW in quantum plasma in the presence of ion beam can be studied analytically and numerically, and the effects of plasma parameters on the phase speed would be understood.

Now, to derive the K–dV equation, we use the stretching coordinates given by Eq. (9). Taking the terms of $\epsilon^{5/2}$, we obtain from Eqs (2)–(7),

$$0 = \frac{\partial\varphi_2}{\partial\xi} - n_{e0}\frac{\partial n_{e2}}{\partial\xi} - n_{e1}\frac{\partial n_{e1}}{\partial\xi} + \frac{H^2}{4}\frac{\partial^3 n_e^{(1)}}{\partial\xi^3} \tag{13}$$

$$\frac{\partial n_{i1}}{\partial\tau} - V_p\frac{\partial n_{i2}}{\partial\xi} + n_{i0}\frac{\partial u_{i2}}{\partial\xi} + \frac{\partial}{\partial\xi}(n_{i1}u_{i1}) = 0 \tag{14}$$

$$\frac{\partial u_{i1}}{\partial \tau} - V_p \frac{\partial u_{i2}}{\partial \xi} + u_{i1}\frac{\partial u_{i1}}{\partial \xi} = -\frac{\partial \varphi_2}{\partial \xi} - \sigma_i n_{i0}\frac{\partial n_{i2}}{\partial \xi} - \sigma_i n_{i1}\frac{\partial n_{i1}}{\partial \xi} \tag{15}$$

$$\frac{\partial n_{b1}}{\partial \tau} - (V_p - u_{b0})\frac{\partial n_{i2}}{\partial \xi} + n_{b0}\frac{\partial u_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{b1}u_{b1}) = 0 \tag{16}$$

$$\frac{\partial u_{b1}}{\partial \tau} - (V_p - u_{b0})\frac{\partial u_{b2}}{\partial \xi} + u_{b1}\frac{\partial u_{b1}}{\partial \xi} = -\mu\frac{\partial \varphi_2}{\partial \xi} - \sigma_b \mu n_{b0}\frac{\partial n_{b2}}{\partial \xi} - \sigma_b \mu n_{b1}\frac{\partial n_{b1}}{\partial \xi} \tag{17}$$

$$\frac{\partial^2 \varphi_1}{\partial \xi^2} = \chi n_{e2} - n_{i2} - (1-\chi)n_{b2} \tag{18}$$

Differentiating Eq. (18) with respect to ξ once more, we get

$$\frac{\partial^3 \varphi_1}{\partial \xi^3} = \chi\frac{\partial n_{e2}}{\partial \xi} - \frac{\partial n_{i2}}{\partial \xi} - (1-\chi)\frac{\partial n_{b2}}{\partial \xi} \tag{19}$$

Putting the first-order values of n_{e1}, n_{i1}, n_{b1}, u_{i1}, u_{b1} in terms of ϕ_1 in Eqs (13)–(18) and then eliminating n_{e2}, n_{i2}, n_{b2}, u_{e2}, u_{i2}, u_{b2}, ϕ_2, the following K–dV equation in quantum plasma having ion beams has been obtained from (19):

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{20}$$

where

$$A = \frac{\chi\mu(\sigma_i - V_p^2)(V_p^2 - \sigma_i)^2(V_b^2 - \mu\sigma_b)^2}{(V_p^2 - \sigma_i)(V_b^2 - \sigma_b)[2\chi\mu V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2 - \sigma_i) - 2\mu V_p(V_b^2 - \mu\sigma_b)]} - \frac{\mu(\sigma_i + 3V_p^2)(V_b^2 - \mu^2\sigma_b) + \chi\mu^2(\mu\sigma_b + 3V_b^2)(\sigma_i - V_p^2 - (1/\chi))(V_p^2 - \sigma_i)^2}{(V_p^2 - \sigma_i)(V_b^2 - \mu\sigma_b)[2\chi\mu V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2 - \sigma_i) - 2\mu V_p(V_b^2 - \mu\sigma_b)]} \tag{20a}$$

$$B = \frac{(H^2 - 4)(V_p^2 - \sigma_i)^2(V_b^2 - \mu\sigma_b)}{4[2\chi V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2 - \sigma_i) - 2V_p(V_b^2 - \mu\sigma_b)} \tag{20b}$$

where $V_b = V_p - u_{b0}$ and the parameters μ, σ_i, σ_b, χ, u_{b0}, H, and V_p are defined earlier.

It is known that the solitary wave structure is formed due to the balance between the dispersive effect and the nonlinear effect. The relative strength of these two effects determines the characteristic of such solitary wave structure. The coefficients A of the nonlinear term and coefficient B of the dispersion term thus play a crucial role in determining the solitary wave structure.

To find the steady-state solution of the K–dV equation [Eq. (20)], the independent variables ξ and τ are transformed into one variable $\eta = \xi - U\tau$ where U is the normalized constant speed of the wave frame. Applying the boundary conditions: as $\eta \to \pm\infty$; (i) $\varphi_1 \to 0$, (ii) $(d\varphi_1/d\eta) \to 0$, (iii) $(d^2\varphi_1/d\eta^2) \to 0$, the solution is obtained as

$$\varphi_1 = \varphi_0 \sec h^2\left(\frac{\eta}{\Delta}\right) \tag{21}$$

where the amplitude φ_0 and width Δ of the solitary structure are given by:

$$\varphi_0 = \frac{3U}{A}, \quad \Delta = \sqrt{\frac{4B}{U}} \tag{22}$$

For the positive value of A, the solitary wave will be compressive and the negative value of A will generate the rarefactive solitary wave. Since A is independent of H, the amplitude of the solitary wave will remain the same for any value of H. On the other hand, the width of the solitary wave depends on H and other plasma parameters σ_i, σ_b, n_{b0}, u_{b0}, μ.

Results and discussions

We have numerically analyzed the behavior of ion acoustic solitary waves in quantum plasma in the presence of an ion beam from the K–dV equation [Eq. (20)] and its solution is given by Eq. (21). It is seen that the structure of solitary waves depends on the nonlinear coefficient (A) and the width depends on dispersion coefficient (B). The characteristics of solitary waves depend on the various plasma parameters, namely, velocity (u_{b0}), concentration (n_{b0}), and temperature (σ_b) of the ion beam. For the numerical estimation we have considered a model quantum plasma and have used the value of quantum diffraction (H) from Hass *et al.* (2003) and Chandra *et al.* (2013); and for ion beam velocity from Kaur *et al.* (2017). The results obtained from numerical estimation for the variations of A and B, the structure and width of solitary waves are given below.

Variation of the coefficients of the nonlinear term and the dispersive term

It is seen from the K–dV equation [Eq. (20)] that the coefficient A of the nonlinear term depends on the plasma parameters, that is μ, σ_i, σ_b, χ, u_{b0} but diffraction parameter H has no effect on A; whereas the coefficient B of dispersive term depends on the plasma parameters including quantum diffraction parameter H. Thus, the nonlinear and dispersion coefficients get modified by the ion beam, quantum diffraction effect, and the quantum statistical effect through electron–ion Fermi temperatures. The nonlinear coefficient A has been numerically estimated for a model quantum plasma for plasma parameters related to ion beam and is graphically shown in Figure 1a–1c.

Figure 1a shows the variation in the nonlinear coefficient (A) with ion beam temperature (σ_b) for different values of ion beam density (n_{b0}) in a quantum plasma having fixed values of other parameters as $\sigma_i = 0.01$, $u_{b0} = 0.5$, $H = 0.1$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam).

It is seen that the nonlinear coefficient (A) becomes negative or positive depending on the values of ion beam temperature. The negative values of A indicate that the solitary waves are rarefactive and, for positive values of A, the solitary waves will be compressive. The nonlinear coefficient A decreases up to the certain value of ion beam temperature ($\sigma_b \approx 1.2$) and then it is suddenly increased to a maximum positive value and finally it decreases slowly with the increase of ion beam temperature keeping a positive value.

Similarly, the variation of A with ion beam velocity (u_{b0}) for different values of velocity ion beam density (n_{b0}) is shown in Figure 1b in a quantum plasma with fixed values of other parameters as $H = 0.1$. $V_p = 1.6$, $\sigma_i = 0.1$, $\sigma_b = 0.1$, $\mu = 1$.

It is seen from Figure 1b that the nonlinear coefficient (A) becomes negative or positive depending on the values of ion beam velocity. Therefore, the solitary waves will be rarefactive (compressive) when $A < 0$ ($A > 0$). It is interesting to see that the nonlinear coefficient A decreases up to the certain value of ion beam velocity ($u_{b0} \approx 1.3$) and then it is suddenly increased to a maximum positive value and finally it starts to decrease with the increase of the ion beam velocity keeping a positive value. Moreover, the nonlinear term is seen to increase with the increase of the ion beam density (n_{b0}).

The variation in the nonlinear coefficient (A) with ion beam velocity (u_{b0}) for different ion beam temperatures (σ_b) can be understood from Figure 1c for a quantum plasma having fixed values of $\sigma_i = 0.01$, $n_{b0} = 0.5$, $H = 0.1$, $V_p = 1.6$, $\mu = 1$.

It is observed that the nonlinear coefficient A becomes positive or negative depending upon the values of ion beam velocity and ion beam temperature. The positive values of A indicate that the solitary wave will be compressive and the negative value of A means the rarefactive solitary will be excited.

Similarly, the dispersion coefficient (B) is numerically estimated for a quantum plasma and its variation with the quantum diffraction parameter H and the parameters related to the ion beam are graphically shown in Figure 2a–2c.

Figure 2(a) shows the variation of B with ion beam velocity (u_{b0}) and H in a quantum plasma having fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is seen that B is significantly affected by H. The coefficient B is negative and it increases with ion beam velocity for $H < 2$ and $B = 0$ for $H = 2$. But, B is positive and it decreases with ion beam velocity for $H > 2$.

In Figure 2b, the variation of B with ion beam temperature for different values of H is shown for a quantum plasma having fixed values of $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

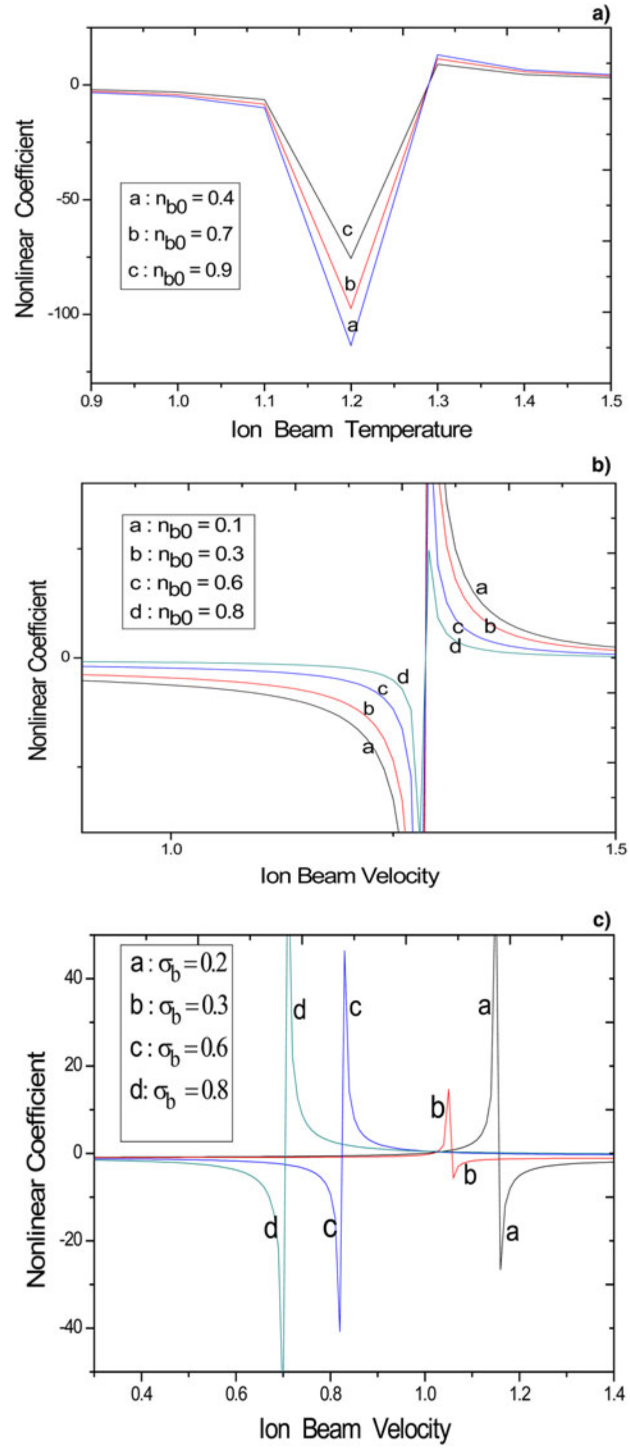

Fig. 1. (a) Variation of Nonlinear coefficient with ion beam temperature (σ_b) for different value of ion beam density (n_{b0}) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam). (b) Variation of nonlinear coefficient with ion beam velocity (u_{b0}) for different value of ion beam density (n_{b0}) in quantum plasma with fixed values of plasma parameters $V_p = 1.6$, $\sigma_i = 0.1$, $\sigma_b = 0.1$, $\mu = 1$ (H^+ ion beam). (c) Variation of nonlinear coefficient with ion beam velocity (u_{b0}) for different value of ion beam temperature (σ_b) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam).

In this case also, B is negative and it increases with ion beam temperature for $H < 2$ and $B = 0$ for $H = 2$. But, B becomes positive and it decreases with ion beam temperature for $H > 2$.

The variation of B with n_{b0} and H is shown in Figure 2c in a quantum plasma having fixed values of $\sigma_i = 0.01$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen that B is negative for $H < 2$ and it increases with the increase of n_{b0}. The coefficient $B = 0$ when $H = 2$. For $H > 2$, the diffraction coefficient B is positive and it decreases with the increase of n_{b0}.

Ion acoustic solitary waves

To understand the nature of ion acoustic solitary waves, the profiles of solitary waves are drawn in the quantum plasma in the presence of an ion beam for different values of velocity, density, and temperature of the beam. In Figure 3a, the structure of solitary waves is shown with a variation of u_{b0} in the quantum plasma having parameters (dimensionless) $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is observed that both compressive and rarefactive solitary waves may be excited in the quantum plasma depending upon the values of u_{b0}. For $u_{b0} = 0.1 - 1.2$, the solitary wave will be rarefactive and the amplitude will decrease with the increase of u_{b0}, but for $u_{b0} = 1.3$ and 1.4, the compressive solitary wave will be excited and the amplitude will increase with the increase of u_{b0}.

The effect of ion beam density on solitary waves is shown in Figure 3b in the quantum plasma with fixed values of $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen that the values of n_{b0} decide to excite the compressive or rarefactive solitary waves. For low values of n_{b0} (=0.1, 0.2, and 0.3), the solitary waves are compressive and the amplitudes are increased with the increase of n_{b0}, but for large values of n_{b0} (0.7, 0.8), the solitary wave will be rarefactive in nature and the amplitudes are decreased with the increase of n_{b0}.

In Figure 3c, the structure of solitary waves for different values of ion beam temperature (σ_b) is shown for a quantum plasma having fixed values of $\sigma_i = 0.01$, $n_{b0} = 0.5$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is seen that both compressive and rarefactive solitary waves may be excited in the quantum plasma and these are dependent on the values of σ_b. For $\sigma_b = 0.0$, 0.2, 0.25, the solitary waves are compressive and the amplitude increases with the increase of σ_b. But when $\sigma_b = 0.3$ and 0.35, the rarefactive solitary waves are excited and the amplitudes are decreased with the increase of σ_b.

Since the width is very important for the study of solitary waves in quantum plasma, we have drawn the profiles of width for different values of ion beam parameters. From Eq. (22), we see the width of solitary depends on H and other plasma parameters σ_i, σ_b, n_{b0}, u_{b0}.

In Figure 4a, the variation in width with σ_b for different values of H is plotted for a quantum plasma having fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen from Figure 4a that the width of solitary waves decreases with the increase of ion beam temperature and it becomes zero at $\sigma_b \approx 0.58$. Moreover, the widths are decreased with the increase of H when $H < 2$. But the widths are increased with the increase of H when $H > 2$.

In Figure 4b, the variation of widths with ion beam velocity (u_{b0}) for different values of H are depicted for a quantum plasma with fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is observed from Figure 4b that the width decreases with the increase of ion beam velocity and it becomes zero at $u_{b0} \approx 1.3$. When $u_{b0} < 1.3$, the width decreases with the increase of H when $H < 2$ and it increases with the increase of H when $H > 2$. But, when $u_{b0} > 1.3$, the width increases with the increase of H.

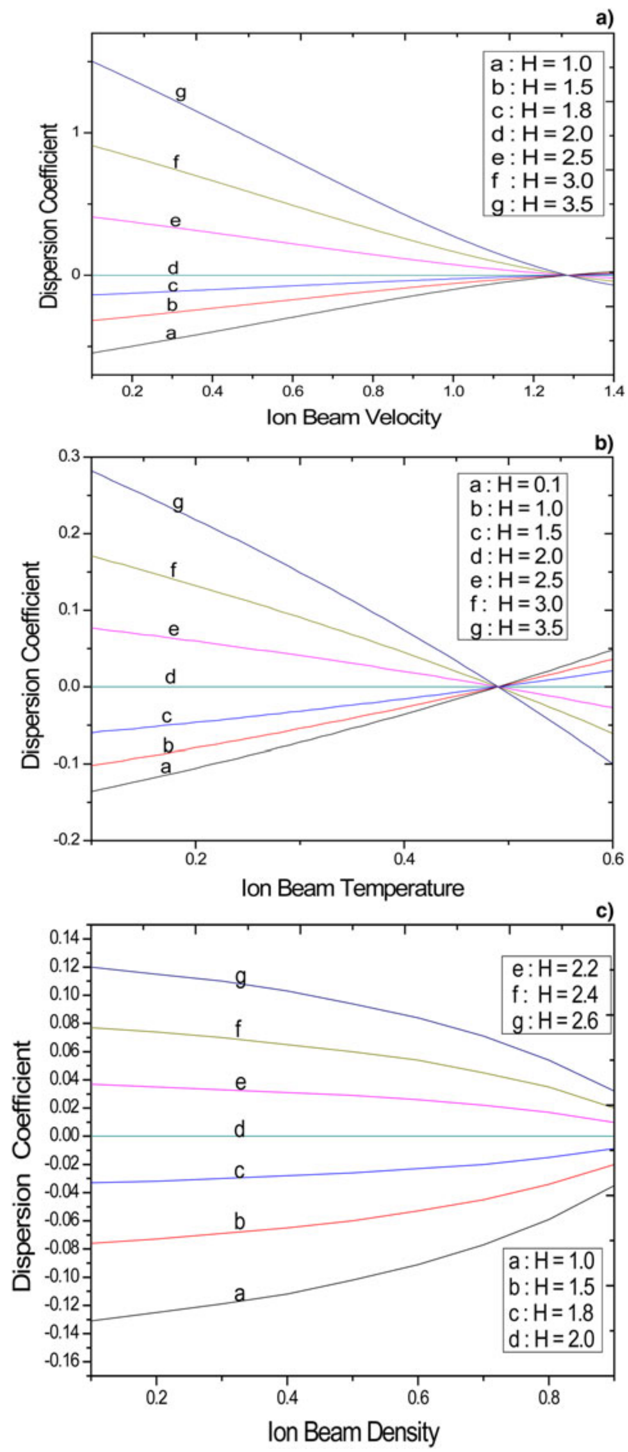

Fig. 2. (a) Variation of dispersion coefficient with ion beam velocity (u_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam). (b) Variation of the dispersion coefficient with ion beam temperature (σ_b) and quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (c) Variation of the dispersion coefficient with ion beam density (n_{b0}) and quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters having $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1.4$ (H^+ ion beam).

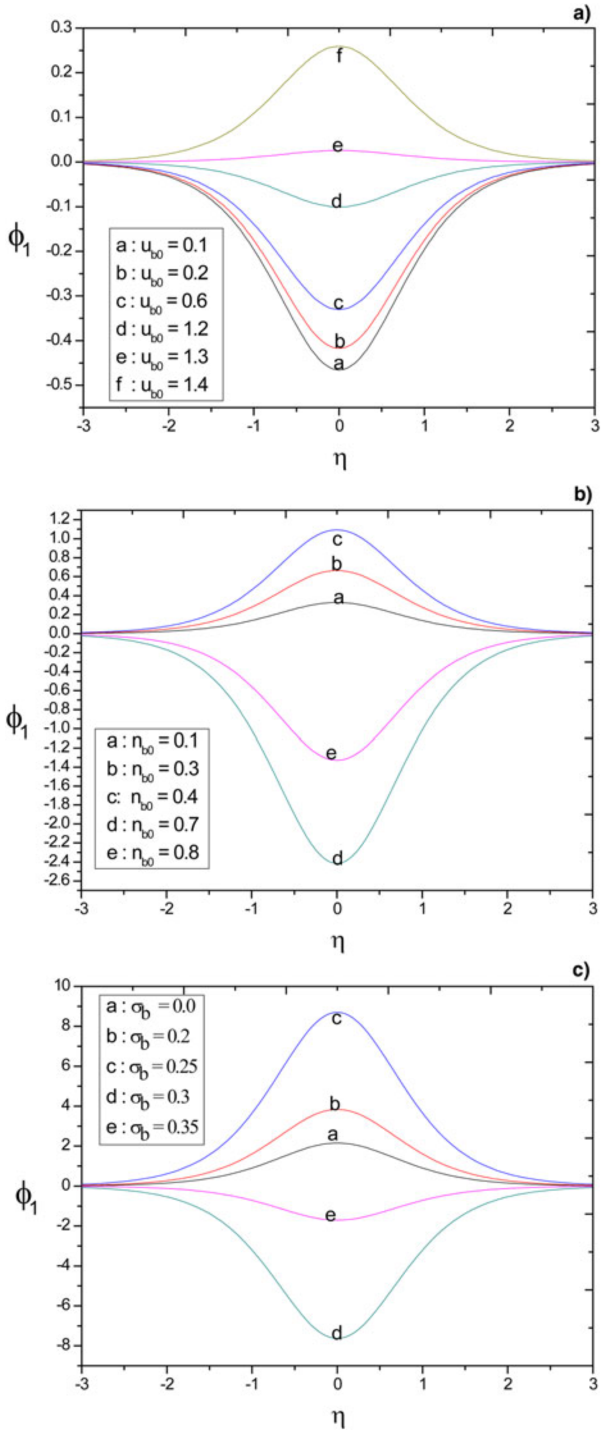

Fig. 3. (a) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam velocity (u_{b0}) with fixed values of plasma parameters $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam), $U = 0.1$. (b) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam density (n_{b0}) with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam), $U = 0.1$. (c) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam temperature (σ_b) with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam), $U = 0.1$.

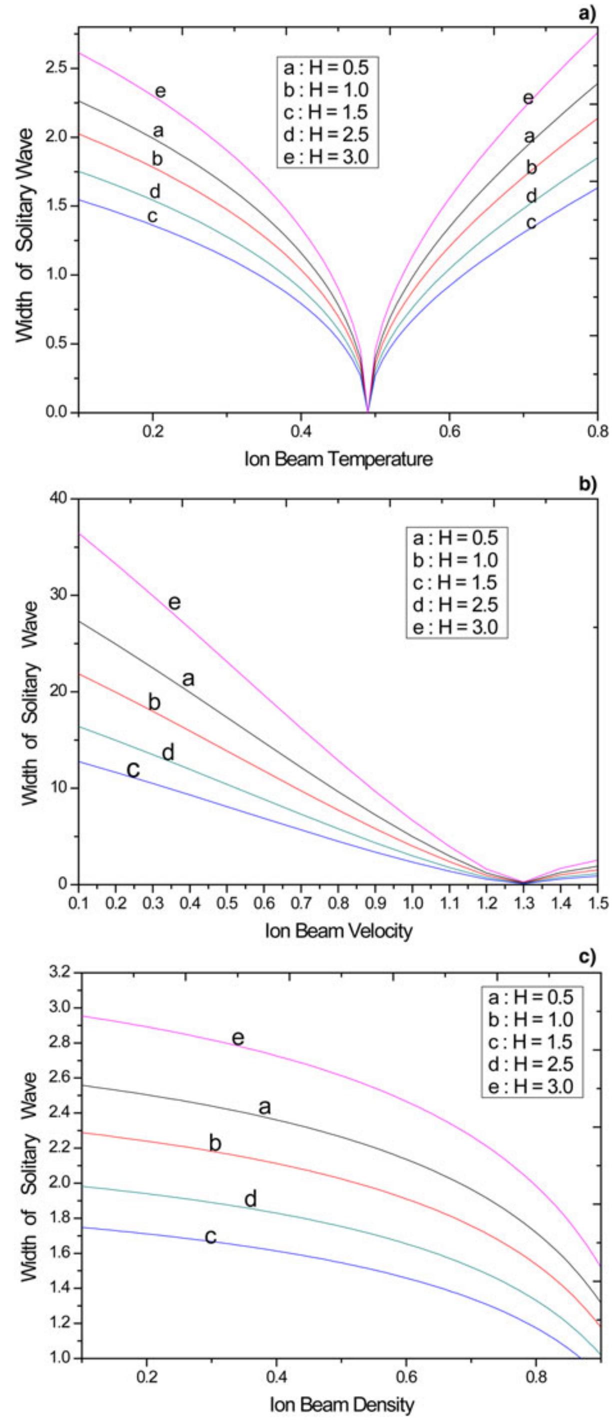

Fig. 4. (a) Variation of width of solitary wave with ion beam temperature (σ_b) and quantum diffraction (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $n_{b0} = 0.5$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (b) Variation of width of solitary waves with ion beam velocity (u_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (c) Variation of width of solitary waves with ion beam density (n_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam).

The variation in the width of solitary waves with ion beam density (n_{b0}) for different values of quantum diffraction parameter (H) is shown in Figure 4c with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

Figure 4c shows that the width of solitary waves decreases with the increase of ion beam density. Moreover, the width decreases with increase of H when $H < 2$ and it increases with the increase of H when $H > 2$.

It is important to point out that the solitary wave will not exist when the coefficient of the nonlinear term of the K–dV equation [Eq. (20)] vanishes for some critical values of the plasma parameters μ, σ_i, σ_b, χ, u_{b0}, and *H*. Assuming $A \approx 0$, we obtain

$$\begin{aligned} &\chi[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)] \\ &\quad = (\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2) \end{aligned} \tag{23}$$

From Eq. (23), the critical values of the plasma parameters can be obtained for the nonexistence of the solitary waves in the quantum plasma. The critical value of ion beam density (N_{bc}) is

$$N_{bc} = 1 - \frac{[(\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2)]}{[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)]} \tag{24}$$

Again, for some critical values *H*, V_p, and V_b, the solitary wave will be non-dispersive if the coefficient $B \approx 0$ in K–dV equation [Eq. (20)]. In this case, $H = 2$, or $V_p = \sqrt{\sigma_i}$, or $V_b = \sqrt{\mu\sigma_b}$.

The variation of critical ion beam density (N_{bc}) with ion beam velocity and ion beam temperature is shown in Figure 5.

It is observed that N_{bc} does not depend on *H* and increases slowly with the increase of u_{b0} and after a certain value of u_{b0}, it is suddenly increased to attain a maximum value. Then, N_{bc} decreases rapidly with the increase of u_{b0}. Moreover, N_{bc} is small for large values of u_{b0}.

But, the solitary waves near the critical density of ion beam in the quantum plasma can be obtained through the derivation of a modified K–dV equation using different stretching coordinates

$$\xi = (x - V_p t) \text{ and } \tau = \varepsilon^3 t \tag{25}$$

The solution of the modified K–dV equation near critical ion beam density will be represented by Paul *et al.* (1996) and Mondal *et al.* (1998)

$$\varphi_{1c} = \varphi_{0c} \sec h(\eta/\Delta') \tag{26}$$

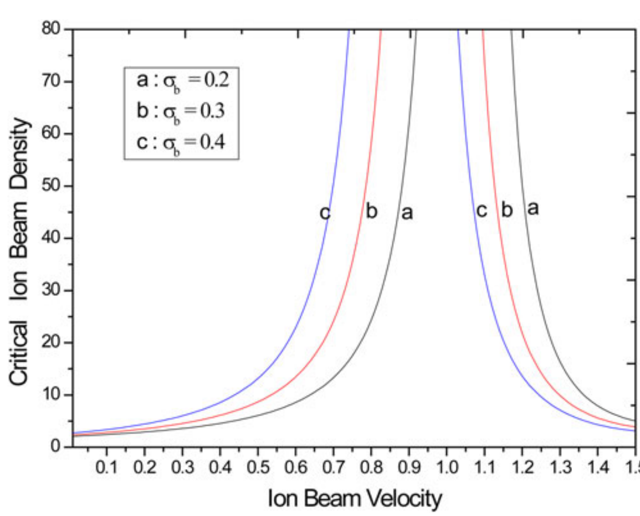

Fig. 5. Variation of critical value of ion beam density with ion velocity (n_{b0}) for different ion beam temperature (σ_b) in quantum plasma with fixed values of plasma parameters $V_p = 1.6$, $\sigma_i = 0.1$, $\mu = 1$ (H^+ ion beam).

where φ_{0c} and Δ′ are the amplitude and width of solitary wave near critical ion beam density. Moreover, double-layer (shock-like) solution of ion acoustic wave in the quantum plasma near critical ion beam density may be obtained from the nonlinear equation combining the K–dV equation and the modified K–dV equation (Paul *et al.*, 1996; Mondal *et al.*, 1998; Kaur *et al.*, 2017),

$$\varphi_d = \frac{\varphi_{0d}}{2}[1 - \tanh(\eta/\Delta'')] \tag{27}$$

where φ_{0d} and Δ″ are the amplitude and the width of DLs.

Summary and conclusions

In this paper, we have used the one-dimensional QHD model to investigate ion acoustic solitary waves in a dense quantum plasma in the presence of ion beam. For the study of the nonlinear behavior of ion acoustic waves, the K–dV equation has been derived by using the reductive perturbation technique. It is seen that the formation and structure of solitary waves are significantly affected by the presence of ion beam in quantum plasma. Ion acoustic solitary waves in the quantum plasma may be compressive or rarefactive depending upon the values of ion beam parameters, that is velocity, density, and temperature of the ion beam. Our results may be useful for understanding the beam–plasma interactions and the formation of nonlinear wave structures in dense quantum plasma. The model includes a quantum statistical effect through the equation of state and quantum diffraction effect through the parameter *H*. It is known that quantum effects in plasma become important when thermal de Broglie wavelength becomes much larger than the average interparticle distance. In most practical situations, quantum effects for the ions may be neglected because of their heavier mass than the electrons.

To the best of our knowledge, no work on the ion acoustic solitary waves in degenerate plasma in the presence of ion beam has been reported till now. In our analysis, the explicit form of solitary waves in the quantum plasma is obtained from the K–dV equation and the structure of solitary waves is graphically discussed for different values of density, velocity, and temperature of the ion beam.

Our main findings are:

i) Both compressive and rarefactive solitary waves may be excited in the quantum plasma depending upon the values of ion beam parameters but the quantum diffraction has no role in it.
ii) Compressive solitary wave will be excited for ion beam velocity $u_{b0} = 1.3$ and 1.4, and amplitude will increase with the increase of u_{b0}; but the solitary wave will be rarefactive for ion beam velocity $u_{b0} = 0.1$–1.2 and the amplitude will decrease with the increase of u_{b0}.
iii) For low values of ion beam density ($n_{b0} = 0.1$, 0.2, and 0.3), the solitary waves will be compressive and the amplitudes are increased with the increase of beam density; but for large values of n_{b0} (0.7, 0.8), the solitary wave will be rarefactive and amplitude decreases with the increase of beam density.
iv) Compressive solitary waves will be excited for ion beam temperature $\sigma_b = 0.0$, 0.2, 0.25 and the amplitude increases with the increase of σ_b; but for $\sigma_b = 0.3$ and 0.35, the solitary waves will be rarefactive in nature and amplitude decreases with the increase of σ_b.

v) For the numerical estimation of A, B, and the electrostatic potential of solitary waves, we have considered H^+ ion beam and the value of $\mu = 1(\mu = m_i/m_b)$. For He^+ beam ($\mu = 1/4$) and Ar^+ beam ($\mu = 1/40$), numerical estimation can be made and in these cases, the structure of solitary will be different. From numerical estimation and graphical representation of solitary waves, it is seen that the solitary waves will be always rarefactive.

vi) The quantum diffraction parameter H plays important roles in the width of solitary waves in the quantum plasma. For $H < 2$ and $H > 2$, the width may be increased or decreased depending upon the values of ion beam parameters.

However, with reference to laboratory and space plasma, interesting results would be obtained on the propagation of electrostatic waves in a bounded system like plasma-filled waveguides. The boundaries in such a system may add additional effects. The wave with no dispersion in an unbounded system may become dispersive in the bounded system; and the stable wave of an unbounded system may become unstable in bounded geometry (Ghosh *et al*., 2014; Paul *et al*., 2016). Moreover, relativistic effect in plasma is important in many practical situations, for example, in space–plasma phenomena (Vette, 1970), the plasma sheet boundary of Earth's magnetosphere (Grabbe, 1989), Van Allen radiation belts (Shprits *et al*., 2013), and laser–plasma interaction experiments (Marklund and Shukla, 2006). Following the works of Das and Paul (1985) in nondegenerate plasma, ion acoustic solitary waves in degenerate (quantum) plasma have been studied by Sahu (2011). Assuming the relativistically degenerate plasma, the nonlinear propagation of electrostatic solitary has been studied by Mamun and Shukla (2010) and others (Akbari-Moghanjoughi, 2011; Masood and Eliasson, 2011). Later, Chandra *et al.* (2013) have considered two-temperature electrons in a relativistically degenerate plasma and have theoretically investigated the electro-acoustic solitary waves. It has been shown that the relativistic degeneracy parameter significantly influences the conditions of the formation and properties of solitary waves. Recently, Hossen and Mamun (2014) have studied theoretically and numerically the nonlinear propagation of modified EAWs in an unmagnetized, collisionless, relativistic degenerate quantum plasma through the derivation of the nonplanar K–dV equation which admits a wave solution for the solitary wave profile. More recently, Paul *et al.* (2019) have considered an ultra-relativistic degenerate dense electron–ion–positron plasma for the study of envelope solitary waves and rogue waves and have obtained significant results which can be further studied in the presence of an electron beam or ion beam following the concept of the present paper.

Some works on the nonlinear propagation of ion and electron acoustic waves in a fully relativistic quantum plasma with ion and electron beams are in progress and these will be reported later.

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References

Abrol PS and Tagare SG (1979) Ion-acoustic solitary waves in an ion-beam-plasma system with nonisothermal electrons. *Physics Letters A* **75**, 74–76.

Agrawal G (1989) *Nonlinear Fiber Optic*. San Diego, CA: Academic, Chaps. 2–5.4.

Akbari-Moghanjoughi M (2010) Effects of ion-temperature on propagation of the large-amplitude ion-acoustic solitons in degenerate electron-positron-ion plasmas. *Physics of Plasmas* **17**, 082315.

Akbari-Moghanjoughi M (2011) Propagation of arbitrary-amplitude ion waves in relativistically degenerate electron-ion plasmas. *Astrophysics and Space Science* **332**, 187–192.

Ali S and Shukla PK (2006) Dust acoustic solitary waves in a quantum plasma. *Phys. Plasmas* **13**, 129902.

Ali S, Moslem WM, Shukla PK and Schlickeiser R (2007) Linear and nonlinear ion-acoustic waves in an unmagnetized electron-positron-ion quantum plasma. *Physics of Plasmas* **14**, 082307.

Ang LK, Kwan TT and Lau YY (2003) New scaling of child-langmuir law in the quantum regime. *Physical Review Letters* **91**, 208303.

Azechi H (2006) FIREX project. *Plasma Physics and Controlled Fusion* **48B**, 267.

Bailung H, Sharma SK and Nakamura Y (2010) Effect of ion beam on the propagation of rarefactive solitons in multicomponent plasma with negative ions. *Physics of Plasmas* **17**, 062103.

Bala P, Gill TS, Bains AS and Kaur H (2017) Ion-acoustic dressed solitary structures in two component plasma with Tsallis-nonthermal velocity distribution of electrons. *Indian Journal of Physics* **91**, 1625–1634.

Baluku TK and Hellberg MA (2012) Ion acoustic solitons in a plasma with two-temperature kappa-distributed electrons. *Physics of Plasmas* **19**, 012106.

Barnes WL, Dereux A and Ebbesen TW (2003) Surface plasmon sub wavelength optics. *Nature* **424**, 824–830.

Chandra S, Paul SN and Ghosh B (2012) Linear and non-linear propagation of electron plasma waves in quantum plasma. *Indian Journal of Pure & Applied Physics* **50**, 314–319.

Chandra S, Paul SN and Ghosh B (2013) Electro-acoustic solitary waves in relativistically degenerate quantum plasma with two-temperature electrons. *Astrophysics and Space Science* **343**, 213–219.

Chattopadhyaya S and Paul SN (2012) Existence of compressive and rarefactive solitary waves in presence of cold positive and negative ions with drift for single temperature plasma. *The African Review of Physics* **7**, 003289.

Danehkar A (2018) Electron beam-plasma interaction and electro-acoustic solitary waves in a plasma with suprathermal electrons. *Plasma Physics and Controlled Fusion* **60**, 065010.

Das R (2012) Effect of ion temperature on small-amplitude ion acoustic solitons in a magnetized ion-beam plasma in presence of electron inertia. *Astrophysics and Space Science* **341**, 543–549.

Das R and Deka P (2015) Korteweg-de Vries solutions in high relativistic electron beam plasma. *International Journal of Engineering Research* **6**, 864–870.

Das GC and Paul SN (1985) Ion-acoustic solitary waves in relativistic plasmas. *Physics of Fluids* **28**, 823–825.

Das GC and Singh KI (1991) Spherical solitons in ion-beam plasma. *Contributions to Plasma Physics* **31**, 15–25.

Das GC, Singh KI and Karmakar B (2011) Solitons in ion-beam plasmas bounded by cylindrical geometry. *Canadian Journal of Physics* **67**, 609–613.

Dip PR, Hossen MA, Salahuddin M and Mamun AA (2017) Electro-acoustic solitary waves and double layers in a quantum plasma. *European Physical Journal D* **71**, 52.

El-Labany SK (1995) Propagation of ion-acoustic solitons in a warm ion beam plasma system. *Journal of Plasma Physics* **54**, 285–293.

Gell Y and Roth I (1977) The effects of an ion beam on the motion of solitons in an ion beam-plasma system. *Plasma Physics* **19**, 915.

Ghosh KK, Paul B, Das C and Paul SN (2008) Analytical study of the ion-acoustic solitary waves in a drift plasma consisting of two-temperature electrons. *Journal of Physics A: Mathematical and Theoretical* **41**, 335501.

Ghosh B, Paul SN and Banerjee S (2014) Instability of electron plasma waves in an electron-hole bounded quantum dusty plasma, world academy of science, engineering and technology. *International Journal of Mathematical, Computational, Physical and Quantum Engineering* **8**, 954.

Gill TS, Bala P, Kaur H, Saini NS, Bansal S and Kaur J (2004) Ion-acoustic solitons and double-layers in a plasma consisting of positive and negative ions with non-thermal electrons. *European Physical Journal D* **31**, 91–100.

Goldman MV (1983) Progress and problems in the theory of type III solar radio emission. *Solar Physics* **89**, 403–442.

Grabbe C (1989) Wave propagation effects of broadband electrostatic noise in the magneto tail. *Journal of Geophysical Research* **94**, 17299.

Haas H, Garcia LG and Goedert J (2003) Quantum ion-acoustic waves. *Physics of Plasmas* **10**, 3858.

Hoffmann RA and Evans DS (1968) Field-aligned electron bursts at high latitudes observed by OGO 4. *Journal of Geophysical Research* **73**, 6201–6214.

Hossen MR and Mamun AA (2014) Modeling of modified electro-acoustic solitary waves in a relativistic degenerate plasma. *Journal of the Korean Physical Society* **65**, 2045–2052.

Huibin L and Kelin W (2009) Solitons in an ion-beam plasma. *Journal of Plasma Physics* **44**, 151–165.

Ikezi H (1973) Experiments on ion acoustic solitary waves. *Physics of Fluids* **16**, 1668.

Karmakar B, Das GC and Singh I (1988) Ion-acoustic solitary waves in ion-beam plasma with multiple-electron-temperatures. *Plasma Physics and Controlled Fusion* **30**, 1167.

Kaur N, Singh K and Saini NS (2017) Effect of ion beam on the characteristics of ion acoustic Gardner solitons and double layers in a multicomponent superthermal plasma. *Physics of Plasmas* **24**, 092108.

Kaur N, Singh K, Ghai Y and Saini NS (2018) Nonplanar dust acoustic solitary and rogue waves in an ion beam plasma with superthermal electrons and ions. *Plasma Science and Technology* **20**, 074009.

Killian TC (2007) Ultra cold neutral plasmas. *Science* **316**, 705–8.

Kremp D, Bornath T, Bonitz M and Schlanges M (1999) Quantum kinetic theory of plasmas in strong laser fields. *Physical Review E* **60**, 4725.

Krivoruchko SM, Fainberg VB, Shapiro VD and Shevchenko VI (1975) Solitary charge density waves in a magnetoactive plasma. *Soviet JETP* **40**, 1039–1043.

Lonngren KE (1983) Soliton experiment in plasma. *Plasma Physics* **25**, 943.

Madelung E (1927) Quantentheorie in hydrodynamischer Form. *Zeitschrift für Physik* **40**, 322–326.

Mamun AA and Shukla PK (2010) Solitary waves in an ultra relativistic degenerate dense plasma. *Physics of Plasmas* **17**, 104504.

Manfredi G, Hervieux PA, Yin Y and Crouseilles N (2009) Collective electron dynamics in metallic and semiconductor nanostructures. In Massobrio C, Bu-lou H and Goyenex C (eds), *Advances in the Atomic-Scale Modeling of Nanosystems and Nanostructured Materials*. Heidelberg: Springer, p. 1.

Marklund M and Shukla PK (2006) Nonlinear collective effects in photon-photon and photon-plasma interactions. *Reviews of Modern Physics* **78**, 591–640.

Masood W and Eliasson B (2011) Electrostatic solitary waves in a quantum plasma with relativistically degenerate electrons. *Physics of Plasmas* **18**, 034503.

Misra AP and Adhikary NC (2011) Large amplitude solitary waves in ion-beam plasmas with charged dust impurities. *Physics of Plasmas* **18**, 122112.

Misra AP and Bhowmik C (2007) Nonplanar ion-acoustic waves in a quantum plasma. *Physics Letters A* **369**, 90.

Mondal KK, Paul SN and Roychowdhury A (1998) Ion-acoustic solitons and shocks in negative ion-beam relativistic plasma. *IEEE Transactions on Plasma Sciences* **26**, 987–994.

Mushtaq A and Khan SA (2007) Ion acoustic solitary wave with weakly transverse perturbations in quantum electron-positron-ion plasma. *Physics of Plasmas* **14**, 052307.

Paul SN, Mondal KK and Roychowdhury A (1996) Effects of negative ions on the ion acoustic solitons and double layers in a relativistic plasma. *Fizika B* **5**, 73–84.

Paul SN, Das C, Paul I, Bandyopadhyay B, Chattopadhyaya S and De SS (2012) Ion acoustic solitary waves in an electron-ion-positron plasma. *Indian Journal of Physics* **86**, 545–553.

Paul I, Bhattacharya SK, Paul SN and Chatterjee A (2016) Instability of ion-acoustic wave in a quantum dusty plasma bounded in finite geometry. *Journal of Physical and Chemical Sciences* **4**, 1–4.

Paul SN, Chatterjee A and Paul I (2017) Nonlinear propagation of ion-acoustic waves in self-gravitating dusty plasma consisting of non-isothermal two-temperature electrons. *Indian Journal of Physics* **91**, 101–107.

Paul SN, Roychowdury A and Paul I (2019) Modulation instability of bright envelope soliton and rogue waves in ultra-relativistic degenerate dense electron-ion-positron plasma. *Plasma Physics Reports* **45**, 1011–1025.

Saberian E, Esfandyari-Kalejahi A, Rastkar-Ebrahimzadeh A and Afsari-Ghazi M (2013) Propagation of ion-acoustic solitons in an electron beam-superthermal plasma system with finite ion-temperature: Linear and fully nonlinear investigation. *Physics of Plasmas* **20**, 032307.

Sah OP and Manta J (2009) Nonlinear electro-acoustic waves in quantum plasma. *Physics of Plasmas* **16**, 032304.

Sahu B (2011) Quantum ion-acoustic solitary waves in weak relativistic plasma. *Pramana* **76**, 933–944.

Schamel H (1973) A modified Korteweg-de Vries equation for ion acoustic wavess due to resonant electrons. *Journal of Plasma Physics* **9**, 377–387.

Shapiro SD and Teukolsky SA (1983) *Black Holes, White Dwarfs and Neutron Stars*. New York, NY: John Wiley & Sons.

Shprits YY, Subbotin D, Drozdov A and Usanova M (2013) Unusual stable trapping of the ultra relativistic electrons in the Van Allen radiation belts. *Nature Physics* **9**, 699–703.

Tagare SG (1973) Effect of ion temperature on propagation of ion-acoustic solitary waves of small amplitudes in collisionless plasma. *Plasma Physics* **15**, 1247.

Vette JI (1970) *Summary of Particle Population in the Magnetosphere.* Dordrecht: Reidel, p. 305.

Washimi H and Taniuti T (1966) Propagation of ion-acoustic solitary waves of small amplitude. *Physical Review Letters* **17**, 996–998.

Yajima T and Wadati M (1990) Solitons in electron beam plasma. *Journal of the Physical Society of Japan* **59**, 237–3248.

Yashvir , Tiwari RS and Sharma SR (1988) Dressed ion acoustic soliton in an ion-beam plasma system. *Canadian Journal of Physics* **66**, 824–829.

Zank GP and McKenzie JF (1998) Solitons in an ion-beam plasma. *Journal of Plasma Physics* **39**, 183–191.

Modulational instability of high frequency surface waves on warm plasma half-space

Basudev Ghosh, Sailendra Nath Paul, Chandra Das, and Indrani Paul

Abstract: Modulational instability of high frequency surface waves on a plasma half-space is investigated taking into account finite temperature effects. The second harmonic generated through nonlinear self-interaction of the waves is found to carry a fraction of the surface wave energy into the bulk of the plasma. This is found to influence the modulational instability mechanism. To describe the nonlinear evolution of the wave, a nonlinear Schrödinger equation is derived in which the coefficient of the nonlinear term becomes a complex quantity. From this evolution equation we derive the condition of instability and show that high-frequency surface waves on a warm plasma half-space are modulationally unstable.

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Résumé : Nous étudions l'instabilité de modulation des ondes de surface de haute fréquence d'un demi-espace de plasma, en tenant compte des effets de température finie. Nous trouvons que la seconde harmonique, générée par une auto-interaction non linéaire, transporte une fraction de l'énergie de l'onde de surface vers l'intérieur du plasma et observons que ceci influence le mécanisme d'instabilité de modulation. Afin de décrire l'évolution non linéaire de l'onde, nous dérivons une équation d'onde non linéaire de Schrödinger, dans laquelle le coefficient du terme non linéaire devient une quantité complexe. À partir de cette équation, nous déterminons les conditions d'instabilité et montrons que les ondes de surface de haute fréquence sur un demi-espace de plasma chaud ont une instabilité de modulation inhérente.

[Traduit par la Rédaction]

1. Introduction

In recent years considerable interest has been aroused in the study of the nonlinear behaviour of surface waves in bounded plasmas [1–16]. This is due to the fact that the surface wave modes promise a wide range of applications in many fields, such as plasma heating by large amplitude waves, laser–plasma interactions, plasma diagnostics, microwave electronics, plasma technology, plasma spectroscopy, nanotechnology, and surface science. Nonlinear behaviour of surface waves on a cold plasma half-space has been studied by a number of authors. Its frequency spectrum extends from $\omega = 0$ to $\omega_e/\sqrt{2}$ where ω_e is the electron plasma frequency. These waves have both electrostatic and electromagnetic characters with the former dominating at the upper limit of the frequency spectrum [17]. Gradov and Stenflo [18] considered electrostatic nondispersive surface waves on a cold homogeneous plasma. On the other hand, Yu and Zhelyazkov [19] included electromagnetic effects to provide finite dispersion at a frequency range where surface waves are usually treated electrostatically. Recently, Ghosh et al. [20] made a detailed electromagnetic treatment of the nonlinear behaviour of surface waves on a cold plasma half-space throughout the whole spectrum of its existence. Inclusion of thermal effect broadens the surface wave's frequency spectrum [21] and makes the electrostatic surface waves dispersive even for homogeneous plasma. It can provide finite dispersion necessary for the formation of surface wave solitons. So, it remains interesting to make detailed analysis regarding the nonlinear behaviour of surface waves taking into account the thermal motion of electrons. In the present paper, we make detailed analysis of the nonlinear behaviour of high frequency surface waves on a plasma half-space taking into account the thermal motion of the electrons. By the method of multiple scales [22] we derive a nonlinear Schrödinger equation whose nonlinear term is found to be complex. From this equation instability criteria are found and it is shown that high frequency surface waves on a warm plasma half-space are modulationally unstable. The instability growth rate is found to increase with increase of wave number and temperature. The quadratic nonlinearities in the system equations are found to generate second harmonics a part of which decays, as well, exponentially away from the surface while the other part carries a fraction of the surface wave energy into the bulk of plasma. So, the instability will have to grow in competition with the continuous energy extraction by the second harmonics.

2. Basic equations

We are interested in the nonlinear behaviour of surface waves at the upper frequency range of their existence where they may be treated as electrostatic [17]. So the basic equations governing the propagation of such high frequency electrostatic waves in hot electron plasma are the following:

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$$\frac{\partial n}{\partial t} + N_0 \nabla \cdot \boldsymbol{u} = -\nabla \cdot (n\boldsymbol{u}) \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} - \frac{e}{m_e}\nabla\phi + \frac{V_e^2}{N_0 + n}\nabla n = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} \tag{2}$$

$$\nabla^2\phi = 4\pi e n \tag{3}$$

where n is the perturbation in the uniform density N_0 of the electrons, $\boldsymbol{u}$ is the fluid velocity of electrons, V_e^2 is the mean square thermal velocity, ϕ is the electric potential, and other symbols have their usual meanings. Ions are assumed to form a uniform neutralizing background (which is correct when $\omega \gg \omega_p$). We also assume that $\omega \gg k_\parallel V_e$. The plasma is assumed to occupy the half space $x > 0$ and be bounded by a vacuum. For vacuum ($x < 0$) we use the well-known Laplace's equation

$$\nabla^2\phi_v = 0 \tag{4}$$

where the subscript v corresponds to vacuum.

The boundary conditions to be used are the continuity of electric potential and normal electric displacement across the plasma–vacuum interface. As we are considering warm plasma, the normal component of the fluid velocity of electrons may be assumed to be zero on the interface [23].

3. Nonlinear analysis

Following the usual procedure [24, 25] we make the following Fourier expansion of the field quantities as

$$W = \varepsilon^2 W_0 + \sum_s^\infty \varepsilon^s\left[W_s \exp(is\psi) + W_s^* \exp(-is\psi)\right] \tag{5}$$

where $\psi = kz - \omega t$ (ω and k are, respectively, linear frequency and wavenumber); ε is a smallness parameter; W_0, W_s, and W_s^* are functions of x, $\xi = \varepsilon(z - C_g t)$; $\tau = \varepsilon^2 t$ ($C_g = d\omega/dk$, the group velocity); and W stands for the field quantities n, u_x, u_z, ϕ, and ϕ_v.

Substituting (5) into (1)–(4) and equating on both sides the coefficients of $\exp(i\psi)$, $\exp(2i\psi)$, and terms independent of ψ we get three sets of equations that we call I, II, and III. To solve these equations we make the following perturbation expansion for the field quantities W_1, W_2, and W_0, which are denoted by W_j (j = 1, 2, 0):

$$W_j = W_j^{(1)} + \varepsilon W_j^{(2)} + \varepsilon^2 W_j^{(3)} + \cdots \tag{6}$$

Solving the lowest order equations obtained from I after substituting (6) we get

$$\phi^{(1)} = \frac{\alpha}{\gamma(\omega_e^2 - \omega^2)}\left(-\omega_e^2 k \mathrm{e}^{-\gamma x} + \gamma\omega^2 \mathrm{e}^{-kx}\right)$$
$$n_1^{(1)} = -\frac{\alpha e N_0 k\left(\gamma^2 - k^2\right)}{\gamma m_e\left(\omega_e^2 - \omega^2\right)}\mathrm{e}^{-\gamma x}$$
$$u_{x1}^{(1)} = \frac{i\alpha e\omega k}{m_e\left(\omega_e^2 - \omega^2\right)}\left(\mathrm{e}^{-\gamma x} - \mathrm{e}^{-kx}\right)$$
$$u_{z1}^{(1)} = \frac{\alpha e\omega k}{m_e\gamma\left(\omega_e^2 - \omega^2\right)}\left(k\mathrm{e}^{-\gamma x} - \gamma\mathrm{e}^{-kx}\right)$$
$$\phi_v^{(1)} = \alpha \mathrm{e}^{kx} \tag{7}$$

where ω and k satisfy the following linear dispersion relation:

$$\gamma\left(2\omega^2 - \omega_e^2\right) = \omega_e^2 k \tag{8}$$

where

$$\gamma^2 = k^2 + \frac{\omega_e^2 - \omega^2}{V_e^2}$$

ω_e is the electron plasma frequency, and α appearing in (7) represents the amplitude of the wave field and is a function of ξ and τ. We seek an evolution equation for α.

At low temperatures (8) gives approximately

$$\omega = \frac{\omega_e}{\sqrt{2}}\left(1 + \frac{kV_e}{\sqrt{2}\omega_e}\right) \tag{9}$$

This clearly shows that the effect of thermal motion of the electrons is to broaden the frequency spectrum of high frequency surface waves in semi-infinite plasma.

Similarly, solving the lowest order equations obtained from II and III we get the following solutions for the second harmonic and zeroth harmonic quantities, respectively:

$$\phi_2^{(1)} = \alpha^2\left(C_0 \mathrm{e}^{i\gamma_2 x} + \sum_{j=1}^3 C_j \mathrm{e}^{-p_j x}\right)$$
$$n_2^{(1)} = \frac{\alpha^2}{4\pi e}\left[-\left(\gamma_2^2 + 4k^2\right)C_0 \mathrm{e}^{i\gamma_2 x} + \sum_{j=2,3} C_j\left(p_j^2 - 4k^2\right)\mathrm{e}^{-p_j x}\right]$$
$$u_{x2}^{(1)} = -\frac{\alpha^2 e}{2\omega m_e}\left(\frac{4\omega^2\gamma_2}{\omega_e^2}C_0 \mathrm{e}^{i\gamma_2 x} + i\sum_{j=1}^3 p_j D_j \mathrm{e}^{-p_j x}\right)$$
$$u_{z2}^{(1)} = -\frac{\alpha^2 e}{2\omega m_e}\left(\frac{8\omega^2 k}{\omega_e^2}C_0 \mathrm{e}^{i\gamma_2 x} + 2k\sum_{j=1}^3 D_j \mathrm{e}^{-p_j x}\right) \tag{10}$$

and

$$\phi_0^{(1)} = \alpha\alpha^* a_2 k^2 \sum_{j=1}^4 A_j \mathrm{e}^{-p_j x}$$
$$n_0^{(1)} = \frac{\alpha\alpha^* a_2 k^2}{4\pi e}\sum_{j=1}^4 p_j^2 A_j \mathrm{e}^{-p_j x}$$
$$u_{x0}^{(1)} = 0$$
$$u_{z0}^{(1)} = \frac{\alpha\alpha^* a_2 e k^2\gamma^2}{m_e\omega_e^2 C_g}\left(-\mathrm{e}^{-p_1 x} - \mathrm{e}^{-p_2 x} + 2\mathrm{e}^{-p_3 x}\right) \tag{11}$$

where

$$p_1 = 2k \qquad p_2 = 2\gamma \qquad p_3 = \gamma + k$$
$$\gamma_2^2 = \frac{4\omega^2 - \omega_e^2}{V_e^2} - 4k^2 \qquad p_4 = \frac{\omega_e}{V_e}$$
$$C_2 = \frac{a_2 k^2\left(\omega_e^2 - \omega^2\right)\left(2\omega^2 + \omega_e^2\right)}{6\omega^2\omega_e^2 V_e^2}$$

$$C_3 = -\frac{a_2 k^2\gamma(\gamma - k)(3\gamma + 5k)}{V_e^2(\gamma + 3k)\left(p_3^2 + \gamma_2^2\right)} \qquad a_2 = \frac{e\omega^2\omega_e^2}{m_e\gamma^2\left(\omega_e^2 - \omega^2\right)^2}$$

$$D_2 = \frac{\left(4\omega^2 - 3\omega_e^2\right)C_2}{\omega_e^2} + \frac{k^2\left(\gamma^2 - k^2\right)a_2}{2\omega^2}$$

$$D_3 = \frac{\left[\omega_e^2 - \left(p_3^2 - 4k^2\right)V_e^2\right]C_3}{\omega_e^2} + \frac{k^2\gamma(k-\gamma)a_2}{\omega_e^2}$$

$$C_1 = D_1 = -\frac{p_2C_2 + p_3C_3}{p_1} + \frac{i4\omega^2\gamma_2C_0}{p_1\omega_e^2}$$

$$C_0 = \frac{2p_2D_2 + 2p_3(D_3 - C_2) - (p_1 + p_3)C_3}{p_1 - i\gamma_2\left(1 - 8\omega^2/\omega_e^2\right)}$$

$$A_1 = \frac{2\gamma^2}{\omega_e^2 - 4k^2V_e^2} \qquad A_2 = \frac{\omega_e^2k^2 - \gamma^2\left(\omega_e^2 - 2\omega^2\right)}{\omega^2\left(\omega_e^2 - 4\gamma^2V_e^2\right)}$$

$$A_3 = \frac{p_2p_3}{p_3^2V_e^2 - \omega_e^2} \quad \text{and} \quad A_4 = -\sum_{j=1}^{3}A_j \tag{12}$$

Solution for the second harmonic field quantities shows that a part of it decays in the usual way away from the interface while the other part represents a wave propagating obliquely away from the plasma–vacuum interface, carrying a fraction of the surface wave energy into the bulk of the plasma.

Now, collecting the coefficients of ε^3 from both sides of I and substituting perturbation expansion (6) we get equations for the first harmonic quantities in the third order, which can be put in the following matrix form:

$$\begin{bmatrix} -i\omega & N_0\dfrac{\partial}{\partial x} & ikN_0 & 0 \\ \dfrac{V_e^2}{N_0}\dfrac{\partial}{\partial x} & -i\omega & 0 & -\dfrac{e}{m_e}\dfrac{\partial}{\partial x} \\ \dfrac{ikV_e^2}{N_0} & 0 & -i\omega & \dfrac{\partial^2}{\partial x^2} - k^2 \\ -4\pi e & 0 & 0 & \dfrac{\partial^2}{\partial x^2} - k^2 \end{bmatrix} \begin{bmatrix} n_1^{(3)} \\ u_{x1}^{(3)} \\ u_{z1}^{(3)} \\ \phi_1^{(3)} \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} \qquad x > 0 \tag{13}$$

and

$$\left(\frac{\partial^2}{\partial x^2} - k^2\right)\phi_{v1}^{(3)} = M_{v1} \qquad x < 0 \tag{14}$$

where

$$M_1 = -\frac{\partial n_1^{(1)}}{\partial \tau} + C_g\frac{\partial n_1^{(2)}}{\partial \xi} - N_0\frac{\partial u_1^{(2)}}{\partial \xi} - \frac{\partial}{\partial x}\left(n_0^{(1)}u_{x1}^{(1)} + n_1^{*(1)}u_{x2}^{(1)} + n_2^{(1)}u_{x1}^{*(1)}\right) - ik\left(n_0^{(1)}u_{z1}^{(1)} + n_1^{(1)}u_{z0}^{(1)} + n_1^{*(1)}u_{z2}^{(1)} + n_2^{(1)}u_{z1}^{*(1)}\right)$$

$$M_2 = -\frac{\partial u_{x1}^{(1)}}{\partial z} + C_g\frac{\partial u_{x1}^{(2)}}{\partial \xi} - \frac{\partial}{\partial x}\left(u_{x2}^{(1)}u_{x1}^{(1)*}\right) - ik\left(u_{z0}^{(1)}u_{x1}^{(1)} + 2u_{z1}^{(1)*}u_{x2}^{(1)} - u_{z2}^{(1)}u_{x1}^{(1)*}\right) + \frac{V_e^2}{N_0^2}\frac{\partial}{\partial x}\left(n_1^{(1)}n_0^{(1)} + n_1^{*(1)}n_2^{(1)}\right) - \frac{V_e^2}{N_0^3}\frac{\partial}{\partial x}\left(n_1^{(1)^2}n_1^{*(1)}\right)$$

$$M_3 = -\frac{\partial u_{z1}^{(1)}}{\partial \tau} + C_g\frac{\partial u_{z1}^{(2)}}{\partial \xi} - \frac{V_e^2}{N_0}\frac{\partial n_1^{(2)}}{\partial \xi} + \frac{e}{m_e}\frac{\partial \varphi_1^{(2)}}{\partial \xi} - \left(u_{x1}^{(1)}\frac{\partial u_{z0}^{(1)}}{\partial x} + u_{x2}^{(1)}\frac{\partial u_{z1}^{(1)*}}{\partial x} + u_{x1}^{(1)*}\frac{\partial u_{z2}^{(1)}}{\partial x}\right) - ik\left(u_{z0}^{(1)}u_{z1}^{(1)} + u_{z1}^{(1)*}u_{z2}^{(1)}\right) + \frac{ikV_e^2}{N_0^2}\left(n_0^{(1)}n_1^{(1)} + n_1^{*(1)}n_2^{(1)}\right) - \frac{ikV_e^2}{N_0^3}\left(n_1^{(1)^2}n_1^{*(1)}\right)$$

$$M_4 = -\frac{\partial^2\phi_1^{(1)}}{\partial \xi^2} - 2ik\frac{\partial \phi_1^{(2)}}{\partial \xi}$$

and

$$M_{v1} = -\frac{\partial^2\phi_{v1}^{(1)}}{\partial \xi^2} - 2ik\frac{\partial \phi_{v1}^{(2)}}{\partial \xi} \tag{15}$$

The boundary conditions to be used at $x = 0$ are

$$\phi_1^{(3)} = \phi_{v1}^{(3)} \qquad \frac{\partial \phi_1^{(3)}}{\partial x} = \frac{\partial \phi_{v1}^{(3)}}{\partial x} \quad \text{and} \quad u_{x1}^{(3)} = 0 \tag{16}$$

Now we multiply (13) by the row matrix $[f_1, f_2, f_3, f_4]$ and integrate the resulting equation with respect to x between the limits 0 to ∞. Similarly, we multiply (14) by the function f_{v1} and integrate the resulting equation with respect to x between the limits –∞ to 0. On adding the two equations thus obtained we get the following equation:

$$\sum_{j=1}^{4}\int_0^{\infty} f_jM_j\mathrm{d}x + \int_{-\infty}^{0} f_{v1}M_{v1}\mathrm{d}x = 0 \tag{17}$$

where we have chosen the functions f_1 and f_{v1} to satisfy the following equations and boundary conditions:

$$\begin{bmatrix} -i\omega & \dfrac{V_e^2}{N_0}\dfrac{\partial}{\partial x} & \dfrac{-ikV_e^2}{N_0} & -4\pi e \\ -N_0\dfrac{\partial}{\partial x} & -i\omega & 0 & 0 \\ ikN_0 & 0 & -i\omega & 0 \\ 0 & \dfrac{e}{m_e}\dfrac{\partial}{\partial x} & \dfrac{-iek}{m_e} & \dfrac{\partial^2}{\partial x^2}-k^2 \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad x > 0 \qquad (18)$$

$$\left(\frac{\partial^2}{\partial x^2}-k^2\right)f_{v1} = 0 \qquad x < 0 \qquad (19)$$

At $x = 0$, $f_4 = f_{v1}$, $\partial f_4/\partial x = \partial f_{v1}/\partial x$, $f_z = 0$

$$f_j \to 0 \quad \text{as} \quad x \to \infty \quad \text{and} \quad f_{v1} \to 0 \quad \text{as} \quad x \to -\infty \qquad (20)$$

The solutions of (18) and (19) satisfying boundary conditions (20) are the following:

$$\begin{aligned} f_1 &= \frac{4\pi ie\omega A}{\gamma\left(\omega_e^2-\omega^2\right)}\left(\gamma e^{-kx} - ke^{-\gamma x}\right) \\ f_2 &= \frac{4\pi eN_0kA}{\left(\omega_e^2-\omega^2\right)}\left(e^{-kx} - e^{-\gamma x}\right) \\ f_3 &= \frac{4\pi ieN_0kA}{\gamma\left(\omega_e^2-\omega^2\right)}\left(\gamma e^{-kx} - ke^{-\gamma x}\right) \\ f_4 &= \frac{A}{\gamma\left(\omega_e^2-\omega^2\right)}\left(\gamma\omega^2 e^{-kx} - k\omega_e^2 e^{-\gamma x}\right) \\ f_{v1} &= Ae^{kx} \end{aligned} \qquad (21)$$

where A is some arbitrary constant independent of x.

It will be shown subsequently that (17) leads to the desired nonlinear Schrödinger equation. The linear parts of M_j and M_{v1} involved in (17) contain first harmonic quantities in the second order, which we have yet to determine. To deal with these quantities we follow the procedure adopted by Blenerhassett [26]. According to this procedure we take the help of a similar but linear problem with a slightly different wavenumber $k' = k + \varepsilon$ and frequency $\omega' = \omega(k + \varepsilon)$. The problem is defined by the following equations:

$$\begin{bmatrix} -i\omega' & N_0\dfrac{\partial}{\partial x} & ik'N_0 & 0 \\ \dfrac{V_e^2}{N_0}\dfrac{\partial}{\partial x} & -i\omega' & 0 & -\dfrac{e}{m_e}\dfrac{\partial}{\partial x} \\ ik'\dfrac{V_e^2}{N_0} & 0 & i\omega' & -\dfrac{iek'}{m_e} \\ -4\pi e & 0 & 0 & \dfrac{\partial^2}{\partial x^2}-k'^2 \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad x > 0 \qquad (22)$$

$$\left(\frac{\partial^2}{\partial x^2}-k'^2\right)F_{v1} = 0 \qquad x < 0 \qquad (23)$$

and the boundary conditions

$$F_4 = F_{v1} \qquad \frac{\partial F_4}{\partial x} = \frac{\partial F_{v1}}{\partial x} \qquad F_2 = 0 \quad \text{at} \quad x = 0$$

$$\begin{aligned} &F_j \to 0 \qquad j = 1, 2, 3, 4 \quad \text{as} \quad x \to \infty \\ &F_{v1} \to 0 \quad \text{as} \quad x \to -\infty \end{aligned} \qquad (24)$$

To solve this problem we make the following perturbation expansions:

$$\begin{aligned} F_j &= F_j^{(1)} + \varepsilon F_j^{(2)} + \varepsilon^2 F_j^{(3)} + \cdots \\ F_{v1} &= F_{v1}^{(1)} + \varepsilon F_{v1}^{(2)} + \varepsilon^2 F_{v1}^{(3)} + \cdots \\ \omega' &= \omega + \varepsilon C_g + \varepsilon^2 \frac{1}{2}\frac{dC_g}{dk} + \cdots \end{aligned} \qquad (25)$$

Substituting these expansions in (22)–(24) and then equating on both sides the coefficients of like powers of ε we get a sequence of equations and boundary conditions for the elements $F_j^{(l)}$ and $F_{v1}^{(l)}$ (l = 1, 2, 3). It is found that the elements $F_j^{(1)}$ and $F_{v1}^{(1)}$ satisfy the same set of equations and boundary conditions as those satisfied by the elements $A_j^{(1)} \equiv [n_1^{(1)}, u_{x1}^{(1)}, u_{z1}^{(1)}, \varphi_1^{(1)}]$ and $\varphi_{v1}^{(1)}$. So we can make the following identification:

$$F_j^{(1)} = A_j^{(1)} \qquad F_{v1}^{(1)} = \varphi_{v1}^{(1)} \qquad (26)$$

If we set

$$F_j^{(2)} = \alpha\psi_j^{(2)} \qquad F_{v1}^{(2)} = \alpha\psi_{v1}^{(2)} \qquad A_j^{(2)} = -i\frac{\partial\alpha}{\partial\xi}X_j^{(2)}$$

$$\varphi_{v1}^{(2)} = -i\frac{\partial\alpha}{\partial\xi}X_{v1}^{(2)} \qquad (27)$$

then we find that the elements $\psi_j^{(2)}$ and $\psi_{v1}^{(2)}$ satisfy the same set of equations and boundary conditions as those satisfied by the elements $X_j^{(2)}$ and $X_{v1}^{(2)}$. So we can make the following identification:

$$\psi_j^{(2)} = X_j^{(2)} \qquad \psi_{v1}^{(2)} = X_{v1}^{(2)} \qquad (28)$$

Setting $F_j^{(3)} = \alpha\psi_j^{(3)}$ and $F_{v1}^{(2)} = \alpha\psi_{v1}^{(3)}$ we find that $\psi_j^{(3)}$ and $\psi_{v1}^{(3)}$ satisfy (13), (14), and boundary conditions (16) as satisfied by the quantities $A_j^{(3)}$ and $\varphi_{v1}^{(3)}$ with the exceptions that the quantities M_j and M_{v1} are replaced by N_j and N_{v1} which are given by

$$\begin{aligned} N_1 &= i\left(C_gX_1^{(2)} - N_0X_3^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\frac{1}{\alpha}A_1^{(1)}\right) \\ N_2 &= i\left(C_gX_2^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\frac{1}{\alpha}A_2^{(1)}\right) \\ N_3 &= i\left(C_gX_3^{(2)} - \frac{V_e^2}{N_0}X_1^{(2)} + \frac{ie}{m_e}X_4^{(2)} + \frac{1}{2}\frac{dC_g}{dk}\frac{1}{\alpha}A_3^{(1)}\right) \\ N_4 &= 2kX_4^{(2)} + \frac{1}{\alpha}A_4^{(1)} \end{aligned}$$

and

$$N_{v1} = 2kX_{v1}^{(2)} + \frac{1}{\alpha}\varphi_{v1}^{(1)} \qquad (29)$$

Following the same procedure as adopted in deriving (17) we obtain the following equation from the equations and boundary conditions satisfied by $\psi_j^{(3)}$ and $\psi_{v1}^{(3)}$:

$$\sum_{j=1}^{4}\int_0^\infty f_j N_j \mathrm{d}x + \int_{-\infty}^{0} f_{v1}N_{v1}\mathrm{d}x = 0 \tag{30}$$

Operation (17) + $i(\partial^2\alpha/\partial\xi^2)$ (30) and some straightforward integrations and algebraic manipulations give the following desired nonlinear Schrödinger equation:

$$i\frac{\partial\alpha}{\partial\tau} + P\frac{\partial^2\alpha}{\partial\xi^2} = (Q_1 - iQ_2)\alpha^2\alpha^* \tag{31}$$

where P, Q_1, and Q_2 are given by

$$P = \frac{\omega V_e^2 + 3kV_e^2 C_g - 4\omega C_g^2}{2\left(2\omega^2 - \omega_e^2 - k^2V_e^2\right)}$$

$$C_g = \frac{\omega k V_e^2}{2\omega^2 - \omega_e^2 - k^2V_e^2} \qquad Q_1 = \frac{I_1}{I_2} \qquad Q_2 = \frac{I_2}{I_3}$$

in which

$$\begin{aligned}
I_1 = {} & \frac{e^2k^3\left(\gamma^2 - k^2\right)^3}{4m_e^2\gamma^2\omega\left(\omega_e^2 - \omega^2\right)} \\
& + \frac{e^2k^3\omega^3\gamma}{m_e^2\left(\omega_e^2 - \omega^2\right)C_g}\left[\frac{k\left(\gamma^2 - k^2\right)}{p_2} + \frac{\left(3\gamma^3 - k^3 - \gamma k p_3\right)}{p_3} + \frac{4k^3 - 3\gamma^3 + \gamma k(k - 2\gamma)}{p_2 + p_3} + \frac{\gamma k^2 + 2\gamma^2 k\left(k - 3\gamma^3\right)}{p_1 + p_3}\right] \\
& - \frac{e^3\omega^3k^2}{m_e^2\gamma\left(\omega_e^2 - \omega^2\right)^2V_e^2}\sum_{j=1}^{4}A_jp_j^2\left[\frac{y_1 + p_jy_2}{p_j + p_2} + \frac{\gamma^2p_j}{p_j + p_1} + \frac{y_3 + p_jy_4}{p_j + p_3}\right] \\
& + \frac{e^3\omega^3k^2}{m_e^2\gamma\left(\omega_e^2 - \omega^2\right)^2V_e^2}\sum_{j=1}^{4}A_jp_j^2\left[\frac{y_1 + p_jy_2}{p_j + p_2} + \frac{\gamma^2p_j}{p_j + p_1} + \frac{y_3 + p_jy_4}{p_j + p_3}\right] \\
& + \frac{e\gamma}{2m_e\omega}\left[\sum_{j=1}^{3}D_j'\left(2k^2\gamma^2 + \frac{x_1 + p_jx_2 + p_j^2x_3}{p_j + p_2} + \frac{x_4 + p_jx_5 + p_j^2x_6}{p_j + p_3}\right) + \frac{4\omega^2}{\omega_e^2}\left(C_{or}Y_r - C_{oi}Y_i\right)\right]
\end{aligned}$$

$$I_2 = \frac{e\omega\gamma\left(p_1^2 + \gamma_2^2\right)\left(C_{or}R_i + C_{oi}R_r\right)}{m_e\omega_e^2} + \frac{e\gamma}{2m_e\omega}\left[\frac{4\omega^2\left(C_{or}Y_i + C_{oi}Y_r\right)}{\omega_e^2} + D_{1i}\left\{2k^2\gamma^2 + \frac{\left(x_1 + p_1x_2 + p_1^2x_3\right)}{p_1 + p_2} + \frac{\left(x_4 + p_1x_5 + p_1^2x_6\right)}{p_1 + p_3}\right\}\right]$$

$$I_3 = 2\gamma^3 - k^3 - \gamma^2k - \frac{p_1p_2}{p_3} \tag{32}$$

where

$$y_1 = 2\gamma^2k - \omega_e^2\left(\gamma^2 + k^2\right)\frac{k}{\omega^2} \qquad y_2 = \frac{\gamma k\left(2\omega^2 - \omega_e^2\right)}{\omega^2}$$

$$y_3 = \frac{\gamma p_3\left(\omega_e^2k - \omega^2\gamma\right)}{\omega^2} \qquad y_4 = \frac{\omega_e^2\gamma k}{\omega} - p_2k - \gamma^2$$

$$z_1 = \frac{\omega_e^2k\left(\gamma^2 + k^2\right)}{\omega^2} \qquad z_2 = \frac{\omega_e^2k\gamma}{\omega^2} \qquad z_3 = \gamma\left(3k^2 + \gamma^2\right)$$

$$z_4 = \gamma p_3 \qquad z_5 = \gamma^2 + \frac{\gamma k\left(\omega^2 - \omega_e^2\right)}{\omega^2} \qquad x_1 = 4k^5$$

$$x_2 = p_1\gamma\left(\gamma^2 + k^2\right) \qquad x_3 = k\left(2\gamma^2 - k^2\right)$$

$$x_4 = 2k^2\gamma\left(\gamma^2 - 3k^2 - 2\gamma k\right) \qquad x_5 = \gamma^2\left(k^2 - \gamma^2\right) - 3\gamma k^2p_3$$

$$x_6 = \gamma\left(k^2 - \gamma^2 - \gamma k\right)$$

$$R_r = \frac{z_1p_2 + z_2\gamma^2}{\gamma^2 + p_2^2} - \frac{z_3p_3 + z_4\gamma^2}{\gamma^2 + p_3^2} + z_5$$

$$R_i = \frac{\gamma_2(z_1 - p_2z_2)}{\gamma_2^2 + p_2^2} - \frac{\gamma_2(z_3 - p_3z_4)}{\gamma_2^2 + p_3^2}$$

$$D_1' = D_{1r} \qquad D_2' = D_2 \qquad D_3' = D_3 \qquad D_1 = D_{1r} + iD_{1i}$$
$$C_0 = C_{or} + iC_{oi}$$

$$Y_i = \frac{\gamma_2\left(x_1 - x_2p_2 - x_3\gamma_2^2\right)}{\gamma_2^2 + p_2^2} + \frac{\gamma_2\left(x_4 - p_3x_5 - \gamma_2^2x_6\right)}{\gamma_2^2 + p_3^2}$$

$$Y_r = \frac{x_1p_2 + \gamma_2^2(x_2 - x_3p_3)}{\gamma_2^2 + p_2^2} + \frac{x_4p_3 + \gamma_2^2(x_5 - x_6p_6)}{\gamma_2^2 + p_3^2} + 2k^2\gamma_2^2 \tag{33}$$

4. Modulational instability criteria

To find the criteria of modulational instability we substitute in (31)

$$\alpha = \alpha_0(1+\widehat{\alpha})e^{-\delta\tau}\exp i\left(-\Delta\omega\tau+\widehat{\theta}\right) \tag{34}$$

where α_o and δ are real constants, $\Delta\omega$ is the amplitude dependent frequency shift; $\widehat{\alpha}$ and $\widehat{\theta}$ are real functions of ξ and τ and represent perturbations in amplitude and phase, respectively.

For simplicity we shall confine our calculations to the initial stage of nonlinear evolution. With this in mind we substitute (34) into (31) and then equating real and imaginary parts we get

$$\frac{\partial\widehat{\alpha}}{\partial\tau}+P\frac{\partial^2\widehat{\theta}}{\partial\xi^2}+2Q_2\alpha_0^2\widehat{\alpha}=0 \tag{35}$$

$$-\frac{\partial\widehat{\theta}}{\partial\tau}+P\frac{\partial^2\widehat{\alpha}}{\partial\xi^2}-2Q_1\alpha_0^2\widehat{\alpha}=0 \tag{36}$$

where we have chosen $\Delta\omega \approx Q_1\alpha_0^2$ and $\delta \approx Q_2\alpha_0^2$.

Assuming ξ and τ dependence of $\widehat{\alpha}$ and $\widehat{\theta}$ to be of the form exp $i(l\xi - \Omega\tau)$ we get from (35) and (36) two algebraic equations in $\widehat{\alpha}$ and $\widehat{\theta}$, from which we get for nontrivial solution,

$$\Omega = -iQ_2\alpha_0^2+\sqrt{-Q_2^2\alpha_0^4+l^2\left(P^2l^2+2PQ_1\alpha_0^2\right)}$$

The first term on the right-hand side is small and is due to the oscillatory part of the second harmonics moving obliquely away from the interface. For a uniform plasma wave train to be modulationally unstable we require $PQ_1 < 0$. Then we can write

$$\Omega = i\left[-Q_2\alpha_0^2+\sqrt{Q_2^2\alpha_0^4+2|PQ_1|\alpha_0^2l^2-P^2l^4}\right] \tag{37}$$

The growth rate of instability, $|\Omega|$, attains a maximum value corresponding to the wave number of perturbation (l_m) given by

$$l_m^2 = \left|\frac{Q_1}{P}\right|\alpha_0^2 \tag{38}$$

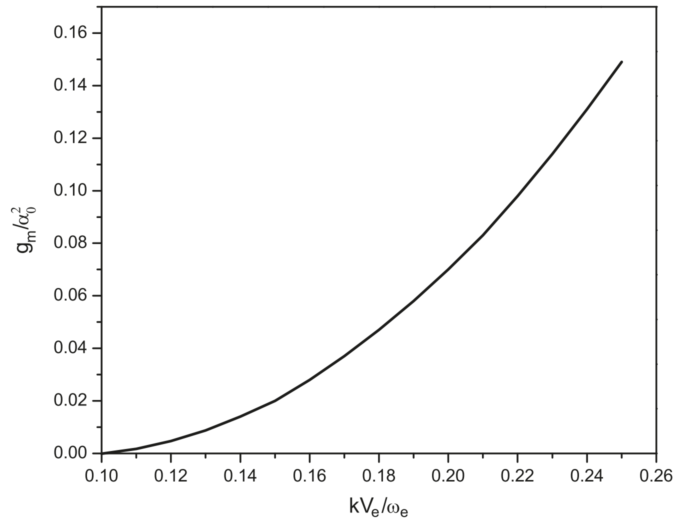
Using this value of l_m the corresponding maximum growth rate of instability during the initial stage of nonlinear evolution is obtained as

$$g_m = \alpha_0^2\left[\sqrt{Q_1+Q_2}-Q_2\right] \tag{39}$$

5. Results and discussions

Numerical calculation shows that $PQ_1 < 0$ for all values of k. So we conclude that high frequency electrostatic surface waves on a warm plasma half-space are modulationally unstable. Also it is found that $Q_2 > 0$ and $|Q_1| \gg Q_2$. So, (39) indicates that the energy extraction by the second harmonics slightly slows down the growth rate of the instability. The growth rate of the instability as given by (39) has been calculated for different values of the dimensionless parameter kV_e/ω_e. The result is shown graphically in Fig. 1. Obviously the modulational instability growth rate increases with increase in thermal velocity of electrons and also with the increase of the wave number k. In fact, thermal motion of the electrons introduces a number of effects. As evident from approximate dispersion relation (9) thermal motion broadens the frequency spectrum of surface waves. Surface waves in cold homogeneous plasma are nondispersive. Dispersion relation (9) shows that inclusion of temperature makes the wave dispersive even in a homogeneous plasma. Thus, it can also provide finite dispersion necessary for the formation of high frequency surface wave envelope solitons. Solutions (10) for the second harmonics show that the inclusion of temperature effect makes a part of the second harmonics generated through nonlinearities to carry a fraction, though small, of wave energy away from the interface into the bulk of the plasma.

Fig. 1. Dependence of maximum (normalized) initial growth rate (g_m/α_0^2) on electron thermal velocity (V_e) and the wavenumber (k).



As pointed out in the previous section, (39) gives only the initial growth rate. During the later stage of evolution the instability growth rate might be somewhat smaller than expected from (39) because the instability will have to grow in competition with the continuous energy extraction by the second harmonic.

We would like to point out here that the preceding analysis could be extended to the whole electromagnetic spectrum of the surface waves by including electromagnetic effects. Physically, however, such extension may not result in any new result as the instability growth rate is largest at high values of wave number, where surface waves behave electrostatically. Our treatment for spatially homogeneous plasma may, however, be extended to include the effects of inhomogeneity. Such an extension may become important in many cases. For example, when a sufficiently strong laser beam interacts with a plasma, the plasma undergoes strong density profile modification due to light pressure near the surface.

Finally we would like to mention that the results presented in the present paper may be of special interest in the surface physics of semiconductors and metals where ions are practically immobile and the boundary is almost rigid.

References

1. M. Georgieva and A. Shivarova. Phys. Scr. **50**, 523 (1994). doi:10.1088/0031-8949/50/5/012.

2. H.J. Lee and S.H. Cho. Plasma Phys. Contr. Fusion, **37**, 989 (1995). doi:10.1088/0741-3335/37/9/005.
3. N.A. Azarenkov, V.O. Girka, and A.E. Sporov. Phys. Scr. **55**, 339 (1997). doi:10.1088/0031-8949/55/3/012.
4. N.F. Cramer, L.K. Yeung, and S.V. Vladimirov. Phys. Plasmas, **5**, 3126 (1998). doi:10.1063/1.873039.
5. H.J. Lee and Y.K. Lim. Plasma Phys. Contr. Fusion, **40**, 1619 (1998). doi:10.1088/0741-3335/40/9/006.
6. D.J. Cooperberg. Phys. Plasmas, **5**, 862 (1998). doi:10.1063/1.872653.
7. K.N. Ostrikov, M.Y. Yu, and N. Azarenkov. Phys. Rev. E, **58**, 2431 (1998). doi:10.1103/PhysRevE.58.2431.
8. M.K. Alam, A.R. Chowdhury, and S.N. Paul. Phys. Plasmas, **6**, 3765 (1999). doi:10.1063/1.873639.
9. L. Stenflo and M.Y. Yu. Phys. Plasmas, **9**, 5129 (2002). doi:10.1063/1.1521715.
10. L. Stenflo and M.Y. Yu. Phys. Plasmas, **10**, 912 (2003). doi:10.1063/1.1555625.
11. J. Kupersztych, M. Raynaud, and C. Riconda. Phys. Plasmas, **11**, 1669 (2004). doi:10.1063/1.1650353.
12. M. Lazar, P.K. Shukla, and A. Smolyakov. Phys. Plasmas, **14**, 124501 (2007). doi:10.1063/1.2825278.
13. B. Ghosh and K.P. Das. Contrib. Plasma Phys. **28**, 97 (1988). doi:10.1002/ctpp.2150280110.
14. H.J. Lee. Phys. Plasmas, **12**, 1 (2005).
15. S.A. Dvinin, A.G. Vologirov, V.V. Mikheev, and V.S. Sviridkina. Fizika Plazmy, **34**, 746 (2008).
16. B.F. Mohamed and M.A. Aziz. Int. J. Plasma Sci. Eng. **2010**, 69304 (2010).
17. A. Shivarova and I. Zhelyazkov. Plasma Phys. **20**, 1049 (1978). doi:10.1088/0032-1028/20/10/007.
18. O.M. Gradov and L. Stenflo. Phys. Fluids, **25**, 983 (1982). doi:10.1063/1.863852.
19. M.Y. Yu and I. Zhelyazkov. J. Plasma Phys. **20**, 18 (1978).
20. B. Ghosh, S.R. Mazumdar, S.K. Bhattacharya, and S.N. Paul. FIZIKA, **A12**, 127 (2003).
21. T. Lindgren, L. Stenflo, N. Kostov, and I. Zhelyazkov. J. Plasma Phys. **34**, 427 (1985). doi:10.1017/S0022377800002981.
22. A. Davey and K. Stewartson. Proc. R. Soc. Lond. A, **388**, 101 (1974).
23. H.C. Barr and T.J.M. Boyd. J. Phys. A, **5**, 1108 (1972). doi:10.1088/0305-4470/5/7/019.
24. A.H. Nayfeh. Perturbation methods. John Wiley & Sons. 1973.
25. T. Taniuti and N. Yajima. J. Math. Phys. **10**, 1369 (1969). doi:10.1063/1.1664975.
26. P.J. Blenerhasset. Philos. Trans. R. Soc. London. Ser. A, **398**, 451 (1980).

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Effects of Boundary of Cylindrical Waveguide and Nonthermal Electrons on the Ion Acoustic Double Layers in Multicomponent Bounded Plasma

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Abstract

Ion acoustic double layers in a multicomponent plasma consisting of positive ions, negative ions, and nonthermal electrons filled in a cylindrical waveguide have been theoretically investigated using pseudopotential technique. The expression of Sagdeev potential for the ion acoustic wave in such a plasma has been derived, and the conditions for the existence of double layers are obtained. The effects of the boundary of cylindrical waveguide and nonthermal electrons on the Sagdeev potential have been graphically discussed. From the nonlinear equation, the solution for double layers of small amplitude ion acoustic waves is obtained, and the structure of double layers is analyzed as function of the plasma parameters. It is seen that finite geometry of bounded plasma, nonthermality of electrons, density of negative ions, and stream velocity of positive and negative ions have significant contribution on the structure of double layers. The double layers may be compressive or rarefactive in plasma depending upon the values of the plasma parameters.

Keywords Double layer · Bounded plasma · Nonthermal electrons · Negative ions

1 Introduction

In the last few years, ion acoustic solitary waves (SWs) and double layers (DLs) have been theoretically studied by various authors considering different parameters in plasma, e.g., ion temperature, two-temperature electrons, positrons, beam ions, and magnetic field. But, the presence of negative ions [1–3] and nonthermal electrons [4–8] in plasma is found to be most interesting on the formation of SWs and DLs. It is noted that most of the works are done in an infinite or unbounded plasma. But, the studies of SWs and DLs in the bounded plasma are important for the laboratory experiments [9–14]. Considering a cylindrically bounded plasma composed of ions and ion beams along with multiple-temperature electrons, Das et al. [15] have derived the Korteweg-deVries equation for the study of ion acoustic SWs using the perturbation method. It has been shown that the interaction of isothermality on the propagation of radially ingoing ion acoustic waves in various plasmas exerts a drastic modifying effect on the existence and behavior of SWs. Later, Das and Singh [16] have studied the nonlinear propagation of ion acoustic waves considering a generalized multicomponent plasma bounded by cylindrical and spherical geometries. They have shown that at critical density of negative ions where the nonlinearity of the Korteweg-deVries (K-dV) equation vanishes, ion acoustic SWs are described by a modified K-dV (mK-dV) equation. Near the critical density, neither the K-dV nor the mK-dV equation is sufficient to describe fully the ion acoustic waves, and thus, it is essential to derive further the mK-dV (fmK-dV) equation in the vicinity of this critical density. Using a reductive perturbation theory, Labany et al. [17] have studied cylindrical ion acoustic SWs in a warm plasma with negative ions and multiple-temperature electrons. Subsequently, the effect of higher order nonlinearity and finite geometry has been considered by Ghosh and Das [18], and it has been shown that the effect of higher order nonlinearity is to increase the amplitude and to decrease the width of ion acoustic SWs compared with the values predicted by first-order theory. Moreover, higher

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order nonlinearity and finite boundary effects give rise to W-shaped ion acoustic SWs. Recently, Kaur [19] has numerically investigated the cylindrical and spherical ion acoustic SWs in a plasma composed with ions, Tsallis distributed electrons and positrons. They have also studied numerically the effect of nonextensivity and nonplanar geometry on the amplitudes and width of the potential structures of ion acoustic SWs. Very recently, using the reductive perturbation method, Sahu and Roy [20] have studied the nonlinear propagation of electron acoustic waves in an unmagnetized dissipative quantum plasma in unbounded planar geometry and bounded nonplanar geometry. It is found that a nondimensional parameter, the ratio of equilibrium density of cold to hot electron components, has crucial role on the formation of both compressive and rarefactive SWs. It is also found that electron acoustic SWs in nonplanar geometry significantly differ from the planar geometry.

It is to be mentioned that the works of above authors on the SWs in unbounded plasma have been done using the perturbation method. But, Sayal and Sharma [21] have analyzed theoretically the ion Langmuir plasma oscillations in a plasma filled in a cylindrical waveguide using fluid equations and using effective potential method. It is shown that propagation of such modes is possible because of the finite radius of the cylindrical waveguide. Moreover, in the nonlinear regime, small as well as large amplitude compressive SWs may be formed in the bounded plasma. Later, Sayal and Sharma [22] have theoretically investigated soliton formation in an electron beam propagating in a plasma-filled cylindrical waveguide. Using pseudopotential technique, Bhattacharya et al. [23] have studied ion acoustic SWs in a multicomponent bounded plasma containing positive ions, negative ions, and nonthermal electrons. It is seen that when one considers nonthermal electrons and negative ions, both compressive and rarefactive solitons will be excited. But, in absence of negative ions, SWs will not exist in the bounded plasma for any value of nonthermal parameter of electrons. Later, Mondal et al. [24] have theoretically studied linear and nonlinear propagation of ion acoustic waves in a multicomponent plasma consisting of electrons, positive ions, and negative ions bounded in a cylindrical waveguide. The effect of nonlinearity on the ion acoustic wave is investigated through the derivation of the effective potential. It is shown that dimensions of the cylindrical system containing plasma have a positive influence on the stability of ion acoustic waves.

However, the DLs in the bounded plasma system have not been studied by any authors till now. Since the negative ions and the nonthermal electrons have important contributions on the excitation of ion acoustic SWs in plasma, we are interested here to study theoretically the DLs in a multicomponent plasma bounded in finite geometry. We have used pseudopotential method to derive the nonlinear equation and obtained the DL solution in bounded multicomponent plasma. The profiles of Sagdeev potential and DLs are drawn and discussed for different values of nonthermal parameter of electrons, concentration of negative ions, and finite geometry of the bounded plasma.

2 Basic Equations

We consider a perfectly conducting cylinder of radius R filled uniformly with a collisionless, unmagnetized, non-relativistic plasma and having an infinite axial uniform magnetic field. This plasma consists of cold positive ions, negative ions, and nonthermal electrons. It is assumed that the ions have streaming velocities along the axis of the cylinder taken to be x-axis. The plasma particles are assumed to move only along the axis of the waveguide, which we choose as x-axis of an (r, θ, x) cylindrical coordinate system. So, there will be no interaction with the corresponding empty waveguide modes that have only transverse electric field components (H-modes), and we shall consider only the E-modes. The set of equations governing the dynamics of such a bounded plasma can be written in non-dimensional form [21, 24]:

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s u_s)=0 \tag{1}$$

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}=-\frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x} \tag{2}$$

$$\nabla_\perp \varphi+\frac{\partial^2\varphi}{\partial x^2}=n_e-\sum\psi_s n_s \tag{3}$$

where

$$\nabla_\perp^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{4}$$

$\nabla_\perp$ is the transverse Laplacian, and the subscript “s” stands for positive ion and negative ion, s = i for positive ions, s = j for negative ions; n_s, u_s, m_s are, respectively, the density, velocity, and mass of positive ions and negative ions; n_e is the number density of electrons, $\psi_i = 1, \psi_j = -1$, $Q_i = 1$, $Q_j = Q = m_j/m_i$; the velocities are normalized by $(K_B T_e/m_i)^{1/2}$, the densities by equilibrium ion density, the electrostatic potential φ by $(K_B T/e)$, all length by the Debye length $(K_B T_e/4\pi n_0 e^2)^{1/2}$; $K_B$is the Boltzmann constant.

We assume the boundary conditions

$$n_s \to n_{so},\, u_s \to u_{so},\, \phi \to 0 \ at\ |x| \to \propto \tag{5}$$

The charge neutrality condition of the plasma is $n_{io} = 1 + n_{jo}$, where n_{i0} and n_{j0} are the equilibrium densities of positive ions and negative ions.

Now we consider the electron distribution which includes the population of nonthermal electrons. A nonthermal plasma, cold plasma, or non-equilibrium plasma is a plasma which is not in the thermodynamic equilibrium, because the electron temperature is much hotter than the temperature of heavy species (ions and neutrals). As only electrons are thermalized, their Maxwell-Boltzmann velocity distribution is very different than the ion velocity distribution. When one of the velocities of a species does not follow a Maxwell-Boltzmann distribution, the plasma is said to be non-Maxwellian. In many astrophysical and space plasma scenarios, plasma particles are not in thermodynamic equilibrium but have long-range interactions. The systems with long-range interactions are believed to exhibit non-Maxwellian distributions. External forces acting on plasma or interaction of wave particle may lead to the formation of non-Maxwellian distributions. The presence of highly energetic plasma particles may also destroy the equilibrium state of plasma. Nonthermal distribution of electrons with excess energetic particles can be modeled by the Cairn's distribution [25, 26]. Cairns et al. [25, 26] have considered a plasma with nonthermal electrons and cold ions and have shown that it is possible to obtain both compressive and rarefactive SWs. This nonthermal model can explain the ion acoustic structures with density depression in the magnetosphere observed by Freja [27] and Viking [28] satellites. In fact, the study of non-Maxwellian plasma is crucial to the understanding of space, astrophysical, and laboratory plasma dynamics.

The nonthermal electron distribution function is chosen as [25, 26]

$$f_e(V) = \frac{n_o}{(1+3p)\sqrt{2\pi V_e^2}}\left[1+p\left(\frac{V^2}{V_e^2}-2\phi\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{V^2}{V_e^2}-2\phi\right)\right] \tag{6}$$

where ϕ is the normalized electrostatic potencial, $V_e = \sqrt{K_B T_e/m_e}$ and m_e is the mass of a electron.

From (Eq. 6), the normalized electron density n_e is obtained as

$$n_e = \left(1-\beta\phi+\beta\phi^2\right)\exp(\phi) \tag{7}$$

where $\beta = \frac{4p}{1+3p}$

β measures the deviation from thermalized state and p determines the number of nonthermal electrons in the plasma. $\beta = 0$ corresponds to the Boltzmann distribution of electrons.

3 Nonlinear Equation and Double Layer Solution

Now we assume that the radial behavior of the perturbed quantities is described by the lowest power of Bessel function. Following Mondal et al. [24], we take the perturbed quantities in the form of $J_0(k_\perp r)f(x,t)$, where $f(x,t) = N_s(x,t)$, $U_s(x,t)$, $\cdots$ and $k_\perp = p_{0n}/R$. Integrating the nonlinear equations over r from 0 to R after multiplying with $rJ_0(k_\perp r)$we get

$$\frac{\partial N_s}{\partial t} + n_{s0}\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial N_s}{\partial x} + \alpha N_s\frac{\partial U_s}{\partial x} + \alpha U_s\frac{\partial N_s}{\partial x} = 0 \tag{8}$$

$$\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial U_s}{\partial x} + \alpha U_s\frac{\partial U_s}{\partial x} = -\frac{\psi_s}{Q_s}\frac{\partial \varphi}{\partial x} \tag{9}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \varphi = N_e - \sum \psi_s n_s \tag{10}$$

where

$$\alpha = \frac{\left(\int_0^R J_0^3 r dr\right)}{\left(\int_0^R J_0^2 r dr\right)} \tag{11}$$

To obtain the expression of Sagdeev potential from Eqs. (8)–(10), we use the transformation $\eta = x - Mt$ where M is the velocity ion acoustic wave. Then, from Eqs. (8)–(10), we obtain

$$U_s = \frac{N_s(M-u_{s0})}{n_{s0}+\alpha N_s} \tag{12}$$

$$(M-u_{s0})U_s - \frac{\alpha}{2}U_s^2 = \left(\frac{\psi_s}{Q_s}\right)\varphi \tag{13}$$

From (10) and (12), we get

$$N_s = \frac{n_{s0}}{\alpha\left[-1+\left\{1-\frac{2\alpha\varphi}{Q_s(M-u_{s0})^2}\right\}^{-\frac{1}{2}}\right]} \tag{14}$$

Therefore, the Poisson Eq. (10) becomes

$$\frac{\partial^2 \varphi}{\partial \eta^2} = \varphi + \left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi) - \sum\left[\frac{\psi_s n_{s0}}{\alpha\left(-1+\left\{1-\frac{2\alpha\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}}\right)}\right] \tag{15a}$$

$$= -\frac{\partial V}{\partial \varphi} \tag{15b}$$

where V is the Sagdeev potential [29].

$$\begin{aligned} V = {} & \left[(1+3\beta) + \sum\frac{\psi_s n_{s0}}{\alpha^2}Q_s(M-u_{s0})^2\log(2)\right] - \\ & \frac{\varphi^2}{2} - \left[(1+3\beta) - 3\beta\varphi + \beta\varphi^2\right]\exp(\varphi) \\ & -\sum\frac{\psi_s n_{s0}}{\alpha^2}Q_s(M-u_{s0})^2\left[\begin{array}{l}\frac{1}{2}\left\{1-\frac{2\alpha\varphi}{Q_s(M-u_{s0})^2}\right\} - \left\{1-\frac{2\alpha\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}} \\ +\log\left(1+\left\{1-\frac{2\alpha\varphi}{Q_s(M-u_{s0})^2}\right\}^{\frac{1}{2}}\right)\end{array}\right] \end{aligned} \tag{16}$$

Now, expanding the r.h.s of Eq. (15a) in the power series of φ and taking the terms up to φ^3 for small amplitude ion acoustic wave, we obtain

$$\frac{d^2\varphi}{d\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + \cdots\cdots \tag{17}$$

where

$$A_1 = \left\{(2-\beta) + \frac{(Qn_{i0}-n_{j0})}{Q(M-u_0)}\right\} \tag{18a}$$

$$A_2 = \left\{\frac{1}{2} + \frac{3\alpha(Q^2n_{i0}-n_{j0})}{Q^2(M-u_0)^2}\right\} \tag{18b}$$

$$A_3 = \left\{\frac{1+3\beta}{6} + \frac{5\alpha(Q^3n_{i0}-n_{j0})}{Q^3(M-u_0)^6}\right\} \tag{18c}$$

and

$u_{i0} = u_{j0} = u_0$, $\psi_i = 1$, $\psi_j = -1$, $Q_i = 1$, $Q_j = Q = m_j/m_i$.

Now, integrating (15a), we get

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2 + V(\varphi) = 0 \tag{19}$$

In deriving Eq. (19), we have used the boundary conditions $\varphi \to 0$, $d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0$.

Equation (19) describes the motion of a pseudoparticle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $V(\varphi)$. The first term on the left hand side of Eq. (19) can be regarded as the kinetic energy of the pseudoparticle. Since the kinetic energy is always nonnegative, it follows that $V(\varphi) \le 0$ for the entire motion, zero being the maximum value of $V(\varphi)$. Equation (19) shows that $V'(\varphi)$ is the force on the particle at the position φ. Equation (19) may also be regarded as an anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the DL solution of Eq. (19), the Sagdeev potential $V(\varphi)$ should be negative between $\varphi = 0$ and φ_m, where φ_m is some extremum value of the potential φ, called the amplitude of the DLs. The DL solutions must additionally satisfy the following boundary conditions:

$$\text{(i) } V(\varphi) = 0, V'(\varphi) = 0 \text{ and } V''(\varphi) < 0\varphi = 0 \tag{20}$$

$$\text{(ii) } V(\varphi) = 0, V'(\varphi) = 0 \text{ and } V''(\varphi) < 0\ \varphi = \varphi_{\max}(\neq 0) \tag{21}$$

$$\text{(iii) } V(\varphi) < 0, \text{for } 0 < |\varphi| < |\varphi_m| \tag{22}$$

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudoparticle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity vanish. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m.

Applying the boundary conditions (21) and (22) in (17), we find that

$$\varphi_m = -2A_2/3A_3 \tag{23}$$

and

$$V(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{24}$$

The DL solution of (17) satisfying (22) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh\left[\sqrt{\left(A_3/8\varphi_m\right)}\eta\right]\right) \tag{25}$$

The thickness of the DLs is

$$w = 2\sqrt{A_3/8|\varphi_m|} \tag{26}$$

It is seen that the nature, amplitude, and width of DLs depend on the plasma parameters, i.e., on nonthermal parameter (β), density of negative ions (n_{j0}), drift velocity (u_0) of ions, and finite geometry (α) of bounded plasma through the coefficients A_2 and A_3.

It is to be noted that Eq. (23) relates the amplitude φ_m of DLs to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right hand side of Eq. (17). The DLs can only exist for positive value of A_3, i.e., for plasma parameters satisfying the following conditions for β, α, and n_{j0}:

$$\beta > \left\{\frac{1}{3} + \frac{10\alpha(Q^3n_{i0}-n_{j0})}{Q^3(M-u_0)^6}\right\} \tag{27a}$$

or

$$\alpha > \left\{\frac{Q^3(1+3\beta)(M-u_0)^6}{30(Q^3n_{i0}-n_{j0})}\right\} \tag{27b}$$

or

$$n_{j0} > Q^3(M-u_{s0})^6\left\{n_{i0} + \frac{(1+3\beta)}{30\alpha}\right\} \tag{27c}$$

where $M - u_{s0} > 0$.

From Eqs. (25) and (23), it follows that since A_3 is positive for the existence of DLs, the sign of A_2 will determine the nature of the DLs. If A_2 is positive (negative), a rarefactive (compressive) DL is formed. Since A_2 is independent of β, the nature of DLs, whether compressive or rarefactive, remains invariant under changes in β. However, the density of negative ions has effective role on the nature of DLs in the plasma. There is a critical value of negative ion for compressive and rarefactive DLs.

For compressive DLs, A_2 is negative, i.e.,

$$n_{j0} > \left[\frac{Q^2(M-u_0)^2}{3\alpha}\left\{\frac{1}{2}+\frac{3n_{i0}\alpha}{(M-u_0)^2}\right\}\right] \tag{28a}$$

and for rarefactive DLs, A_2 is positive, i.e.,

$$n_{j0} < \left[\frac{Q^2(M-u_0)^2}{3\alpha}\left\{\frac{1}{2}+\frac{3n_{i0}\alpha}{(M-u_0)^2}\right\}\right] \tag{28b}$$

It is evident that there is a critical value of negative ion density (n_{j0C}) for the occurrence of compressive and rarefactive DLs in the bounded plasma,

$$n_{j0C} = \left[\frac{Q^2(M-u_0)^2}{3\alpha}\left\{\frac{1}{2}+\frac{3n_{i0}\alpha}{(M-u_0)^2}\right\}\right] \tag{29}$$

when $M-u_{s0}>0$.

The value of n_{j0C} can be numerically estimated if the values of other plasma parameters Q, β, α, M, u_0, etc., are known.

4 Results and Discussions

We now investigate numerically the ion acoustic DLs in a multicomponent cold drifting plasma composed of positive ions, negative ions, and nonthermal electrons filled in a cylindrical waveguide. Our analysis reveals that both compressive and rarefactive DLs exist in the plasma under consideration for selected ranges of the plasma parameters. To see the nature of DLs in the bounded plasma, we have considered different types of negative and positive ions, e.g., (O^-, Ar^+) ions, (Cl^-, He^+) ions, and (O^-, H^+) ions which exist in space and also can be produced in laboratory. Moreover, we consider a plasma having physically admissible values of β, i.e., $0 \le \beta < 4/3$.

4.1 Sagdeev Potential in the Bounded Plasma

The nature of Sagdeev potential V is important for the existence of DLs in the bounded plasma. So, we have drawn the profiles of V for different values of α and β with fixed values of other plasma parameters n_{j0}, M, u_{i0}, Q.

4.1.1 The effect of *α*

To start with, we have plotted the Sagdeev potential V shown in Fig. 1 (a), (b), and (c) for different values of α of the bounded plasma having (O^-, Ar^+) ions for fixed values of other plasma parameters. It is seen from Fig. 1 (a) that Sagdeev potential V for different values of α of the bounded plasma for fixed values of $\beta=0.14$, $n_{j0}=0.12$, $u_0=0.1$, and $M=1.98$ are $V>0$ for $\alpha=2.02$, $V=0$ for $\alpha=2.12$, and $V<0$ for $\alpha=2.22$, 2.32. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ions when $\alpha=2.12$.

Figure 1 (b) shows that Sagdeev potential V for different values of α parameter of bounded plasma for fixed values of $\beta=0.1$, $n_{j0}=0.12$, $u_0=0.1$, and $M=2$ are $V>0$ for $\alpha=2.12$, $V=0$ for $\alpha=2.22$, and $V<0$ for $\alpha=2.1$, 2.2. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ion when $\alpha=2.22$.

From Fig. 1 (c), it is seen that Sagdeev potential V for different values of α parameter of the fixed values of $n_{j0}=0.2015$, $\beta=5.065$, $M=4$, and $u_0=0.1$ are $V>0$ for $\alpha=-4.5$, $V=0$ for $\alpha=-5.88$, and $V<0$ for $\alpha=-7.26, -8.64$. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ions when $\alpha=-5.88$.

4.1.2 The effect of *β*

The effect of nonthermal parameter β of electrons on the Sagdeev potential V in the bounded plasma having (O^-, Ar^+) ions for fixed values of other plasma parameters are shown in Fig. 2 (a), (b), and (c). In Fig. 2 (a), it is observed that the Sagdeev potential V for different values of β for fixed values of $\alpha=2.2$, $n_{j0}=0.12$, $M=1.98$, and $u_0=0.1$ is $V>0$ for $\beta=0.4$, $V=0$ for $\beta=0.3$, and $V<0$ for $\beta=0.2$ and $\beta=0.1$. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ions when $\beta=0.3$.

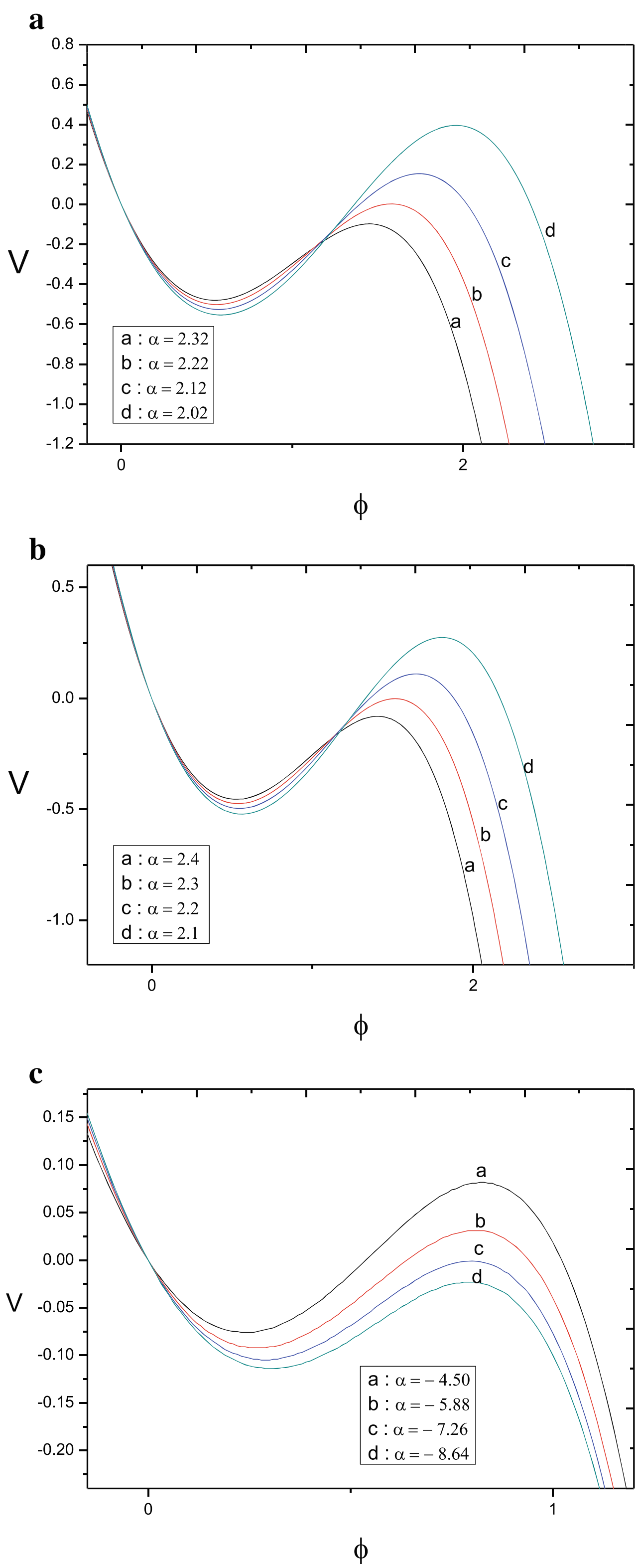
Figure 2 (b) shows that the Sagdeev potential V for different values of β in the bounded plasma for fixed values of $\alpha=2.1$, $n_{j0}=0.12$, $M=1.98$, and $u_0=0.1$ is $V>0$ for $\beta=0.48$, $V=0$ for $\beta=0.36$, and $V<0$ for $\beta=0.24$ and $\beta=0.12$. So, the DL is formed in the bounded plasma having (O^-, Ar^+) ions when $\beta=0.36$.

It is seen from Fig. 2 (c) that the Sagdeev potential V for different values of β in the bounded plasma for fixed values of $\alpha=-2.8$, $n_{j0}=0.12$, $M=4.6$, and $u_0=0.1$ is $V>0$ for $\beta=3.434$ and $\beta=3.851$, $V=0$ for $\beta=3.017$, and $V<0$ for $\beta=3.183$ and $\beta=2.6$. So, the DLs are formed in the bounded plasma having (O^-, Ar^+) ions when $\beta=3.017$.

4.2 Profiles of Double Layers in the Bounded Plasma

4.2.1 The effect of *α*

The effects of finite geometry of bounded medium (α) on the DLs in plasma having (O^-, Ar^+) ions and (Cl^-, He^+) ions have been numerically estimated, and the profiles are drawn keeping fixed values of other plasma parameters. It is seen from Fig. 3 (a) that the DLs are rarefactive in plasma having (O^-, Ar^+) ions, and the amplitudes are increased with the decrease



◀ **Fig. 1** (a) (color online).The profiles of Sagdeev potential for different values of parameter α of the bounded plasma having (O^-, Ar^+) ions for fixed values of $\beta = 0.14$, $n_{j0} = 0.12$, $u_0 = 0.1$, and $M = 1.98$. Curves a, b, c, and d refer to four different values $\alpha = 2.32$, 2.22, 2.12, and 2.02, respectively. (b) (color online). The profiles of Sagdeev potential for different values of parameter α of the bounded plasma having (O^-, Ar^+) ions for fixed values of $\beta = 0.1$, $n_{j0} = 0.12$, $u_0 = 0.1$, and $M = 2$. Curves a, b, c, and d refer to four different values $\alpha = 2.4$, 2.3, 2.2, and 2.1, respectively. (c) (color online). The profiles of Sagdeev potential for different values of parameter α of the bounded plasma having (O^-, Ar^+) ions for fixed values of $n_{j0} = 0.2015$, $\beta = 5.065$, $M = 4$, and $u_0 = 0.1$. Curves a, b, c, and d refer to four different values $\alpha = -4.50$, -5.88, -7.26, and -8.64, respectively

of α. But, Fig. 3 (b) shows that DLs in plasma having (Cl^-, He^+) ions may be compressive or rarefactive depending on the values of α. The DLs are compressive for $\alpha = -2.10, -1.99$, and rarefactive for $\alpha = -1.88, -1.77$. The amplitudes of compressive DLs are decreased with the decrease of α, and for rarefactive DLs, the amplitudes are increased with decrease of α.

4.2.2 The effect of β

The effects of β of nonthermal electrons on the DLs in plasma having Q = 0.4 (O^-, Ar^+) and Q = 16 (O^-, H^+) have been numerically estimated, and the profiles are drawn keeping fixed values of other plasma parameters, i.e., dimension of finite geometry of bounded plasma, Mach number, ion stream velocity, and concentration of negative ions. It is seen from Fig. 4 (a) that the DLs are rarefactive in plasma having (O^-, Ar^+) ions, and the amplitudes of DLs are increased with the increase of β for fixed values of $\alpha = -2$, $M = 2$, $u_0 = 0.2$, $n_{j0} = 0$. But, Fig. 4 (b) shows that the DLs in plasma having (O^-, H^+) ions are compressive, and the amplitude of DLs is increased with the increase of β.

4.3 Width of Double Layers in the Bounded Plasma

It is well known that the width of DLs in plasma is important for the experimental studies. So, we have numerically estimated the width of DLs using Eq. (26), and the variation of width of DLs is graphically discussed for different parameters of the bounded plasma system.

4.3.1 The effect of α

The variation of DL width with negative ion density (n_{j0}) for different values of α is shown in Fig. 5 (a) and (b) for the plasma having (O^-, Ar^+) ions and (Cl^-, He^+) ions. It is seen from Fig. 5 (a) for the plasma having (O^-, Ar^+) ions that the DL width decreases with the increase of n_{j0} up to a critical negative ion density n_{j01}; then, it increases steadily with the increase of n_{j0}. Moreover, the DL width is larger for large value α. The DL width in plasma with (Cl^-, He^+) ions is shown in Fig. 5 (b). It is observed that the DL width decreases

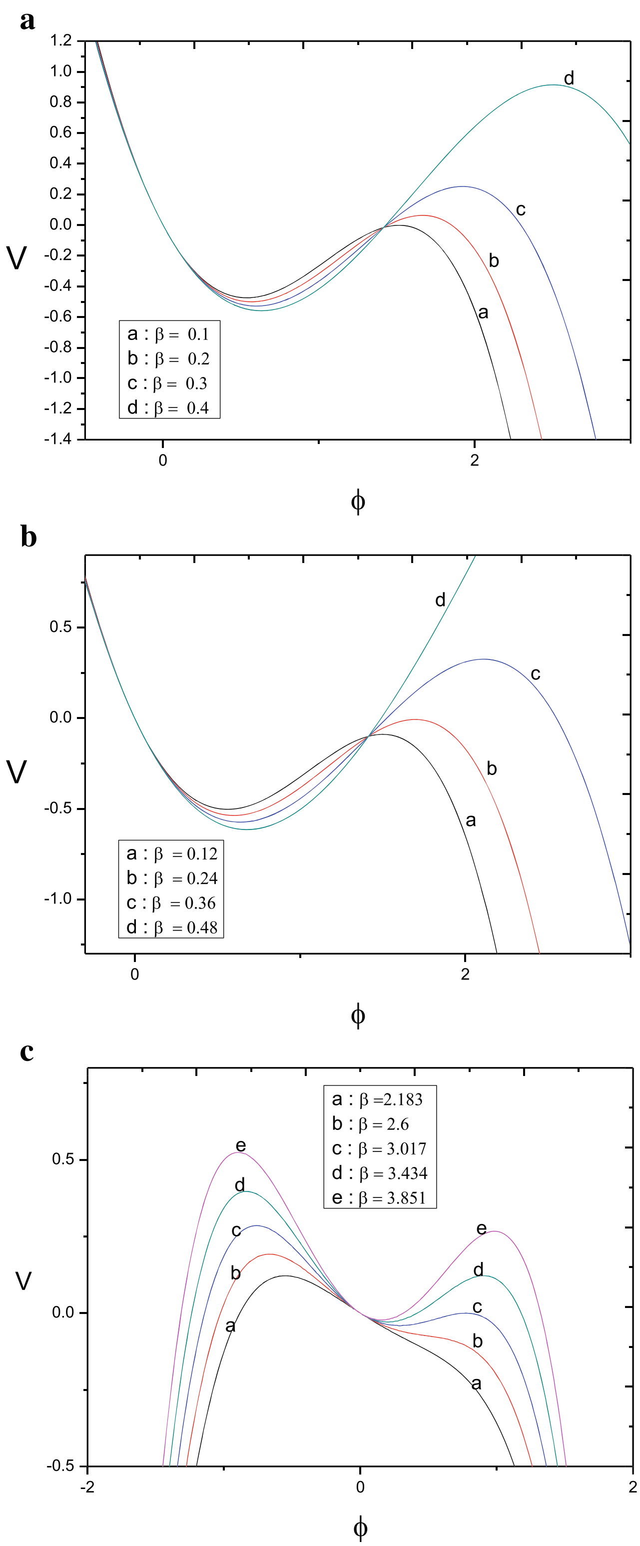


Fig. 2 (a) (color online). The profiles of Sagdeev potential for different values of nonthermal parameter β of electrons in the bounded plasma having (O^-, Ar^+) ions for fixed values of $\alpha = 2.2$, $n_{j0} = 0.12$, $M = 1.98$, and $u_0 = 0.1$. Curves a, b, c, and d refer to four different values $\beta = 0.1$, 0.2, 0.3, and 0.4, respectively. (b) (color online). The profiles of Sagdeev potential for different values of nonthermal parameter β of electrons in the bounded plasma having (O^-, Ar^+) ions for fixed values of $\alpha = 2.1$, $n_{j0} = 0.12$, $M = 1.98$, and $u_0 = 0.1$. Curves a, b, c, and d refer to four different values $\beta = 0.12$, 0.24, 0.36, and 0.48, respectively. (c) (color online). The profiles of Sagdeev potential for different values of nonthermal parameter of electrons β in the bounded plasma having (O^-, Ar^+) ions for fixed values of $\alpha = -2.8$, $n_{j0} = 0.12$, $M = 4.6$, and $u_0 = 0.1$. Curves a, b, c, and d refer to five different values $\beta = 2.183$, 2.6, 3.017, 3.434, and 3.851, respectively

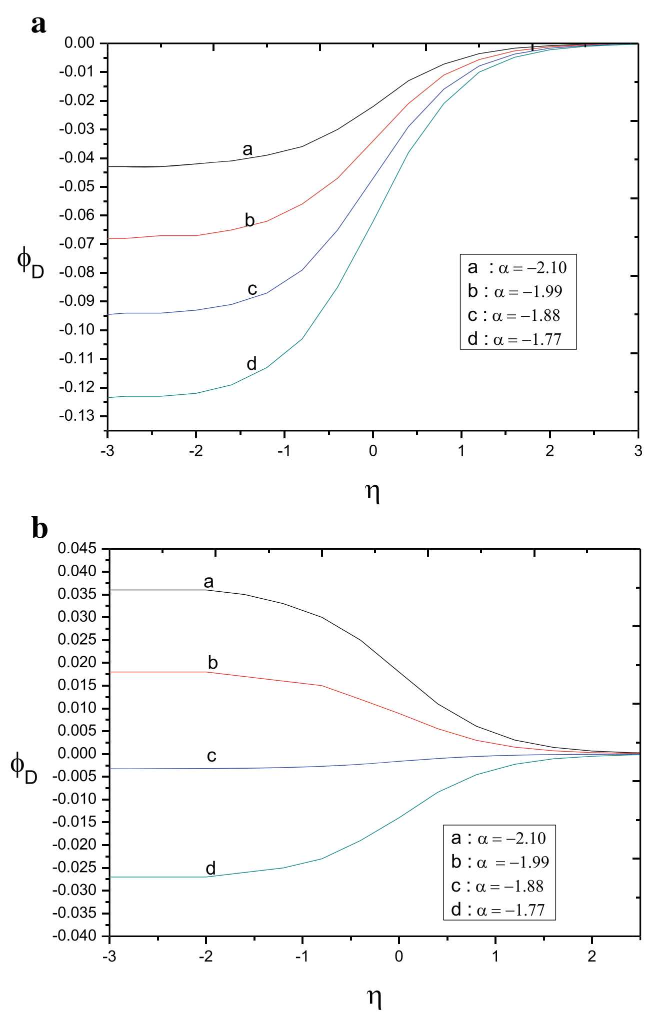


Fig. 3 (a) (color online). The profile of double layers for different values of α in the bounded plasma having (O^-, Ar^+) ions for fixed values of $n_{j0} = 0.03$, $M = 2$, $u_0 = 0.15$, and $\beta = 0.5$. Curves a, b, c, and d refer to four different values $\alpha = -2.10$, -1.99, -1.88, and -1.77, respectively. (b) (color online). The profiles of double layers for different values of α in the bounded plasma having (Cl^-, He^+) ions for fixed values of $n_{j0} = 0.03$, $M = 2$, $u_0 = 0.15$, and $\beta = 0.5$. Curves a, b, c, and d refer to four different values $\alpha = -2.10$, -1.99, -1.88, and -1.77, respectively

with n_{j0}, but for small α, $\alpha = -1.77$, the DL width increases with n_{j0} and attain a maximum value at negative ion density n_{j01}; then, it decreases with the increase of n_{j0}. Moreover, the DL width is large for small value of α.

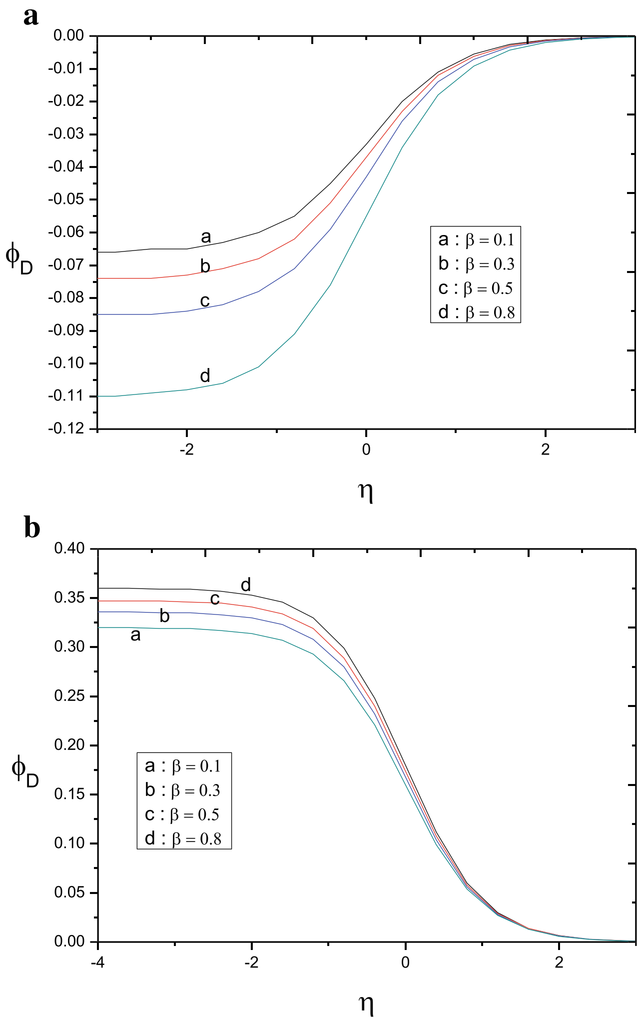


Fig. 4 (a) (color online). The profiles of double layer for different values of nonthermal parameter β in the bounded plasma having (O^-, Ar^+) ions for fixed values of $\alpha = -4$, $M = 2$, $u_0 = 0.2$, and $n_{j0} = 0.1$. Curves a, b, c, and d refer to four different values $\beta = 0.1$, 0.3, 0.5, and 0.8, respectively. (b) (color online). The profile of double layers for different values of nonthermal parameter β in the bounded plasma having (O^-, H^+) ions for fixed values of $\alpha = -4$, $M = 2$, $u_0 = 0.2$, and $n_{j0} = 0.1$. Curves a, b, c, and d refer to four different values $\beta = 0.1$, 0.3, 0.5, and 0.8 respectively

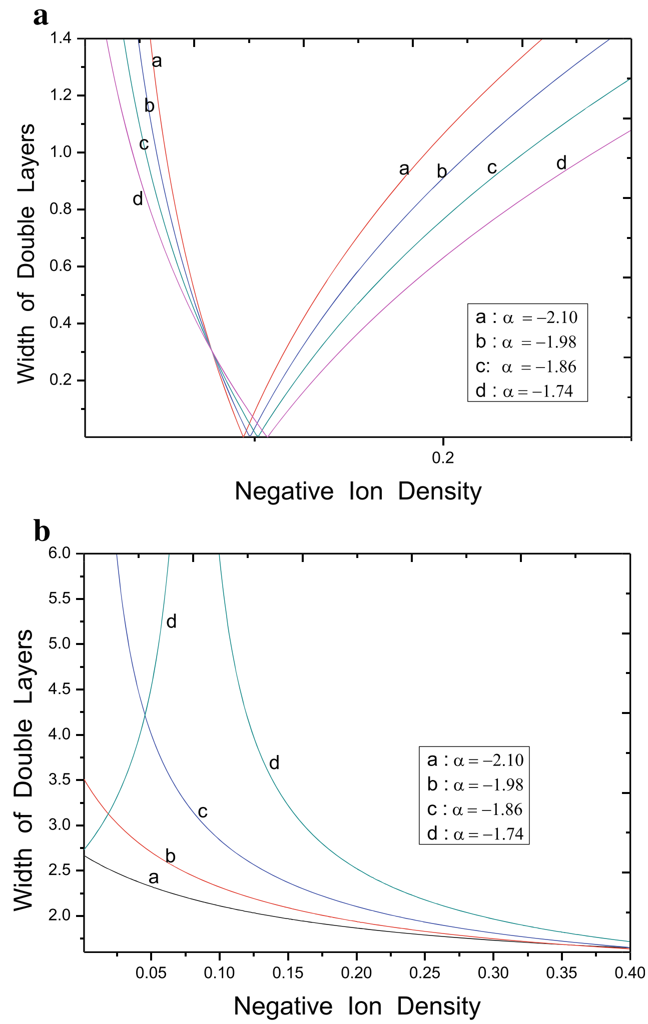


Fig. 5 (a) (color online). Variation of width of double layers with negative ion density n_{j0} for different values of α in the bounded plasma having (O^-, Ar^+) ions for fixed values of $M = 2$, $\beta = 0.1$, and $u_0 = 0.2$. Curves a, b, c, and d refer to four different values $\alpha = -2.10$, -1.98, -1.86, and -1.74 respectively. (b) (color online). Variation of width of double layers with negative ion density for different values of α in the bounded plasma having (Cl^-, He^+) ions for fixed values of $M = 2$, $\beta = 0.1$, and $u_0 = 0.2$. Curves a, b, c, and d refer to four different values $\alpha = -2.10$, -1.98, -1.86, and -1.74, respectively

4.3.2 The effect of β

Similarly, the variation of DL width with negative ion density (n_{j0}) for different values of nonthermal parameter of electrons (β) is shown in Fig. 6 (a) and (b) for the plasma having (O^-, Ar^+) ions and (Cl^-, He^+) ions. In Fig. 6 (a), it is seen that the DL width decreases with increase of n_{j0} up to a negative ion density n_{j01}; then, it increases with the increase of n_{j0} up to a certain maximum value of DL width at negative ion density n_{j02}., after that, the DL width decreases and attains certain values at negative ion density n_{j03}; then, it increases with the increase of n_{j0}. Moreover, at n_{j01} the DL width is smaller for larger value of β, at n_{j03}; the DL width is smaller for larger value of β, and at is observed from Fig. 6 (b) that the DL width increases with increase of n_{j0}, and the DL width is larger for larger value of β.

5 Summary and Conclusion

Considering the multicomponent plasma composed of cold positive ions, negative ions, and warm nonthermal electrons bounded in a finite geometry cylindrical tube, we have theoretically studied the excitation of ion acoustic DLs in the plasma. It has been shown that the formation of DLs in the bounded plasma depends on the plasma parameters, i.e., finite

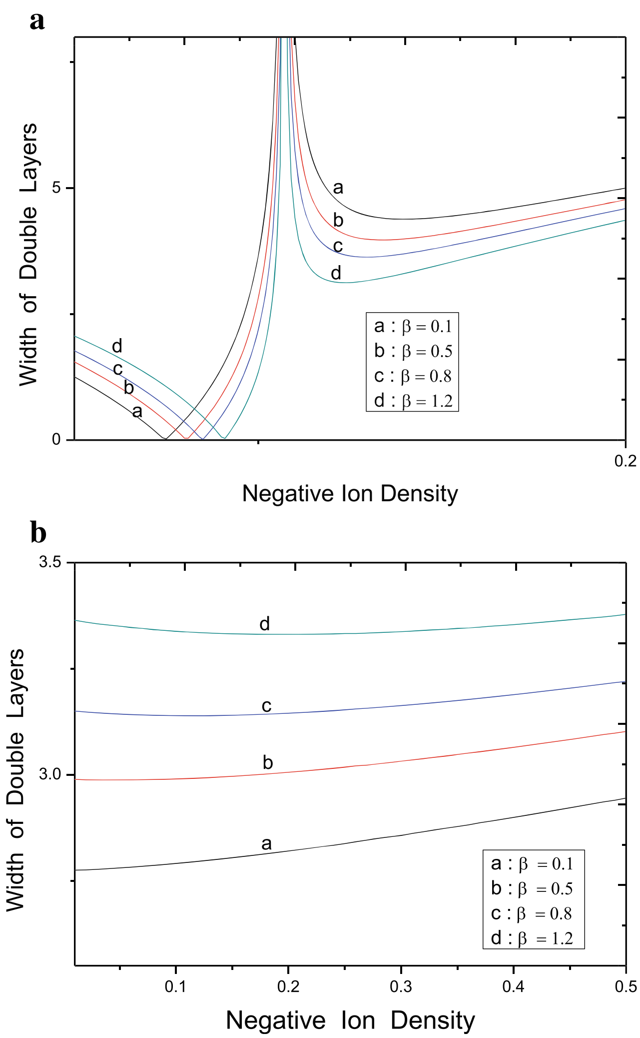


Fig. 6 (a) (color online). Variation of width of double layers with negative ion density for different values of nonthermal parameter β in the bounded plasma having (O^-, Ar^+) ions for fixed values of $M = 2$, $\alpha = -4$, and $u_0 = 0.2$. Curves a, b, c, and d refer to four different values $\beta = 0.1$, 0.5, 0.8, and 1.2 respectively. (b) (color online).Variation of width of double layers with negative ion density for different values of β in the bounded plasma having (O^-, H^+) ions for fixed values of $M = 2$, $\alpha = -4$, and $u_0 = 0.2$. Curves a, b, c, and d refer to four different values $\beta = 0.1$, 0.5, 0.8, and 1.2, respectively

geometry of the bounded medium, nonthermal parameters of electrons, density of negative ions, and stream velocity of ions. Numerical estimation is made taking different types of positive and negative ions in the plasma, and the profiles of DLs are drawn. The DLs in the bounded multicomponent plasma may be compressive or rarefactive depending upon the values of the plasma parameters.

Our main findings are:

- The DLs are rarefactive in plasma having (O^-, Ar^+) ions and the amplitudes are increased with the increase of α. But, the DLs in plasma having (Cl^-, He^+) ions may be compressive or rarefactive depending on the values of α. The DLs are compressive for $\alpha = -2.10$, -1.99, and rarefactive for $\alpha = -1.88$, -1.77. Moreover, the amplitudes of compressive DLs are decreased with the decrease of α, and for rarefactive DLs, the amplitudes are increased with the decrease of α.
- The DLs are rarefactive in plasma having (O^-, Ar^+) ions, and the amplitudes are increased with the increase of β. But, the DLs in plasma having (O^-, H^+) ions are compressive, and the amplitude of DLs is increased with the increase of β.
- The DL width in plasma having (O^-, Ar^+) ions decreases with the increase of n_{j0} up to a critical negative ion density n_{j01}; then, it increases steadily with the increase of n_{j0}. Moreover, the DL width is larger for larger value of α. In plasma with (Cl^-, He^+) ions, the DL width decreases with n_{j0}, but for small α, $\alpha = -1.77$, the DL width increases with n_{j0} and attains a maximum value at negative ion density n_{j01}, then it decreases with the increase of n_{j0}. Moreover, the DL width is larger for smaller values of α.
- The DL width in plasma having (O^-, Ar^+) ions decreases with the increase of n_{j0} up to a negative ion density n_{j01}; then, it increases steadily with the increase of n_{j0} up to certain maximum value at negative ion density n_{j02}., after that, the DL width decreases and attains certain values at negative ion density n_{j03}; then, it increases with n_{j0}. At n_{j01}, the DL width is smaller for larger values of β, at n_{j03}; the DL width is small for large value β. But, in a plasma with (Cl^-, He^+) ions, the DL width increases with the increase of n_{j0}, and the DL width is larger for larger values of β.

Finally, we would like to point out that the type of plasma considered in the present work exists in space and also can be produced in the laboratory. Following our present analysis, ion acoustic DLs can be studied in the bounded plasma having different types of electrons, namely, two-temperature electrons, superthermal electrons, and Tsallis-distributed electrons, and also in dusty plasma. In the present study, we have not considered the temperatures of positive ions and negative ions and the static magnetic field in the plasma. Considering these parameters more interesting, results may be obtained which would give new information on the DLs in the bounded plasma. The DLs in plasma have been observed experimentally [11–14]. But, there is no information about experimental observation on the DLs in a bounded multicomponent plasma having nonthermal electrons. So, our theoretical results may not be verified/compared with the experimental observations.

To study the ion acoustic double layers in bounded multicomponent plasma, we have used the pseudopotential technique. Using cylindrical coordinates and perturbation method, one can study ion acoustic double layers in the bounded plasma following Das et al. [15], Labany et al. [17], and also Paul et al. [30] and Mondal et al. [31], using Cartesian coordinates. We shall study this problem in the near future.

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References

1. S. Watanabe, J. Phys. Soc. Jpn. **53**, 950 (1984)
2. M.K. Mishra, R.S. Tiwari, J.K. Chawla, Phys. Plasmas **19**, 062303 (2012)
3. K. Kumar, M.K. Mishra, AIP Adv. **7**, 115114 (2017)
4. R.A. Cairns, A.A. Mamun, R. Bingham, P.K. Shukla, Phys. Scr. **T63**, 80 (1996)
5. B. Ghosh, S.N. Paul, C. Das, I. Paul, S. Banerjee, Braz. J. Phys. **43**, 28–33 (2013)
6. B. Ghosh, S. Banerjee, S.N. Paul, Indian J. Pure Appl. Phys. **51**, 488–493 (2013)
7. I. Paul, S. Chandra, S. Chattopadhyay, S.N. Paul, Indian J. Phys. **90**, 1195–1205 (2016)
8. S. Chattopadhyay, Sri Lanka J. Phys. **20**, 1–16 (2019)
9. K.E. Lonngren, Plasma Phys. **25**, 943–982 (1983)
10. Y. Nakamura, H. Sugai, Chaos Solitons Fractals **7**, 1023–1031 (1996)
11. S. Torvén, *Formation of double layers in laboratory plasmas*, Wave Instabilities in Space Plasmas (1979), pp. 109–128
12. N. Hershkowitz, Review of recent laboratory double layer experiments. Space Sci. Rev. **41**, 351–391 (1985)
13. M.A. Raddu, Phys. Rep. **178**, 25–97 (1989)
14. C. Charles, Plasma Sources Sci. Technol. **16**, R1–R25 (2007)
15. G.C. Das, K.I. Singh, B. Karmakar, Can. J. Phys. **67**(6), 609–613 (1989)
16. G.C. Das, K.H. Ibohanbi, Singh, Aust. J. Phys. **44**(5), 523–534 (1991)
17. S.K. EL-Labany, S.A. EL-Warraki, W.M. Moslem, J. Plasma Phys. **63**(**4**), 343–353 (2000)
18. B. Ghosh, K. Das, J. Plasma Phys. **40**, 545 (1988)
19. S. Kaur, Spherical and cylindrical ion-acoustic solitary waves in electron-positron-ion plasmas, Ph.DThesis-2017, Punjab Agricultural University
20. B. Sahu, D. Roy, Phys. Plasmas **24**, 112705 (2017)
21. V.K. Sayal, S.R. Sharma, Phys. Scr. **42**, 475 (1990)
22. V.K. Sayal, S.R. Sharma, Phys. Lett. A **149**, 155–158 (1990)
23. S.K. Bhattacharya, S.N. Paul, B. Chakraborty, Indian J. Phys. **77B**(3), 327–334 (2003)
24. K.K. Mondal, S.K. Bhattacharya, S.N. Paul, FIZIKA **A 13**, 77–88 (2004)
25. R.A. Cairns, R. Bingham, R.O. Dendy, C.M.C. Nairn, P.K. Shukla, A.A. Mamun, J. Phys, IV France **5**, C6–C43 (1995)
26. R.A. Cairns, A.A. Mamun, R. Bingham, R. Bostrom, R.O. Dendy, C.M.C. Nairn, P.K. Shukla, Geophys. Res. Lett. **22**, 2709 (1995)
27. R. Bostrom, G. Gustafsson, B. Holback, G. Holmgren, H. Koskinen, P. Kintner, Phys. Rev. Lett. **61**, 82 (1988)
28. P.O. Dovner, A.I. Erikson, R. Bostrom, B. Holback, Geophys. Res. Lett. **21**, 1827 (1994)
29. R.Z.Sagdeev, in Reviews of Plasma Physics, ed. by M.A. Leontovich (Consultants Bureau, New York, 1996), **4**, p. 23
30. S.N. Paul, K.K. Mondal, A. Roychowdhury, FIZIKA **B 5**, 73–84 (1996)
31. K.K. Mondal, S.N. Paul, A. Roychowdhury, IEEE Trans. Plasma Sci. **26**, 987–994 (1998)

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ORIGINAL PAPER

Effects of nonthermal electrons and ion beams on ion-acoustic double layers in warm ion plasma

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Abstract: The effects of nonthermal electrons and ion beams on the ion acoustic double layers in warm ion plasma have been studied using the pseudopotential technique. Sagdeev potential of ion-acoustic wave in the plasma has been derived and the solution of ion acoustic double layers has been obtained. The critical value of nonthermal parameter of electrons for existence of double layers is obtained and discussed. The profiles of double layers in the plasma are drawn taking different values of density and temperature of ion beams, and the nonthermal parameter of electrons obeying the critical condition. It has been shown that both compressive and rarefactive double layers are excited in the plasma depending on the values of nonthermal electrons and ion beam parameters. The width of double layers is also graphically discussed. The results are new and may be useful in understanding the acceleration processes of charged particles in space plasma.

Keywords: Ion acoustic wave; Nonthermal electron; Ion beam; Sagdeev potential; Double layers

1. Introduction

In recent years, there is a great deal of interest on the study of nonlinear propagation of waves in plasma consisting of electron/ion beams because of its importance in understanding the nonlinear electrostatic disturbances in magnetosphere and solar plasma [1, 2]. The nonlinear structure of plasma is changed considerably in presence of electron/ion beams. Krivoruchku et al. [3] have shown that an electron beam in plasma can change the propagation of the Trivelpiece-Gould solitons. Later, it is shown by Gell et al. [4] that compressive solitary waves are excited in an electron beam plasma system. However, the effects of ion beams are also important to excite the ion acoustic solitary waves (IASWs) in plasma. Abrol and Tagare [5] have obtained a modified Koteweg-deVries (KdV) equation for IASWs in presence of ion beams in plasma composed of cold ions, beam ions and nonisothermal electrons. They have critically discussed the effect of resonant electrons, trapped and free electrons, plasma-ion to beam-ion mass ratio and beam-ion concentration on the amplitude and width of IASWs. Later, Karmakar et al. [6] have obtained the solution of IASWs in ion-beam plasma with multiple-electron-temperatures through the derivation of a modified KdV equation. It is seen that the existence solitons and its behaviour depends effectively on the ion beam as well as on the multiple electron temperatures. The solitons might be large amplitude with the addition of a small percentage of ion-beam concentration or by the increase of electron-temperatures. Subsequently, El-Labany [7] has used the reductive perturbation method (RPM) for investigating the IASWs in an ion beam plasma system consisting of warm ions and isothermal electrons in presence of warm ion beams. He has obtained the KdV and KdV-type equations for first- and second-order perturbed potentials. For obtaining the soliton solution, he has used the renormalization method to remove the secular terms in nonlinear equation. It is found that both the amplitude and the width of solitons are strongly affected by the ion temperatures as well as the velocity of ion beam. Das [8] have investigated the effect of ion temperature on small-amplitude IASWs in a magnetized ion-beam plasma in presence of electron inertia. It is shown from the KdV equation that both compressive and rarefactive solitons will exist in a magnetized plasma model consisting of ions, electrons and positive ion beams. Both fast and slow modes are found to

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exist due to the presence of ion temperature in the plasma. Moreover, the amplitude of soliton decreases with increase in temperature for the mass ratio of beam-ion to warm plasma-ion greater than two. The amplitude becomes maximum when the wave propagates parallel with the direction of the magnetic field. The investigation further revealed that though both compressive and rarefactive solitons exist for slow-mode, only compressive soliton exist for the fast-mode. Later, Das and Deka [9] have considered relativistic electron beam for the study of small amplitude IASWs in plasma composed of positive ion, electron and electron beams. Using the RPM, they have derived the KdV equation from which it is found that both compressive and rarefactive solitons are excited in such plasma. Kaur et al. [10] have investigated the propagation characteristics of dust acoustic solitary and rogue waves in an unmagnetized ion beam plasma with electrons and ions following kappa-type distribution in nonplanar geometry. The RPM is employed to derive the cylindrical/spherical KdV equation, which is further transformed into the standard KdV equation by neglecting the geometrical effects. Using new stretching coordinates, nonlinear Schrödinger equation has also been derived from the standard KdV equation to study the different order rational solutions of dust acoustic rogue waves. The impact of various physical parameters on the characteristics of dust acoustic solitary waves is elaborated specifically in nonplanar geometry. Further, the effects of ion beam and superthermality of electrons/ions on the characteristics of rogue waves are studied. It is predicted that the results obtained may be useful in comprehending a variety of phenomena in Earth's magnetosphere polar cap region where the presence of positive ion beam has been detected and also in other regions of space/astrophysical environments where dust along with superthermal electrons and ions exist. Recently, Paul et al. [11] have studied nonlinear propagation of ion-acoustic waves in unmagnetized quantum (degenerate) plasma in the presence of an ion beam using the one-dimensional quantum hydrodynamic model. They have derived the KdV equation using the RPM and have obtained the solution of IASWs. The critical value of ion beam density for the nonexistence of solitary wave is numerically estimated and its variation with velocity and temperature of ion beam has been discussed graphically. It is seen that formation and structure of solitary waves are significantly affected by the ion beam in quantum plasma. The solitary waves on the quantum plasma may be compressive or rarefactive depending upon the values of velocity, concentration and temperature of the ion beam.

On the other hand, the nonthermal electrons in plasma also give rise to many interesting characteristics in nonlinear excitation of IASWs and IADLs. The nonthermal electron plasma (cold plasma or non-equilibrium plasma) is a plasma which is not in thermodynamic equilibrium, because the electron temperature is much hotter than the temperature of heavy species (ions and neutrals). To explain the observational results of the Freja [12] and Viking [13] satellites it has been assumed by Cairns et al. [14, 15] that the electron distribution to be nonthermal in the magnetosphere [16]. Considering the presence of nonthermal electrons in plasma several authors have theoretically studied the IASWs [17–19]. Gill et al. [20] have studied the properties of IASWs and IADLs in a plasma consisting of warm positive and negative ions with different concentration of masses, charged states and nonthermal electrons using small amplitude approximation. The KdV and modified KdV equations are derived using the RPM. Existence of IASWs and IADLs is explored over a wide range of parameter space. The role of nonthermal electrons characterized by finite value of nonthermal parameter is also investigated. It is observed that for a particular value of nonthermal parameter, there is a transition from compressive to rarefactive solitons. However, when nonthermal parameter is increased beyond a critical value, no IADLs are obtained. The significance of relative ion masses is also investigated. Later, Bhattacharyya et al. [21] have investigated the effects of non-thermal electrons on the IASWs in a bounded plasma using Sagdeev potential technique (SPT) and have shown that both compressive and rarefactive solitons would be excited for any value of nonthermal electron parameter in plasma. Subsequently, Das et al. [22] have studied the nonlinear behaviour of ion-acoustic waves in a plasma consisting of warm adiabatic ions and nonthermal electrons having vortex-like velocity distribution function, immersed in a uniform and static magnetic field using the RPM. They have obtained the condition for the existence of DLs solution in such plasma. Later, Paul et al. [23] have obtained exact analytical solutions of the nonlinear equations governing the propagation of IASWs in a plasma consisting of warm streaming positive ions and nonthermal electrons with necessary and sufficient conditions for the existence of IASWs. The critical values of nonthermal parameter, ion temperature and velocity of IASWs have been numerically evaluated and discussed the dependence on the plasma parameters. They have analysed the structure of the solitary waves as a function of the nonthermal electron parameter which measures the deviation from the thermalized state. Later, Ghosh et al. [24] have used the SPT to investigate theoretically the effect of nonthermal electrons and warm negative ions on IASWs in multicomponent drifting plasma. Subsequently, the IADLs in such plasma have been theoretically investigated by Ghosh et al. [25]. The conditions for the existence of IADLs which control the formation and nature of DLs are obtained. The effects of nonthermal electrons, negative ion concentration, and negative ion temperature on the formation and the structure of IADLs are also investigated. Ghosh and Banerjee [26] have studied the IASWs in collision less unmagnetized plasma composed of nonthermal electrons and positrons, warm drifting positive ions using the SPT. The nonthermal parameters and temperatures are shown to play significant roles on the formation and nature of solitary waves. It is shown that the nature, amplitude and width of lower- and higher-order solitons depend sensitively on these plasma parameters. It is also found that stronger nonthermality leads to shorter and wider solitons. The dependence of Sagdeev potential profile on different plasma parameters has also been examined. Paul et al. [27] have considered higher nonlinear and dispersive effect for the study higher order IASWs in plasma consisting of cold ions and nonthermal electrons using the SPT and Bogoliubov–Mitropolosky method. It is seen that shape of higher-order IASWs in such plasma are W-type. Using the SPT, Gao et al. [28] have studied the IADLs and ion-acoustic super solitons in a plasma with Maxwellian cold electrons, nonthermal hot electrons, and fluid ions.

But, few authors have studied the double layers (DLs) in an electron/ion beam plasma system. Taibany [29] have studied electro-acoustic solitary waves and double layers with an electron beam and phase space electron vortices in space plasmas using the RPM. He has considered a plasma composed of a cold electron fluid, hot electrons obeying trapped/vortex-like distribution, warm electron beam, and stationary ions. In the vicinity of the isothermal population, a nonlinear evolution equation with mixed nonlinearity is obtained. Its solution gives a (compressive/rarefactive) soliton or a compressive double layer (DL) depending on the system parameters. Later, Shan and Saleem [30] have investigated IADLs in an unmagnetized and collisionless plasma consisting of cold positive ions, q-nonextensive electrons, and a cold electron beam. Small amplitude DLs solution is obtained by expanding the Sagdeev potential truncated method. The effects of entropic index q, speed and density of cold electron beam on double layer structures are discussed. In ion beam plasma system, Reddy et al. [31] have developed a general analysis for small amplitude IADLs and IASWs, taking into account any number of ion beams and their charges, together with cold and hot electrons. For auroral plasma parameters, the analysis predicts the excitation of fast and slow hydrogen (as well as oxygen) beam-acoustic modes, which can be either rarefactive DLs or rarefactive or compressive solitons. Variations in the temperature and beam speed of the hydrogen (oxygen) beam can lead to the conversion of modes from originally rarefactive DLs to rarefactive solitons and finally to compressive solitons. Fast and slow hydrogen beam-acoustic modes are the first to be excited. The excitation of fast and slow oxygen modes is usually possible for larger values of beam speeds or beam temperatures. The typical width and speed of the nonlinear modes are in good agreement with the observations of DLs and SWs by the *S*3-3 and *Viking* satellites. In a dusty plasma, Das et al. [32] have studied the formation of large-amplitude DLs in a plasma whose constituents are electrons, ions, warm dust grains and positive ion beam using the SPT. It is found that both the temperature of dust particles and ion beam temperature play significant roles in determining the region of the existence of double layers. Later, Kaur et al. [33] have derived the KdV, modified KdV and Gardner equations using the RPM and have studied the acoustic Gardner SWs and DLs in a multi-component superthermal plasma having the ion beams. The characteristic features of both compressive and rarefactive nonlinear excitations from the solution of these equations are discussed. The results have been compared with the observation of the He^+ ion beam in polar cap region near solar maximum by the Dynamics Explorer-1 satellite. The characteristics of DLs are also studied in detail below the critical density of cold electrons. It is predicted that the results may be useful for the observation of nonlinear excitations in laboratory and ion beam driven plasmas in the polar cap.

Recently, Paul et al. [34] have theoretically investigated the IADLs in a multicomponent plasma consisting of positive ions, negative ions and nonthermal electrons filled in a cylindrical wave guide using the SPT. It is seen that the finite geometry of bounded plasma, nonthermality of electrons, density of negative ions, stream velocity of positive and negative ions have significant contributions on the structure of IADLs. The DLs may be compressive or rarefactive in the plasma depending upon the values of the plasma parameters.

From the works of above authors, it is seen that a lot of theoretical works have been done on the IASWs in plasma having ion beams and nonthermal electrons. But, IADLs in nonthermal plasma having ion beams have not been studied by any authors. So, we are interested here to study the IADLs using the SPT in a multicomponent plasma composed of warm positive ions and beam ions together with nonthermal electrons. For solving the problem, we have derived the expression of Sagdeev potential and have obtained the nonlinear equation for the propagating ion acoustic waves. The solution for IADLs is obtained and graphically discussed for different values of density, velocity and temperature of the ion beam, and nonthermality of electrons. The widths of the IADLs are also discussed graphically.

2. Formulation

We assume, a collision less unmagnetized plasma consisting of warm positive ions, warm beam ions, and nonthermal electrons. Moreover, we consider that the ion beam velocity is constant in unperturbed state and ion beams are in the direction of propagation of waves. The normalized basic equations in unidirectional propagation for such plasma in dimensionless form are:

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s u_s)=0 \tag{1}$$

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}+\frac{\sigma_s}{Q_s n_i}\frac{\partial p_i}{\partial x}=\psi_s\frac{Z_s}{Q_s}\frac{\partial \varphi}{\partial x} \tag{2}$$

$$\frac{\partial p_s}{\partial t}+u_s\frac{\partial p_s}{\partial x}+3p_s\frac{\partial u_s}{\partial x}=0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+\sum_s \psi_s Z_s n_s \tag{4}$$

where the subscript s= i and b denote positive ions and beam ions; n_s, u_s, p_s, σ_s are, respectively, the density, velocity, pressure and temperature of positive ion and beam ions. The concentration of nonthermal electrons is represented by n_e, $Q_b=m_b/m_i$, $Q_i=1$, m_i and m_b are the mass of positive ions and beam ions, $\psi_i=\psi_b=-1$, Z_i and Z_b are the charges of ions and ion beam and negative. For single charged ions, $Z_i=1$, multiple charged ion beams $Z_b=Z$, Z is the number of positive charges of ion beams. The velocities are normalized by the ion-acoustic speed $(K_B T_e/m_i)^{1/2}$, the densities by the equilibrium ion density n_0, all lengths by the Debye length $(K_B T_e/4\pi n_0 e^2)^{1/2}$, the time variable by the ion plasma period $\omega_p^{-1}=(m_i/4\pi n_0 e^2)^{1/2}$ and the potential φ by $(K_B T_e/e)$ where, T_e is the temperature of electron, K_B is the Boltzmann constant and other symbols have their usual meanings.

The nonthermal electron distribution function is chosen as [14, 15]

$$f_e(V)=\frac{n_o}{(1+3\alpha)\sqrt{2\pi V_e^2}}\left[1+\alpha\left(\frac{v}{V_e^2}-2\phi\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{v}{V_e^2}-2\phi\right)\right] \tag{5}$$

where $\alpha(\geq 0)$ is a parameter that determines the proportion of the fast energetic electrons, v is the velocity of electrons in phase space and is normalized by the average thermal speed $V_e=\sqrt{K_B T_e/m_e}$ and m_e is the mass of electron.

The normalized density of nonthermal electron n_e is

$$n_e=\left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi) \tag{6a}$$

$$\text{where}\quad \beta=\frac{4\alpha}{1+3\alpha} \tag{6b}$$

β measures the deviation from thermalized state. The parameter β cannot exceed a certain range $(0\leq\beta\leq 1.3)$, beyond which α is negative which is not physically acceptable. When $\beta\to 0$, Eq. (5) gives the Boltzmann distribution of electrons.

For obtaining the solution of IADLs from Eqs. (1) to (4), we use the transformation $\eta=x-Vt$, where V is the velocity of ion acoustic waves and we assume the boundary conditions:

$$n_i\to n_{io}, n_b\to n_{bo}, u_i\to 0, u_b\to u_{bo}, p_i\to p_{io}, p_b \to p_{bo}, \varphi\to 0 \quad \text{at} \quad |x|\to\propto \tag{7}$$

The charge neutrality condition in the plasma is $n_{i0}=1-Zn_{b0}$.

By using Eq. (4) and the boundary conditions (7), we get from Eqs. (1) to (6)

$$\begin{aligned}\frac{d^2\phi}{d\eta^2}&=\left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi)\\&-\frac{n_{io}^{\frac{3}{2}}}{\sqrt{12\sigma_i p_{io}}}\left[\sqrt{\left\{\left(V+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}-\sqrt{\left\{\left(V-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}\right]\\&-\frac{Zn_{bo}^{\frac{3}{2}}Q^{\frac{1}{2}}}{\sqrt{12\sigma_b p_{bo}}}\left[\sqrt{\left\{\left(V-u_{bo}+\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^2-\frac{2Z}{Q}\varphi\right\}}-\sqrt{\left\{\left(V-u_{bo}-\sqrt{\frac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^2-\frac{2Z}{Q}\varphi\right\}}\right]\end{aligned} \tag{8}$$

$$=-\frac{d\psi}{d\varphi} \tag{9}$$

where $\psi(\phi)$ is known as the Sagdeev Potential and it is given by

$$\begin{aligned}\psi(\phi) &= (1+3\beta) - [(1+3\beta) - 3\beta\varphi + \beta\varphi^2]\exp(\varphi)\\
&- \frac{n_{\mathrm{io}}^{\frac{3}{2}}}{6\sqrt{3\sigma_{\mathrm{i}}p_{\mathrm{io}}}}\left[\left\{\left(V+\sqrt{\frac{3\sigma_{\mathrm{i}}p_{\mathrm{io}}}{n_{\mathrm{io}}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left\{\left(V-\sqrt{\frac{3\sigma_{\mathrm{i}}p_{\mathrm{io}}}{n_{\mathrm{io}}}}\right)^2 - 2\varphi\right\}^{\frac{3}{2}} - \left(V+\sqrt{\frac{3\sigma_{\mathrm{i}}p_{\mathrm{io}}}{n_{\mathrm{io}}}}\right)^3 + \left(V-\sqrt{\frac{3\sigma_{\mathrm{i}}p_{\mathrm{io}}}{n_{\mathrm{io}}}}\right)^3\right]\\
&- \frac{Q^{\frac{3}{2}}n_{\mathrm{bo}}^{\frac{3}{2}}}{6\sqrt{3\sigma_{\mathrm{b}}p_{\mathrm{bo}}}}\left[\left\{\left(V-u_{\mathrm{bo}}+\sqrt{\frac{3\sigma_{\mathrm{b}}p_{\mathrm{bo}}}{Qn_{\mathrm{bo}}}}\right)^2 - \frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}} - \left\{\left(V-u_{\mathrm{bo}}-\sqrt{\frac{3\sigma_{\mathrm{b}}p_{\mathrm{bo}}}{Qn_{\mathrm{bo}}}}\right) - \frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}} + \left\{\left(V-u_{\mathrm{bo}}+\sqrt{\frac{3\sigma_{\mathrm{b}}p_{\mathrm{bo}}}{Qn_{\mathrm{bo}}}}\right)^3\right\} - \left\{\left(V-u_{\mathrm{bo}}-\sqrt{\frac{3\sigma_{\mathrm{b}}p_{\mathrm{bo}}}{Qn_{\mathrm{bo}}}}\right)^3\right\}\right]\end{aligned} \tag{10}$$

Integrating (9), we obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\eta}\right)^2 + \psi(\varphi) = 0 \tag{11}$$

where we have used the boundary conditions $\varphi \to 0, \mathrm{d}\varphi/\mathrm{d}\eta \to 0$ and $\mathrm{d}^2\varphi/\mathrm{d}\eta^2 \to 0$.

Equation (11) describes the motion of a pseudoparticle of unit mass with velocity $\mathrm{d}\varphi/\mathrm{d}\eta$ and position φ in a potential $\psi(\varphi)$. The first term of Eq. (11) can be regarded as the kinetic energy of the pseudoparticle and where as the second term is the potential energy of same particle at that instant. Since the kinetic energy is always nonnegative, it follows that $\psi(\varphi) \le 0$ for the entire motion, zero being the maximum value of $\psi(\varphi)$. Equation (11) shows that $\mathrm{d}\psi/\mathrm{d}\varphi$ is the force on the particle at the position φ. Equation (11) may also be regarded as anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the double layer solution of Eq. (11), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi = 0$ and φ_{m}, where φ_{m} is some extreme value of the potential φ, called the amplitude of the DL. For the double layer solutions, $\psi(\varphi)$ must additionally satisfy the following boundary conditions:

(i) $\psi(\varphi) = 0, \mathrm{d}\psi/\mathrm{d}\varphi = 0$ and $\mathrm{d}^2\psi/\mathrm{d}\varphi^2 < 0$ at $\varphi = 0$, (12a)

(ii) $\psi(\varphi) = 0, \mathrm{d}\psi/\mathrm{d}\varphi = 0$ and $\mathrm{d}^2\psi/\mathrm{d}\varphi^2 < 0$ at $\varphi = \varphi_{\max}(\neq 0)$ (12b)

(iii) $\psi(\varphi) < 0,$ for $0 < |\varphi| < |\varphi_{\mathrm{m}}|$ (12c)

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudoparticle is not reflected at $\varphi = \varphi_{\mathrm{m}}$ because the pseudoforce and the pseudovelocity are vanished. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_{m}.

3. Solution for the double layers

Now we expand the r.h.s of (8) and (10) in power series of ϕ and we obtain

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + A_4\varphi^4 + A_5\varphi^5 + \cdots \tag{13}$$

$$A_1=\left[\begin{array}{l}(1-\beta)-\dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-1}-\left(V-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-1}\right\}\\ -\dfrac{Z^2 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{1}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V-u_{bo}+\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-1}-\left(V-u_{bo}-\sqrt{\dfrac{3\sigma_b p_{jo}}{Qn_{bo}}}\right)^{-1}\right\}\end{array}\right] \tag{14a}$$

$$A_2=\frac{1}{2}\left[\begin{array}{l}1-\dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}-\left(V-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}\right\}\\ -\dfrac{Z^3 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V-u_{bo}+\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-3}-\left(V-u_{bo}-\sqrt{\dfrac{3\sigma_b p_{jo}}{Qn_{bo}}}\right)^{-3}\right\}\end{array}\right] \tag{14b}$$

$$A_3=\frac{1}{6}\left[\begin{array}{l}(1+3\beta)+\dfrac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5}-\left(V-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5}\right\}\\ +\dfrac{3Z^4 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V-u_{bo}+\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-5}-\left(V-u_{bo}-\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-5}\right\}\end{array}\right] \tag{14c}$$

$$A_4=\frac{1}{24}\left[\begin{array}{l}(1+8\beta)-\dfrac{15n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-7}-\left(V-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-7}\right\}\\ -\dfrac{15Z^5 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{7}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V-u_{bo}+\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-7}-\left(V-u_{bo}-\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-7}\right\}\end{array}\right] \tag{14d}$$

$$A_5=\frac{1}{120}\left[\begin{array}{l}(1+15\beta)+\dfrac{105n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-9}-\left(V-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-9}\right\}\\ +\dfrac{105Z^6 n_{bo}^{\frac{3}{2}}}{2Q^{\frac{9}{2}}\sqrt{3\sigma_b p_{bo}}}\left\{\left(V-u_{bo}+\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-9}-\left(V-u_{bo}-\sqrt{\dfrac{3\sigma_b p_{bo}}{Qn_{bo}}}\right)^{-9}\right\}\end{array}\right] \tag{14e}$$

And

$$\psi(\varphi)=-\frac{1}{2}A_1\varphi^2-\frac{1}{3}A_2\varphi^3-\frac{1}{4}A_3\varphi^4-\frac{1}{5}A_4\varphi^5-\frac{1}{6}A_5\varphi^6-\cdots \tag{15}$$

For small amplitude ion acoustic waves ($\varphi<1$), we take the terms up to φ^3 of Eq. (13) to get the solution of IADLs in plasma. Applying boundary conditions (12a), (12b) and (12c), we obtain

$$\varphi_m=-\frac{2A_2}{3A_3} \tag{16}$$

And

$$\psi(\varphi)=-\frac{A_3}{4}\varphi^2(\varphi_m-\varphi)^2 \tag{17}$$

The small amplitude IADLs solution of (11) satisfying (17) is obtained as

$$\varphi_D=\frac{1}{2}\varphi_m\left(1-\tanh\left[\sqrt{(A_3/8\varphi_m)}\eta\right]\right) \tag{18}$$

The width of double layers is given by

$$W_{dL}=2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{19}$$

It is seen that width of IADLs depends on the plasma parameters, e.g. ion beam density, ion beam temperature, ion beam velocity and nonthermal parameter of electrons. Equation (18) relates the amplitude φ_m of the IADLs to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (13). It is important to be noted that the IADLs in plasma can only exist for positive value of A_3, i.e. $A_3>0$. To satisfy the condition, the plasma parameters should have some critical values. The nonthermal parameter β must satisfy the condition $\beta < \beta_C$, β_C being the critical value of β. For the existence of IADLs, the parameter β_C obtained from (14c) is

$$\beta_C = \left(\frac{1}{3}\right)\left[\frac{3n_{i0}}{2\sigma_1}\left\{(V-\sigma_1)^{-5} - (V+\sigma_1)^{-5}\right\} + \frac{3n_{b0}}{2Q^{\frac{5}{2}}\sigma_2}\left\{(V-u_{bo}-\sigma_2)^{-5} - (V-u_{bo}+\sigma_2)^{-5}\right\} - 1\right] \tag{20}$$

where $\sigma_1 = \sqrt{3\sigma_i p_{i0}/n_{i0}}$, $\sigma_2 = \sqrt{3\sigma_b p_{b0}/n_{b0}}$ and $n_{i0} = 1 - Zn_{b0}$.

The parameter β_c for the existence of small amplitude IADLs in plasma can be numerically estimated from (20), if the values of other parameters of plasma are known. Similarly, for the existence of IADLs when $A_3 > 0$, the critical values of ion beam density (n_{b0c}), ion beam velocity (u_{b0c}) and ion beam temperature (σ_{bc}) can be obtained from (14c).

If IADLs are excited in plasma, it is very important to see the nature of DLs (compressive and rarefactive). From Eqs. (16) and (18), it follows that the sign of A_2 determines the nature of IADLs. If A_2 is positive (negative), a rarefactive (compressive) IADL is excited. Since A_2 is independent of β, the nature of IADLs (whether compressive or rarefactive), remains invariant under change of values of β. It is seen from (14b) that A_2 depends on the parameters n_{b0}, u_{b0} and σ_b. For compressive IADLs, $A_2 < 0$, and for rarefactive IADLs, $A_2 > 0$. Expressions for the critical values of $n_{b0}(n_{b0c})$, $u_{b0}(u_{b0c})$ and $\sigma_b(\sigma_{bc})$ for compressive and rarefactive IADLs in warm plasma can be obtained from (14b). In warm plasma, it is difficult to get the expressions of n_{b0c}, u_{b0c} and σ_{bc}. For a cold plasma, $\sigma_i = 0$ and $\sigma_b = 0$, the critical values of n_{b0c} and u_{b0c} for compressive and rarefactive IADLs are obtained from (14b):

$$n_{b0c} = \frac{Q^2(V^4-3)(V-u_{bo})^4}{3[V^4 - Q^2(V-u_{b0})^4]} \tag{21}$$

and

$$u_{b0c} = V\left[1 - \left\{\frac{3n_{b0}}{V^4 - 3 + 3n_{b0}}\right\}^{\frac{1}{4}}\right] \tag{22}$$

When,

(1) $n_{b0} < n_{b0c}$ or $u_{b0} < u_{b0c}$, positive potential (compressive) IADLs will be formed,
(2) $n_{b0} > n_{b0c}$ or $u_{b0} > u_{b0c}$, negative potential (rarefactive) IADLs will be excited.

The critical values of ion beam density (n_{b0c}) and ion beam velocity (u_{b0c}) for the excitation of IADLs in cold plasma can be numerically estimated form (21) and (22), if the values of other parameters of the plasma are known

4. Results and discussions

From the expression (18), we see that the plasma parameters, e.g. ion beam density, ion beam temperature, plasma ion temperature, ion beam velocity and nonthermality of electrons have significant influences on the excitation of IADLs in an ion beam plasma. To ascertain the actual nature of the IADLs, we have numerically evaluated for different parameters of the plasma and have discussed graphically. The Figures in the article illustrate the results of evaluation of the characteristics of the IADLs excitation that occur in an ion beam plasma, and are obtained by using (18). The evaluated curves are the dimensionless dependences of the values of the evaluated characteristics of IADLs excitation, along the axes of the ordinates and abscissas of which the values of the dimensionless quantities in arbitrary units are used. The potential of dimensionless IADLs is plotted along the y axis and the dimensionless space variable in x-axis to form the dimensionless curves shown in the Figures, which are equal in magnitude to the ratio of the corresponding calculated values of the selected characteristic of the IADL excitation on the y-axis and the values of the variable used in the calculation and deposited on the x-axis to the corresponding units of measurement of these quantities, which are chosen to normalize the dimensionless curve.

4.1. Profile of double layers

To start with we have plotted the IADL potential with the plasma parameters: (1) Nonthermal parameter of electron shown in Fig. 1(a), (b); (2) Ion beam density shown in Fig. 2(a), (b); (3) Ion beam temperature shown in Fig. 3(a), (b) and (4) Ion beam velocity shown in Fig. 4(a), (b).

4.1.1. Effect of nonthermal parameter of electron

: The effects of nonthermal electrons on the IADLs in the beam plasma having H^+ ion beam and Ar^+ ion beam have been numerically estimated and the normalized potential of IADLs are shown Fig. 1(a), (b) for the fixed values of other normalized plasma parameters. In the plasma, we have considered the values of nonthermal parameter of electron is less than 1.3 ($\beta < 1.3$).

To see the effects of nonthermal parameter electrons (β) on the IADLs in plasma having H^+ beams and Ar^+ beams, we have followed the conditions for the existence of IADLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand

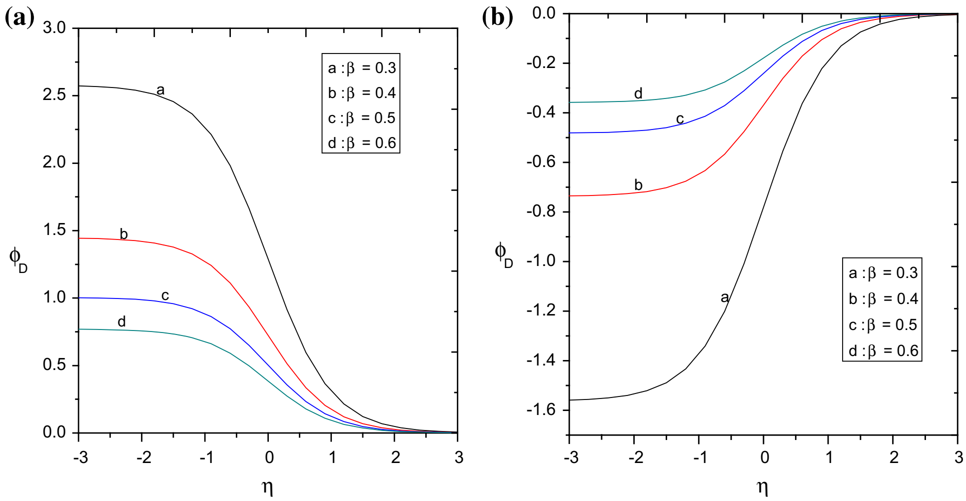

Fig. 1 (**a**) (colour online) Profiles of the IADLs for different nonthermal parameter (β) in H^+ ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.1$ and $u_{b0} = 0.2$; Curves a, b, c, and d refer to four different values $\beta = 0.3$, 0.4, 0.5 and 0.6 respectively. (**b**) Profiles of the IADLs for different nonthermal parameter (β) in Ar^+ ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.3$and $u_{b0} = 0.9$; Curves a, b, c, and d refer to four different values $\beta = 0.3$, 0.4, 0.5 and 0.6, respectively

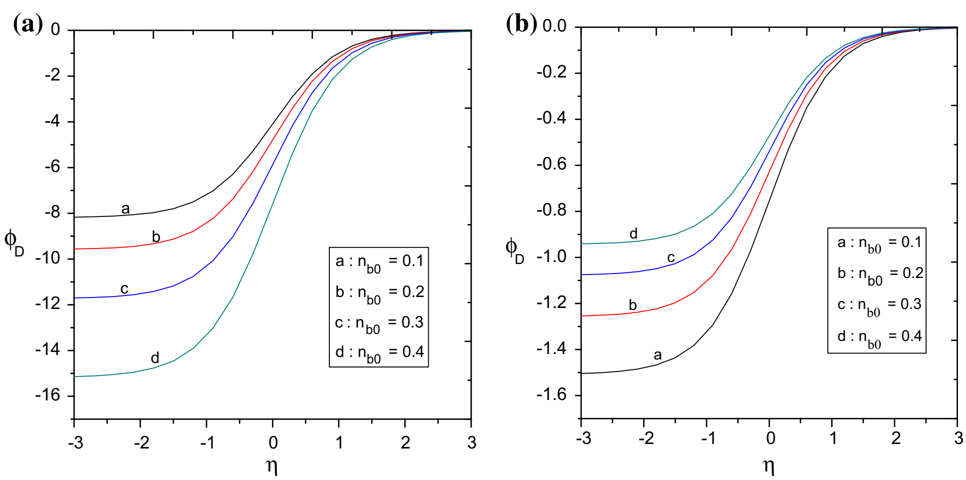

Fig. 2 (**a**) (colour online) Profiles of the IADLs for different ion beam density (n_{b0}) in Hydrogen beam ion plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 1.1$ and $u_{b0} = 0.4$; Curves a, b, c, and d refer to four different values $n_{b0} = 0.1$, 0.2, 0.3 and 0.4 respectively. (**b**) Profiles of the IADLs for different ion beam density (n_{b0}) in Argon beam ion plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 0.3$ and $u_{b0} = 0.4$; Curves a, b, c, and d refer to four different values $n_{b0} = 0.1$, 0.2,0.3 and 0.4 respectively

the nature of IADLs, the amplitude of DLs given by (16) is first numerically estimated using the parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs having H^+ beam is positive and for Ar^+ beam is negative giving rise to compressive and rarefactive IADLs, respectively. In fact, the positive and negative values of amplitude of IADLs depend on the coefficient A_2 of Eq. (13). Using (18) Fig. 1(a), (b) are drawn for the profiles of the electrostatic potential IADLs for the H^+ beams and Ar^+ beams in plasma. It is seen from Fig. 1(a) that the coefficient A_2 is negative. So compressive IADLs are excited in plasma having H^+ beams for $\beta = 0.3$, 0.4, 0.5 and 0.6 with fixed values of the parameters $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.1$ and $u_{b0} = 0.2$. It is also observed that the amplitudes of compressive IADLs are decreased with increase of β. But, for the Ar^+ beam plasma A_2 is positive for $\beta = 0.3$, 0.4, 0.5 and 0.6 with fixed values of parameters $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.9$ for which the amplitude of IADLs becomes negative. For this reason, the IADLs in Fig. 1(b) for Ar^+ beam plasma are rarefactive in nature. It is observed that the amplitudes of rarefactive IADLs are decreased with the increase of nonthermal parameter β.

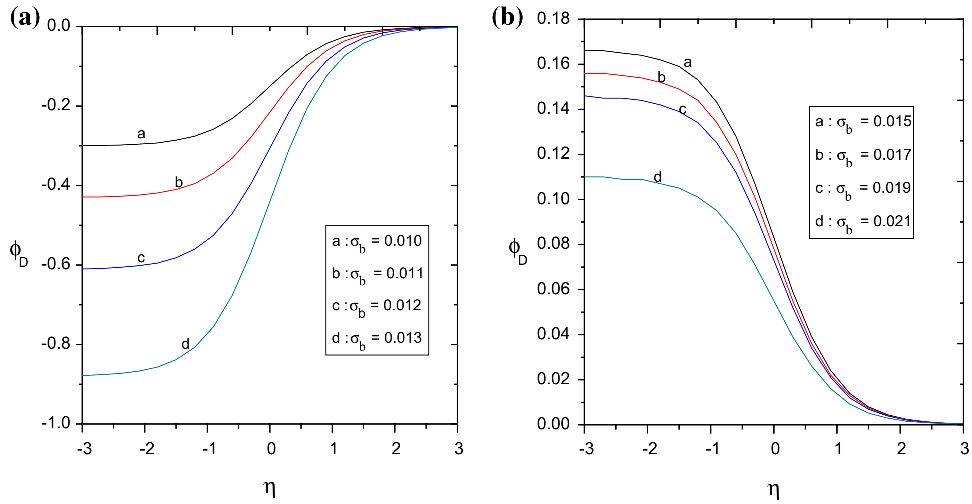

Fig. 3 (**a**) (colour online) Profiles of the IADLs for different ion beam temperature (σ_b) in Hydrogen ion beam plasma having $\sigma_i = 0.01$, $\beta = 1.18$ $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.4$; Curves a, b, c, and d refer to four different values $\sigma_b = 0.01, 0.011, 0.012, 0.013$, respectively. (**b**) Profiles of the IADLs for different ion beam temperature (σ_b) in Argon ion beam plasma having $\sigma_i = 0.001$, $\beta = 1.29$, $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.75$; Curves a, b, c, and d refer to four different values $\sigma_b = 0.015, 0.017, 0.019$, respectively

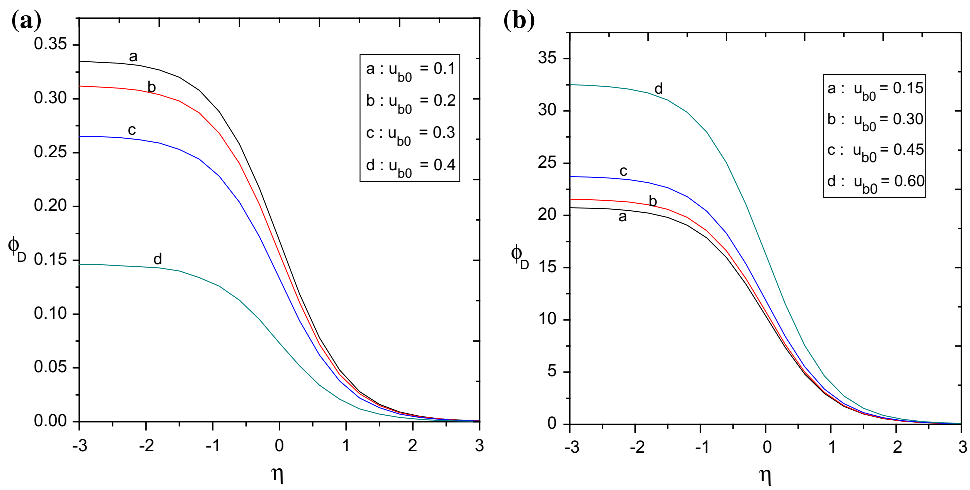

Fig. 4 (**a**) (colour online) Profiles of the IADLs for different ion beam velocity (u_{b0}) in Hydrogen ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 1.2$, $V = 1.6$ and, $n_{b0} = 0.3$; Curves a, b, c, and d refer to four different values $u_{b0} = 0.1$, 0.2, 0.3 and 0.4, respectively. (**b**) Profiles of the IADLs for different ion beam velocity (u_{b0}) in Argon ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 0.1$, $V = 1.6$ and $n_{b0} = 0.1$; Curves a, b, c, and d refer to four different values $u_{b0} = 0.15$, 0.30, 0.45 and 0.60, respectively

4.1.2. Effect of ion beam density

The effects of ion beam density (n_{b0}) on the IADLs in plasma having H^+ beams and Ar^+ beams are understood from the numerical estimation of amplitude and electrostatic potential given by (16) and (18). From numerical estimation using the values of parameters of our model plasma, it is observed that the amplitudes of electrostatic potential IADLs are negative for both H^+ beams and Ar^+ beams giving rise to rarefactive IADLs shown in Fig. 2(a), (b). Figure 2(a) shows that potential of IADLs are negative in plasma having H^+ beams for n_{b0}= 0.1, 0.2, 0.3 and 0.4 with fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 1.1$ and $u_{b0} = 0.4$. It is seen that the amplitudes of rarefactive IADLs for H^+ beams are increased with the increase of ion beam density. Figure 2(b) illustrates the rarefactive IADLs in plasma having Ar^+ beams for n_{b0}= 0.1, 0.2, 0.3 and 0.4 with fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6, \beta = 0.3$, and $u_{b0} = 0.4$. The amplitudes of rarefactive IADLs of Ar^+ beams are decreased with the increase of the ion beam density.

4.1.3. Effect of ion beam temperature

The effects of ion beam temperature on the normalized potential of IADLs are shown in Fig. 3(a), (b) with fixed values of other normalized plasma parameters. In fact, the effect of ion beam temperature on the IADLs in plasma is understood from numerical estimation for the amplitude given by (16) and electrostatic potential from (18). It is seen from numerical estimation that the amplitude for H^+ beams is negative (since A_2 is positive) in our plasma having for $\sigma_b = 0.01$ 0.011, 0.012 and 0.013 with fixed values of $\sigma_i = 0.01$, $\beta = 1.18$ $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.4$. So, the profile of IADLs for H^+ beam plasma is rarefactive in nature. It is also observed from Fig. 3(a) that the amplitudes of rarefactive DLs are increased with the increase of ion beam temperature. In similar way, numerical estimation is made for IADLs for Ar^+ beams in plasma. It is observed that the amplitude of DLs is positive (since A_2 is negative) in our plasma having $\sigma_b = 0.011$, 0.012 and 0.013 with fixed values of $\sigma_i = 0.001$, $\beta = 1.29$ $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.75$. For this reason, the IADLs in Fig. 3(b) are compressive in nature. It is seen that the amplitudes of compressive IADLs in Fig. 3(b) are increased with decrease of ion beam temperature.

4.1.4. *Effect of ion beam velocity*

The effects of ion beam velocity on the normalized potential of IADLs in plasma having H^+ ion beam and Ar^+ ion beam are shown in Fig. 4(a), (b) with fixed values of other normalized plasma parameters. It is seen that amplitudes of double layers for H^+ beams are positive (since A_2 is negative) for $u_{b0} = 0.1$, 0.2, 0.3 and 0.4 in plasma having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 1.2$, $V = 1.6$ and $n_{b0} = 0.3$. So, the IADLs in Fig. 4(a) for H^+ beam are compressive in nature. The amplitudes of compressive IADLs are increased with the decrease of ion beam velocity. Similarly, numerical estimation for amplitude of IADLs for Ar^+ beams in plasma is made from which it is seen that amplitude is also positive (since A_2 is negative) for $u_{b0} = 0.15$, 0.30, 0.45 and 0.60 for Ar^+ ion beams in plasma having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 0.1$, $V = 1.6$ and $n_{b0} = 0.1$. As a result, the IADLs in Fig. 4(b) for Ar^+ beams in plasma become compressive in nature. The amplitudes of compressive IADLs for Ar^+ beams are increased with increase of ion beam velocity.

4.2. Width of double layers

Now, we have plotted the width of IADLs with the ion beam density (n_{b0}) in plasma:

(1) For different values of Nonthermal parameter (β) of electron shown in Fig. 5(a) for H^+ beams, Fig. 5(b) for Ar^+ beams; (2) for different values of ion beam temperature (σ_b) shown in Fig. 6(a) for H^+ beams, and Fig. 6(b) for Ar^+ beams; (3) for different values of Ion beam velocity (u_{b0}) shown in Fig. 7(a) for H^+ beams, and Fig. 7(b) for Ar^+ beams.

It is known that width is important for the study of DLs in plasma because it is important for experimental studies in laboratory and in space plasma. We have numerically estimated the width of IADLs using (19) and its variation for different parameters of an ion beam plasma is graphically discussed. It is to be remembered that for the numerical estimation we have followed the conditions for the existence of IADLs (i.e. A_3 must be positive).

4.2.1. *Effect of nonthermal parameter*

The variation of width of IADLs with ion beam density (n_{b0}) for different values of nonthermal parameter (β) of electrons in plasma having H^+ beams and Ar^+ beams are shown in Fig. 5(a), (b). From Fig. 5(a) for H^+ beams, it is seen that the width of IADLs increases with increase of n_{b0} when $\beta = 0.05$, 0.0.1, 0.15 and 0.2 in a plasma having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, and $u_{b0} = 0.3$. The width of DLs is large for small values of β. But, Fig. 5(b) for Ar^+ beams shows that width of IADLs increases up to certain maximum value when $\beta = 0.2$, 0.25, 0.3, 0.35 having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $u_{b0} = 0.2$; then the width is decreased to a minimum value, and after that it increases steadily with increase of ion beam density. The widths of IADLs are large for small values of nonthermal parameter of electrons.

4.2.2. *Effect of ion beam temperature*

Similarly, the variation of width of IADLs with beam ion density (n_{b0}) for different values of ion beam temperature (σ_b) in plasma having H^+ beams and Ar^+ beams are shown in Fig. 6(a), (b). It is seen from Fig. 6(a) that the width increases up to a certain maximum value with increase of n_{b0} in plasma having H^+ beams when $\sigma_b = 0.20$, 0.25, 0.3, 0.35 having fixed values of $\sigma_i = 0.01$, $V = 1.6, \beta = 0.3$. $u_{b0} = 0.2$; and then it decreases to a minimum value with increase of n_{b0}, after that width increases again with increase of n_{b0}. On the other hand, the variations of width of DLs in plasma having Ar^+ beams are shown in Fig. 6(b). It is seen that the width of IADLs in plasma having Ar^+ beams increases to a maximum value with increase of n_{b0} when values of σ_b, σ_i, 0.01, V, β and u_{b0} are same as in H^+ beam plasma; after the maximum value the width decreases to a minimum value; again it increases to a maximum value and finally it is decreased to a minimum value and remains constant with increase of ion beam density.

4.2.3. *Effect of ion beam velocity*

The variation of width of IADLs with ion beam density (n_{b0}) for different values of ion beam velocity (u_{b0}) in plasma having H^+ beams and Ar^+ beams are shown in Fig. 7(a), (b). It is seen from Fig. 7(a) that the width of DLs increases for H^+ beams with increase of n_{b0} up to a certain maximum value, and then it decreases with increase of n_{b0}; when $u_{b0} = 0.25$, 0.30, 0.35 and 0.40 in a plasma having constant values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 0.1$. From Fig. 7(b) for Ar^+ ion beams it is observed that the width of DLs increases steadily with increase of n_{b0}, when

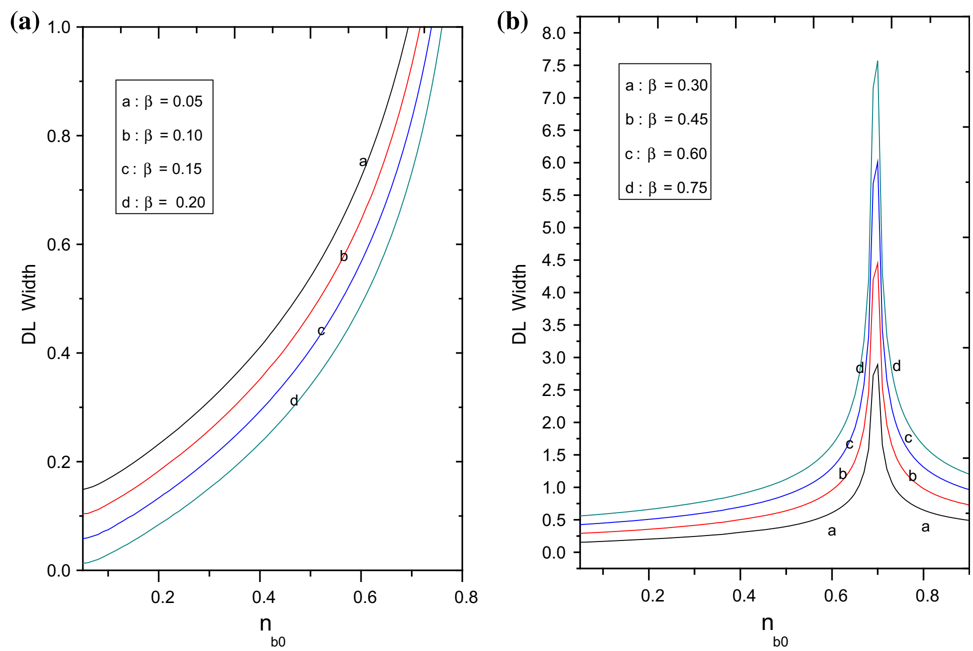

Fig. 5 (**a**) (colour online) Variation of width of IADLs with ion beam concentration for different values of nonthermal parameter (β) in Hydrogen ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $u_{b0} = 0.3$; Curves a, b, c, and d refer to four different values $\beta = 0.05$, 0.1, 0.15 and 0.2, respectively. (**b**) Variation of width of IADLs with ion beam concentration for different values of nonthermal parameter (β) in Argon ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $u_{b0} = 0.2$; Curves a, b, c, and d refer to four different values $\beta = 0.3$, 0.45, 0.6 and 0.75, respectively

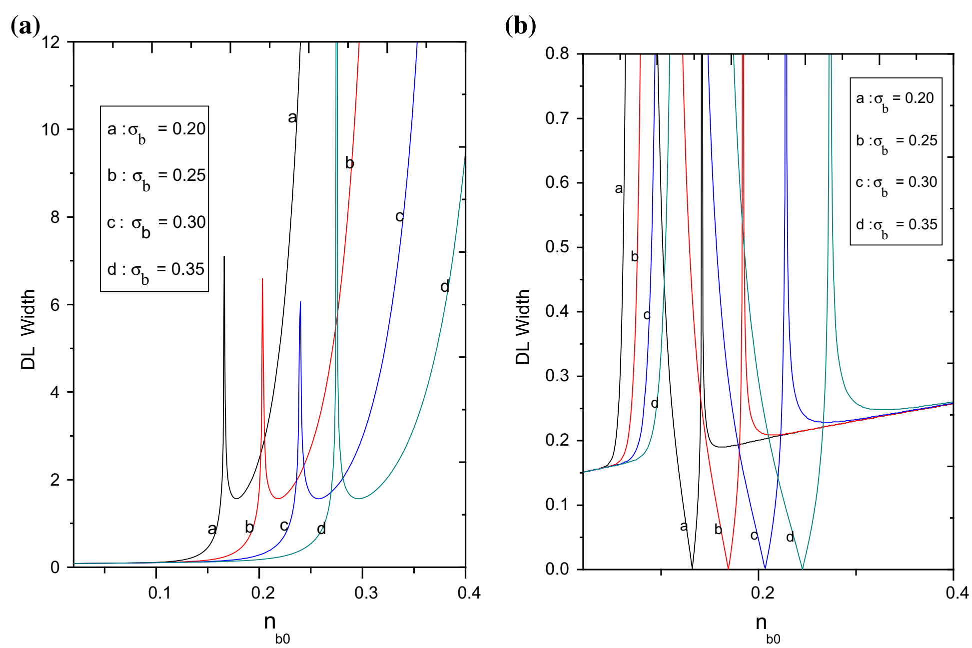

Fig. 6 (**a**) (colour online) Variation of width of IADLs with ion beam concentration for different values of ion beam temperature (σ_b) in Hydrogen beam plasma having $\sigma_i = 0.01$, $V = 1.6, \beta = 0.3$. $u_{b0} = 0.2$; the red, blue, green and magenta lines correspond to $\sigma_b = 0.20$, 0.25, 0.3 and 0.35, respectively. (**b**) Variation of width of IADLs with ion beam concentration for different values of ion beam temperature (σ_b) in Argon beam plasma having $\sigma_i = 0.01$, $V = 1.6$, $\beta = 0.3$ and $u_{b0} = 0.2$; Curves a, b, c, and d refer to four different values $\sigma_b = 0.20, 0.25, 0.30$, and 0.35, respectively

$u_{b0} = 0.25$, 0.30, 0.35 and 0.40 in a plasma having constant values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 1.2$.

5. Conclusions

We have theoretically studied the excitation of IADLs for small amplitude waves in a plasma composed of warm positive ions, warm positive beam ions and warm nonthermal electrons using pseudo potential method. It has been shown that the IADLs in such plasma depend on the plasma parameters, i.e. ion beam density, ion beam temperature, ion beam velocity and nonthermal parameters of electrons. Numerical estimation is made considering a model plasma having beam of ions and the profiles of IADLs including widths are drawn. The IADLs of small amplitude waves in plasma having ion beams ions are compressive or rarefactive depending upon the values of the plasma parameters.

Our findings on IADLs are:

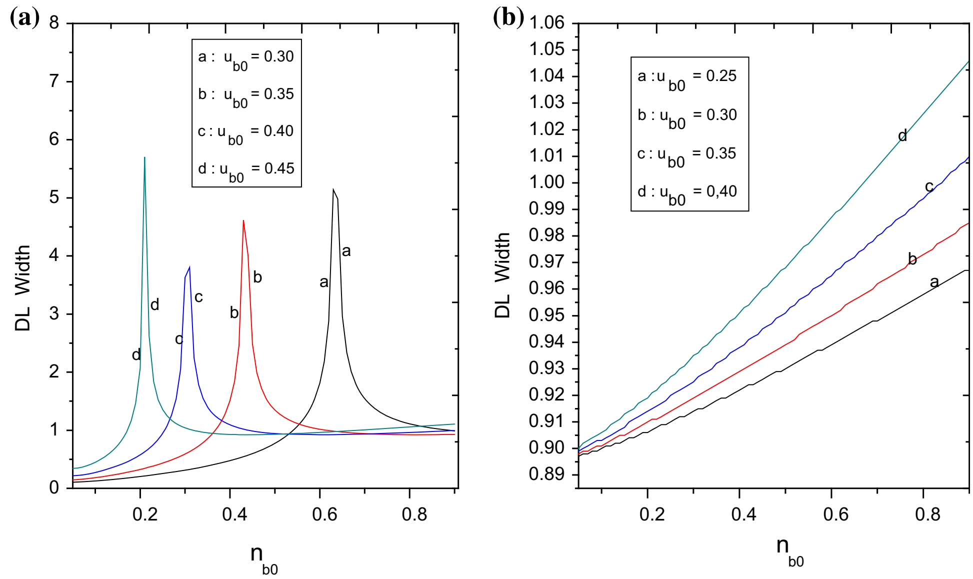

Fig. 7 (**a**) (colour online) Variation of width of IADLs with ion beam density for different values of ion beam velocity (u_{b0}) in Hydrogen ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 0.1$; Curves a, b, c, and d refer to four different values $u_{b0} = 0.25$, 0.30, 0.35 and 0.40, respectively. (**b**) Variation of width of DLs with ion beam density for different values of Argon ion beam velocity (u_{b0}) in Hydrogen ion beam plasma having $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 1.2$; Curves a, b, c, and d refer to four different values ub0 = 0.25, 0.30, 0.35 and 0.40, respectively

- The IADLs are compressive and rarefactive in plasma having H^+ beams and Ar^+ beams depending on the values of nonthermality of electrons and other plasma parameters. The IADLs are compressive in plasma having H^+ beam for the nonthermal parameter $\beta = 0.3$, 0.4, 0.5, 0.6 with fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.1$ and $u_{b0} = 0.2$. The amplitudes of compressive DLs are decreased with increase of β. But, in presence of Ar^+ ion beams in plasma the IADLs are rarefactive for $\beta = 0.3$, 0.4, 0.5, 0.6 when $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.9$. The amplitudes rarefactive DLs are decreased with increase of the values of nonthermal parameter.
- The rarefactive IADLs are excited in plasma having H^+ beams for n_{b0}= 0.1, 0.2, 0.3, 0.4 when $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 1.1$ and $u_{b0} = 0.4$. The amplitudes of rarefactive DLs for H^+ beams are increased with the increase of ion beam density. In plasma having Ar^+ beams, the rarefactive IADLs are also excited for n_{b0}= 0.1, 0.2, 0.3 and 0.4 with fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, $\beta = 0.3$, and $u_{b0} = 0.4$. The amplitudes of rarefactive DLs for Ar^+ beams are decreased with the increase of the ion beam density.
- The IADLs are rarefactive in plasma having H^+ beams for $\sigma_b = 0.01$ 0.011, 0.012, 0.013 when $\sigma_i = 0.01$, $\beta = 1.18$ $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.4$. The amplitudes of rarefactive DLs are increased with the increase of ion beam temperature. But, the IADLs are compressive in plasma having Ar^+ beams for $\sigma_b = 0.011$, 0.012, 0.013, 0.014 when $\sigma_i = 0.001$, $\beta = 1.29$ $V = 1.6$, $n_{b0} = 0.3$ and $u_{b0} = 0.75$. The amplitudes of compressive DLs for Ar^+ beams are increased with decrease of ion beam temperature.
- The IADLs are compressive in plasma having H^+ beams for $u_{b0} = 0.1$, 0.2, 0.3 and 0.4 when $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 1.2$, $V = 1.6$ and $n_{b0} = 0.3$. The amplitudes of compressive DLs for H^+ beams are increased with the decrease of ion beam velocity. In plasma having Ar^+ beams, the IADLs are also compressive for $u_{b0} = 0.15$, 0.30, 0.45 and 0.60 for the fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $\beta = 0.1$, $V = 1.6$ and $n_{b0} = 0.1$. The amplitudes of compressive IADLs for Ar^+ beams are increased with increase of ion beam velocity.

Our findings on the width IADLs are:

- In H^+ beam plasma, the width of IADLs increases with increase of n_{b0} when $\beta = 0.05$, 0.0.1, 0.15, 0.2 having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$, and $u_{b0} = 0.3$. The width is large for small values of nonthermal parameter of electrons. In Ar^+ beam plasma, the width of IADLs increases up to certain maximum value with increase of n_{b0} when $\beta = 0.3$, 0.35, 0.4 and 0.45 having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $u_{b0} = 0.2$. Then, the width decreases to a minimum value with the increase of n_{b0} and the width is large for large value of β.
- In plasma having H^+ beams, the width of IADLs increases up to a certain maximum value with increase of n_{b0} when $\sigma_b = 0.20$, 0.25, 0.3, 0.35 having fixed values of $\sigma_i = 0.01$, $V = 1.6$,$\beta = 0.3$. $u_{b0} = 0.2$. After that width decreases to a minimum value with increase of n_{b0}, then the width increases again with increase of n_{b0}. In plasma having Ar^+ beams, the width of IADLs increases to a maximum value with increase of ion beam density when values of σ_b, σ_i, 0.01, V, β and u_{b0}

are same as in H^+ beam plasma. Then, the width decreases to a minimum value; again it increases to a maximum value and it is decreased to a minimum value and remains constant with increase of ion beam density.

- In plasma having H^+ beams, the width of IADLs increases with increase of u_{b0} up to a certain maximum value, and then it decreases steadily with increase of n_{b0} when u_{b0} = 0.25, 0.30, 0.35 and 0.40 having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $\beta = 0.1$. In Ar^+ ion beam plasma, the width of IADLs increases steadily with increase of n_{b0} when u_{b0} = 0.25, 0.30, 0.35 and 0.40 having fixed values of $\sigma_i = 0.001$, $\sigma_b = 0.003$, $V = 1.6$ and $\beta = 1.2$.

It is to be stated that the plasma considered in present work exists in space and also can be produced in the laboratory. The present study can be useful for the study of DLs in auroral and magnetospheric plasma which can explain the particle acceleration in astrophysical plasmas [35]. Our study is important because of the fact high energy charged particles (electrons and ions) are found to exist in solar flares and upstream solar wind. During solar bursts, evolution of stars and pulsar radiation when relativistic and no relativistic ions and electrons are ejected and enter the space and also the ionosphere [36] follows our present analysis, ion acoustic DLs can also be studied in plasma having electron beams and negative ions. Moreover, electro-acoustic and ion-acoustic DLs can be studied in plasma having electron/ion beams considering different types non-Maxwellian electrons, namely nonisothermal electrons, superthermal electrons, Tsallis distributed electrons, etc. In our present study, static magnetic field in the ion beam plasma has not been considered. Considering the magnetic field and different types of non-Maxwellian electrons in presence of beams (electron/ion) more interesting results on the DLs in plasma would be obtained which may give new information about acceleration process of ionized particles in space and laboratory plasma. The DLs in degenerate plasma in presence of electron/ion beams can also be studied following our present work. It is to be mentioned that for the study of large amplitude DLs in ion beam plasma the terms φ^4, φ^5 etc. of r.h.s of Eq. (13) should be taken into account and the solution of DLs can be obtained using tanh-method [37–40]. The DLs in ion beam plasma can also be studied from the modified KdV equation using the reductive perturbation method [41, 42]

It is to be mentioned the DLs in plasma have been experimentally observed by various authors [43–47]. But, there are no information in experimental observations on the DLs in ion beam plasma consisting of nonthermal electrons. So, at present, our theoretical results cannot be verified/compared with the experimental observations.

6. Remarks

It was first proposed by Hannes Alfven (the developer of magneto hydrodynamics) that the creation of the polar lights or Aurora Borealis is created by accelerated electrons in the magnetosphere of the Earth [48]. Alfven supposed that the electrons were accelerated electrostatically by an electric field localized in a small volume bounded by two charged regions. This so-called DLs would accelerate electrons Earthwards. Many experiments with rockets and satellites have been performed to investigate the magnetosphere and acceleration regions. Sakai [49] has also established the fact that the DLs are responsible for accelerating the particles in auroral region of ionosphere [50]. It is believed that the DLs play a significant role in supplying and accelerating plasma in magnetic coronal funnels and on the formation of solar flares [51]. The DLs are detected in Viking and S3-3 satellite observations in magnetospheric regions [52] and auroral regions [53], respectively. It has also been discovered that the acoustic DLs are responsible for auroral electron precipitation. During the past 2 decades, the formation of DLs has raised interest because of their relevance to cosmic applications [51, 54–56]. It is important to note that double layers can facilitate the transfer of electrical energy into kinetic energy, $dW/dt = I \cdot \Delta V$ where I is the electric current dissipating energy into a double layer with a voltage drop of ΔV. The potential drop across the double layer will accelerate electrons and positive ions in opposite directions. The magnitude of the potential drop determines the acceleration of the charged particles. In strong double layers, this will result in beams or jets of charged particles. The thickness of a double layer is of the order of ten Debye lengths, which is a few centimetres in the ionosphere, a few tens of meters in the interplanetary medium, and tens of kilometres in the intergalactic medium.

However, some important points to be remembered about the DLs and the shocks in plasma. There are two type DLs, (1) Weak DLs and (2) Strong DLs, depending on the relative values of potential drop and the equivalent thermal potentials of the particle populations (free and reflected ions and electrons) present in the surrounding plasma. Also the DLs may be divided into relativistic and non-relativistic layers. Block [57] has proposed from theoretical investigations that electrostatic DLs can be formed provided the following conditions are satisfied:

(1) $K_B.T_C \geq e.\varphi_{DL}$ where K_B is the Boltzmann constant, T_C is the temperature of the coldest plasma bordering the layers, e is the charge of electron and φ_{DL} is the potential drop between the layers.

(2) The electric field strength is much stronger inside the layer and almost zero outside, leading to cancellation of integrated positive and negative charges.
(3) Quasi-neutrality is locally violated between the layers.
(4) Collision free path should be greater than the DL thickness.

In this regard, Langmuir [58] has proposed that space charge separation leading to DLs is due to the balance between two types of forces. Since positive and negative species are accelerated in the same DL potential, their energies are around the same and therefore,

$$\sqrt{\frac{n_e}{n_i}}\frac{u_{De}}{u_{Di}} = \sqrt{\frac{m_i}{m_e}}$$

and if $n_i \approx n_e$ then

$$\frac{u_{De}}{u_{Di}} = \sqrt{\frac{m_i}{m_e}}$$

where u_{Di} and u_{De} are the drift velocities of the ions and the electrons, respectively.

It is observed in laboratory that $m_e u_{De}^2 > 2K_B T_e$ is a condition for maintaining a DL [59]. It is one of the Bohm criteria [60]. The requirement of trapped particles for DLs development arises in order to satisfy the conditions for net vanishing of charge in the DL. If the densities of the electrons and ions are equal on one side, they have to differ on the other side, thereby violating the condition of charge neutrality [61].

In plasma, another class of nonlinear structure is found which is called electrostatic shocks. Although shocks are considered to resemble a DL, but some important differences can be brought out. Goertz [62] has pointed out that the DLs have a large fraction of trapped ions, static in the plasma frame and having small mass transport, whereas shocks are moving, having ion mass transport at supersonic velocities and have only a small fraction of trapped ions. Kan [63] has stated that electric current is essential for DLs formation but not for the shocks. However, slowing down of ions by the high potential side is essential for shocks. In the experiment by Hershkowitz [64] on weak DLs with $e\Delta\varphi/T_e \approx 5$, it is shown that when ions were injected from the high potential side with considerable velocity and density, similar potential structures could be achieved whether or not significant densities of trapped ions are present. This situation may appear as shocks [62] while it may appear as DLs [63].

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References

[1] R A Hoffmann and D S Evans *J. Geophys. Res*. **73**, 6201 (1968).
[2] M V Goldman *Sol. Phys*. **89**, 403 (1983).
[3] S M Krivoruchku, Y B Fainberg, V D Shapiro and V I Shevchenko *Sov. Phys. JETP* **40**, 1039 (1974).
[4] Y Gell and I Roth *Plasma Phys.* **19**, 915 (1977).
[5] P S Abrol and S G Tagare *Phys. Letts. A* **75**, 74 (1979).
[6] B Karmakar, G C Das and K I Singh *Plasma Phys. Control. Fusion* **30**, 1167 (1988)
[7] S K El-Labany *J. Plasma Phys*. **54**, 285 (2009)
[8] R Das *Astrophys. Sp. Sci.* **341**, 543 (2012)
[9] R Das and P Deka *Int. J. Sci. Eng. Res.* **6**, 864 (2015)
[10] N Kaur, K Singh, Y Ghai and N S Saini *Plasma Sci. Technol.* **20**, 074009 (2018)
[11] I Paul, A Chatterjee and S N Paul *Laser Part. Beams* **37(4)**, 1–11 (2019)
[12] L P O Dovner, A I Erikson, R Bostrom and B Holback *Geophys. Res. Lett.* **21**, 1827 (1994).
[13] R Bostrom, G Gustafsson, B Holback, G Holmgren, H Koskinen and P Kintner *Phys*. *Rev. Letts*. **61**, 82 (1988).
[14] R A Cairns, R Bingham, R O Dendy, C M C Nairn, P K Shukla and A A Mamun, *J. Phys.* IV France **5**, C6–43 (1995)
[15] R A Cairns, A A Mamun, R Bingham, R Bostrom, R O Dendy, C M C Nairn and P K Shukla *Geophys. Res. Lett.* **22**, 2709 (1995)
[16] D S Hall et al. *J. Geophys. Res*. **96**, 7869 (1991)
[17] A A Mamun *Euro. Phys. J.* **11**, 143 (2000)
[18] A Bandyopadhyay and K P Das *Phys. Scr.* **61**, 92 (2000)
[19] A Bandyopadhyay and K P Das *J. Plasma Phys*. **68**, 285 (2002)
[20] T S Gill, P Bala, H Kaur, N S Saini, S Bansal and J Kaur *Euro. Phys*. J. D **31**, 91 (2004)
[21] S K Bhattacharyya, S N Paul and B Chakraborty *Indian J. Phys*. **77B**, 327 (2003)
[22] J Das, A Bandyopadhyay and K P Das *J. Plasma Phys.* **74(02)**, 163 (2008)
[23] I Paul, B Ghosh, G Pakira, S N Paul and S Chattopadhyay *Indian J. Phys.* **86**, 395 (2012)
[24] B Ghosh, S Banerjee and S N Paul *Indian J. Pure Appl. Phys*. **51**, 488 (2013)
[25] B Ghosh, S N Paul, C Das, I Paul and S. Banerjee *Braz. J Phys* **3**, 28 (2013)
[26] B Ghosh and S Banerjee, *Indian J. Phys.*, **89(12)**, 1307 (2015)
[27] I Paul, S Chandra, S Chattopadhyay and S N Paul *Indian J Phys.* **90**, 1195–1205 (2016)
[28] D N Gao, J Zhang, Y Yang, W S Duan *Plasma Phys. Rep.* **43**(8), 833–837 (2017)
[29] W F El-Taibany *J. Geophys. Res.* **110**, A01213 (2005)
[30] S Ali Shan and H Saleem *Phys. Lett. A* **378**, 795–799 (2014)
[31] R V Reddy, G S Lakhina and F Verheest *Planet. Space Sci.* **40**, 1055–1062 (1992)
[32] B Das, D Ghosh and P Chatterjee *Pramana J. Phys.* **74**, 973–981 (2010)
[33] N Kaur, K Singh and N S Saini *Phys. Plasmas* **24**, 092108 (2017)
[34] I Paul, A Chatterjee and S N Paul *Braz. J. Phys.* **50**, 272–281 (2020)
[35] P Carlqvist *Double Layers in Space*, Symposium on Plasma Double Layers, Risø National Laboratory, June 16–18, 1982 DK 4000 Roskilde, Denmark (eds.) J Juul Rasmussen, P Michelson pp. 255–273(1982)
[36] S A Kaplan and V N Tsytovich *Plasma Astrophysics*. New York: Pergamum (1973), translated by D. tar Haar
[37] G C Das and J Sarma *Phys. Plasmas* **5**, 63 (1998)
[38] A Bandyopadhyay and K P Das *Phys. Scr.* **61**, 92 (2000)

[39] B Sahu and R Roychowdhury *Phys. Plasmas* **11**, 1947(2004).
[40] S N Paul, A Chatterjee, I Paul and B Ghosh *Int. J. Chem. Phys. Sci*, ISSN:2319-6602,IJCPS **Vol. 4** Special Issue-6, APST–2015, pp 38–42 (2015)
[41] S N Paul, K K Mondal and A Roychowdhury *FIZIKA* **B 5**, 73–84 (1996)
[42] K K Mondal, S N Paul and A Roychowdhury *IEEE Trans. Plasma Sci.* **26**, 987–994 (1998)
[43] Y Nakamura and H Sugai, *Chaos Solitons Fractals* **7**, 1023–1031 (1996)
[44] S Torvén *Formation of Double Layers in Laboratory Plasmas; Wave Instabilities in Space Plasmas*, pp 109–128 (1979)
[45] N Hershkowitz *Space Sci. Rev*. **41**, 351–391 (1985)
[46] H Schamel *Phys. Rep.* (*Rev. Sect. Phys. Lett.*) **140**, 161–191 (1986)
[47] C Charles *Plasma Sources Sci. Technol*. **16**, R1–R25 (2007)
[48] H Alfvén *Tellus* **10**, 104 (1958)
[49] K Saeki, S Iizuka and N Sato *Phys. Rev. Lett* **45**, 1853 (1980)
[50] D E Baldwin and B G Logan *Phys. Rev. Lett*. **43**, 1318 (1979)
[51] H Massey *Negative ions*, 3rd edn. (Cambridge University Press, Cambridge, 1976), p. 663
[52] R A Gottscho and C E Gaebe *IEEE Trans. Plasma Sci.* **PS-14**, 92 (1986)
[53] M Bacal and G W Hamilton *Phys. Rev. Lett*. **42**, 1538 (1979)
[54] S Torven, L Lindberg and R Carpenter *Plasma Phys.* **27**, 143 (1985)
[55] M A Raadu *Phys. Rep*. **178**, 25 (1989)
[56] H Alfven, P Carlqvist *Solar Phys*. **1**, 220 (1967)
[57] L P Block Cosmic Electrodyn. **3**, 349 (1972)
[58] I Langmuir *Phys. Rev*. **33**, 195 (1929)
[59] C K Goertz and G Joyce *Astrophys. Space Sci.* **32**, 165 (1975)
[60] D Bohm *The characteristics of electric discharges in magnetic fields* (Ed. Guthrie and Wokerling Mc: Grew Hill, NY) p 77 *(1949)*
[61] H Schamel *Phys. Rep.* **140**, 161 (1986)
[62] C K Goertz *Rev. Geophys. Space Phys.* **17**, 418 (1979)
[63] J R Kan and L C Lee *J.Geophys. Res*. **85**, 788 (1980)
[64] N J Hershkowitz *Geophys. Res.* **86 A 5**, 3307 (1981)

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EFFECTS OF POSITRONS AND NONTHERMAL ELECTRONS ON THE ION-ACOUSTIC DOUBLE LAYERS IN WARM MULTICOMPONENT PLASMA

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ABSTRACT

The effects of isothermal positrons and nonthermal electrons on the ion-acoustic double layers in warm multicomponent plasma having positive and negative ions are studied using pseudo potential technique.. The expression of Sagdeev potential has been derived and the solution for ion-acoustic double layers has been obtained from the nonlinear equation. The profiles of double layers have been drawn taking different values of plasma parameters. It is observed that both compressive and rarefactive double layers would be excited in presence of positrons and nonthermal electrons in the plasma. The variation of amplitudes and widths of double layers have been discussed graphically for different positron density, positron temperature, nonthermal electrons, negative ion density including drift velocity of ions.

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Keywords: Ion acoustic wave; negative ions; sagdeev potential; double layer; positrons; nonthermal electrons.

1. INTRODUCTION

The study of double layers (DLs) in plasma is important because it is believed that DLs are responsible for accelerating particles in the auroral region of the ionosphere [1]. The DLs play a significant role in supplying and accelerating plasma in magnetic coronal funnels [2], and various theories on the formation of solar flares involve DLs [3]. Viking and S3-3 satellite observations have detected the DLs in the magnetospheric regions [4] and auroral regions [5], respectively. It has also been discovered that the acoustic DLs are responsible for auroral electron precipitation. During the past two decades, the formation of DLs has raised interest because of their relevance to cosmic applications [3,6–8], ion heating in linear turbulent heating devices [9], and confinement of plasma in tandem mirror devices [10]. In last few years, there has been considerable interest in the study of ion-acoustic DLs in multispecies plasmas including the effects of negative ions. The negative ions are produced by the attachment of electrons to neutral particles when an electronegative gas is introduced in an electrical gas discharge. The negative ions are found in D- region of the ionosphere [11], plasma processing reactors [12], and neutral beam sources [13]. Even a small amount of negative ions can have significant effect on the

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formation of nonlinear ion-acoustic wave structures [14].

Merlino and Loomis [15] have first experimentally investigated the formation of strong double layers in a plasma consisting of positive ions, negative ions, and electrons has been investigated in a triple plasma device. When a strong double layer is present in a positive ion/electron plasma, the introduction of negative ions results in the formation of a second double layer which is separated from the first double layer by a region of quasineutral plasma. Subsequently, Jain et al. [16] have theoretically investigated the effect of second-ion species on the characteristics of a large-amplitude ion-acoustic double layers (IADLs or DLs) in a collisionless, unmagnetized plasma consisting of hot and cold Maxwellian populations of electrons and two cold-ion species with different masses, concentrations, and charge states. After deriving the criteria for the existence of large-amplitude DLs, they have found that the presence of a positive-ion impurity does not modify considerably the characteristics of large-amplitude DLs. However, the presence of a negative-ion impurity changes significantly the characteristics of large-amplitude DLs . They have also presented an analytic discussion of small-amplitude DLs using a reductive perturbation method. Later, Mishra et al. [17] have studied DLs in plasma consisting of warm positive- and negative-ion species with different masses, concentrations, and charge states along with electrons with two-electron temperature distributions. It is found that there exist two critical concentrations of negative ions. One of them generally decides the existence of the DLs, whereas the other one decides the nature of the DLs. It is found that the system supports DLs only when the negative-ion concentration is greater than the critical concentration It is also found that below the critical concentration compressiveDLs exist and above it rarefactive DLs exist. Paul et al. [18] have considered the effect of electron inertia for theoretical investigation on ion acoustic solitons (IAS) and DLs in a multicomponent plasma consisting of warm positive ions, negative ions and isothermal electrons using pseudo potential method. It has been found that drift velocity and electron-inertia have significant contribution on the formation of IADLs in plasma having ($H+$, Cl^-), ($H+$, O^-), ($He+$, H^-) and ($He+$, O^-) ions. Baboolal et al. [19] have also investigated IADLs in negative-ion plasmas and have resolved the discrepancy of the results of previous authors on the existence of DLs in plasma.

On the other hand, the non-Maxwellian electrons in plasma give rise to many interesting characteristics in the nonlinear propagation of waves, including the excitation of IAS and DLs in plasma. To explain the observational results of the Freja [20] and Viking [21] satellites it has been assumed by Cairns et al. [22, 23] that the electron distribution to be nonthermal in the magnetosphere [24]. The nonthermal distributions of electrons are commonly found in the auroral zone [25] and in many laboratory plasmas in which wave damping produces electron tails [26]. The mechanism for the formation of nonthermal particle distributions in space plasma is still a research problem. Considering the effects of nonthermal electrons Gill et al [27] have studied the properties of IAS and DLs in a plasma consisting of warm positive and negative ions with different concentration of masses and charged states using small amplitude approximation. They have used the reductive perturbation method to derive KdV and m-KdV equations. The existence of IAS and DLs is explored over a wide range of parameter space. However, when nonthermal parameter of electrons is increased beyond a critical value, no double-layers are obtained. Das et al. [28] have considered the non-thermal electrons to have vortex-like velocity distribution and have derived a combined Schamel's modified Korteweg–de Vries–Zakharov– Kuznetsov (S–KdV–ZK) equation . Then they have studied the existence and stability of DLs in magnetized plasma consisting of warm adiabatic ions and non-thermal electrons. using Sagdeev's pseudopotential technique Das and Chatterjee [29] have studied the formation of large amplitude DLs in dusty plasma with non-thermal electrons and two temperature isothermal ions. It is found that the temperature of the dust particles, non-thermal electrons and ion-temperature all play significant role in determining the region of the existence of DLs. Later, Ghosh et al. [30] have applied pseudo potential technique to study the DLs in a multicomponent plasma consisting of nonthermal electrons and warm positive and negative ions with drift motion. They have obtained the conditions for the existence of such layers which control the formation and nature of DLs. The effects of nonthermal electrons, negative-ion concentration, and negative-ion temperature on the DLs formation and structure are also investigated. It is shown that the nonthermal electrons and the negative ions are shown to contribute significantly to the excitation and structure of the DLs. Recently, Kaur et al. [31] have investigated DLs in an inhomogeneous plasma consisting of Maxwellian and non-thermal distributions of electrons using reductive perturbation technique. They have derived a modified Korteweg–de Vries (mKdV) equation for DLs propagating in a collisionless inhomogeneous plasma. It is observed that the non-thermal parameters affect the amplitude and width of the DLs which further depend on the density. More recently, Paul et al. [32] have theoretically studied DLs in a multicomponent plasma consisting of positive ions, negative ions, and nonthermal electrons filled in a cylindrical waveguide using pseudo-potential technique. The expression of Sagdeevpotential for the IAS in such a plasma has been derived, and the conditions for the existence of DLs are obtained. The effects of the boundary of cylindrical waveguide and nonthermal electrons on the Sagdeev potential have been graphically discussed. From the nonlinear equation, the solution for double layers DLs of small amplitude IAW is obtained, and the structure of DLs is analyzed as function of the plasma parameters. It is seen that finite geometry of bounded plasma, nonthermality of electrons, density of negative ions, and stream velocity of positive and negative ions have significant contribution on the structure of DLs. The DLs may be compressive or rarefactive in plasma depending upon the values of the plasma parameters..Subsequently, Paul et al, [33] have considered the nonthermal electrons and ion beams in plasma for the study of the DLs in warm ion plasma using the pseudopotential technique. The critical value of nonthermal parameter of electrons for existence of DLs is obtained and discussed. The profiles of DLs in the plasma are drawn taking different values of density and temperature of ion beams, and the nonthermal parameter of electrons obeying the critical condition. It has been shown that both compressive and rarefactive DLs are excited in the plasma depending on the values of nonthermal electrons and ion beam parameters. The width of DLs is also graphically discussed. The results are useful in understanding the acceleration processes of charged particles in space plasma.

However, most fascinating results on the DLs are observed in a plasma consisting of positrons in plasma. DLs in electron-ion-positron plasma have different characteristic from that obtained in electron-ion (e.g. nonthermal electrons and ions) plasma. It is believed that electron-positron plasma has its origin in the early stage of universe [34-36] and this has important contribution on the physical processes in galactic nuclei [37], pulsar magnetosphere [38], polar caps of neutron star [39]. It has been observed that nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently from the plasma consisting of electrons and ions [40]. To understand the nature of some characteristics of the DLs in e-i-p plasma Bharuthram [41] have investigated the existence and the properties of arbitrary amplitude double layers in a four-component electron-positron plasma, consisting of two species of hot electrons, a hot and a cold positron species. The results are discussed with applications to the polar-cusp region of pulsars. Later, Mishra et al. [42] have studied DLs in multicomponent plasma with positrons. Using the reductive perturbation method, they have derived the modified Korteweg–de Vries (mKdV) equation for the system and discussed about the DLs solution of the mKdV equation. It is found that there exist two critical concentrations of positrons which decide the existence and nature of the DLs. It is also found that the system supports DLs only when the positron concentration is less than the critical concentration. It is also investigated that for the given sets of parameter values the system supports rarefactive or compressive DLs for different values of positron density. It is predicted that for the given sets of parameter values, the amplitude of rarefactive (compressive) double layer decreases (increases) with the increase of positron concentration but the width of the rarefactive (compressive) double layer increases (decreases). Subsequently, Jain and Mishra [43] have studied the large-amplitude DLs in a collisionless plasma consisting of isothermal positrons, warm adiabatic ions and two-temperature distribution of electrons using the pseudo-potential approach. It is found that for a selected set of physical parameters, the rarefactive DLs exists in the electron–positron–ion plasma. The existence regime of DLs is very sensitive to the plasma parameters, e.g. cold electron concentration and temperature ratio of two electron species. The increase in the finite ion temperature ratio increases the amplitude of the rarefactive double layer. For the study of small-amplitude DLs, they have expanded the Sagdeev potential. It is found that the amplitude of the DLs increases with increase in ion temperature ratio and cold electron concentration. However, for the increase of positron concentration and temperature ratio of positrons to electrons the amplitude of the DLs are decreased.

In many physical situations, it seems therefore more realistic to consider the simultaneous presence of nonthermal electrons and positrons in multicomponent plasma with negative ions for the investigation of DLs n drifting plasmas. To the best of our knowledge, this problem has not been studied by any author till now. We are hence motivated to study the excitation and characteristics of the DLs in a multicomponent drifting plasma composed of nonthermal electrons, isothermal positrons and warm positive as well as negative ions. The paper is organized as follows: In Section 2, the basic equations are given. In Section 3, the Sagdeev pseudo potential is derived, and the double layer solutions are obtained for arbitrary amplitude ion acoustic waves The results and discussions are presented in Section 4.

2. BASIC EQUATIONS

We assume**,** a collisionless unmagnetized plasma consisting of warm positive and negative ions, positrons and nonthermal electrons. Moreover, we consider the stream velocity of positive and negative ions in unperturbed state. The normalized basic equations in unidirectional propagation for such plasma in dimensionless form are:

For positive ions:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}\left(n_i u_i\right)=0 \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial u_i}{\partial x}=0 \tag{3}$$

For negative ions:

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j u_j\right)=0 \tag{4}$$

$$\frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}+\frac{\sigma_j}{Qn_j}\frac{\partial p_j}{\partial x}=\frac{Z}{Q}\frac{\partial \phi}{\partial x} \tag{5}$$

$$\frac{\partial p_j}{\partial t}+u_j\frac{\partial p_j}{\partial x}+3p_j\frac{\partial u_j}{\partial x}=0 \tag{6}$$

Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_p-n_i+Zn_j \tag{7}$$

Where, the subscript e, i, j and p denote electron, positive ions, negative ions and positrons. n_i, u_i, p_i, σ_i and n_j, u_j, p_j, σ_j are respectively the concentration , velocity, pressure and temperature of positive and negative ions, $Q=m_j/m_i$, m_i and m_j are the mass of positive ions and negative ions, Z is number of electrons attached to negative ion, $Z=1$.

The normalized electron density n_e is given by [22,23]

$$n_e=\left(1-\beta\phi+\beta\phi^2\right)e^{\phi} \tag{8}$$

where

$$\beta=\frac{4p}{1+3p} \tag{9}$$

β measures the deviation from thermalized state and p determines the presence of fast particles in the model.
The density of the positrons is $n_p=\chi_p\exp(-\sigma_p\varphi)$

where,

$$\chi_p=n_{p0}/n_{e0}=1-n_{i0}\,,\ \sigma_p=T_e/T_p\,.$$

3. NONLINEAR EQUATIONS AND DOUBLE LAYER SOLUTION

For obtaining the double layer solution from equation (1) to (7) we use the Gallelian transformation $\eta=x-Vt$, where V is the velocity of the solitary waves and we assume the boundary conditions

$$n_i\to n_{io}, n_j\to n_{jo}, u_i\to u_{io}, u_j\to u_{jo}, p_i\to p_{io}, p_j\to p_{jo}, \phi\to 0$$
$$\text{at } |x|\to\infty \tag{10}$$

By using equation (11) and boundary conditions (12) we get from equations (1) to (6)

$$\frac{d^2\phi}{d\eta^2}=\left(1-\beta\varphi+\beta\varphi^2\right)\exp(\varphi)\text{ - }\chi_p\exp(-\sigma_p\varphi)+$$
$$\frac{n_{io}^{\frac{3}{2}}}{\sqrt{12\sigma_i p_{io}}}\left[\sqrt{\left\{\left(V-u_{io}+\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}-\sqrt{\left\{\left(V-u_{io}-\sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2-2\varphi\right\}}\right]+$$
$$\frac{Zn_{jo}^{\frac{3}{2}}Q^{\frac{1}{2}}}{\sqrt{12\sigma_j p_{jo}}}\left[\sqrt{\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2+\frac{2Z}{Q}\varphi\right\}}-\sqrt{\left\{\left(V-u_{jo}-\sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2+\frac{2Z}{Q}\varphi\right\}}\right]$$

$$= -\frac{d\psi}{d\varphi} \tag{11}$$

where $\psi(\phi)$ is known as the Sagdeev Potential.

$$\psi(\phi) = (1+3\beta) - [(1+3\beta) - 3\beta\varphi + \beta\varphi^2]\exp(\varphi) - \frac{\chi}{\sigma_p}[1-\exp(-\sigma_p\varphi)] -$$

$$\frac{n_{io}^{\frac{3}{2}}}{6\sqrt{3\sigma_i p_{io}}} \cdot \left[\begin{array}{l} \left\{\left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left\{\left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^2 - 2\varphi\right\}^{\frac{3}{2}} \\ -\left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^3 + \left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}}\right)^3 \end{array}\right]$$

$$- \frac{Q^{\frac{3}{2}} n_{jo}^{\frac{3}{2}}}{6\sqrt{3\sigma_j p_{jo}}} \left[\begin{array}{l} \left\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^2 + \frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}} - \left\{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right) + \frac{2Z}{Q}\varphi\right\}^{\frac{3}{2}} + \\ \left\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^3\right\} - \left\{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^3\right\} \end{array}\right]. \tag{12}$$

Integating (11) we get

$$\frac{1}{2}\left(\frac{d\varphi}{d\eta}\right)^2 + \psi(\varphi) = 0 \tag{13}$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi \to 0, d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0.$

Equation (13) describes the motion of a pseudoparticle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $\psi(\varphi)$. The first term on the right-hand side of Eq. (13) can be regarded as the kinetic energy of the pseudoparticle. Since the kinetic energy is always nonnegative, it follows that $\psi(\varphi) \le 0$ for the entire motion, zero being the maximum value of $\psi(\varphi)$. Equation (13) shows that $d\psi/d\varphi$ is the force on the particle at the position φ . Equation (13) may also be regarded as an anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the double layer solution of Eq. (13), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi = 0$ and φ_m, where φ_m is some extremum value of the potential φ, called the amplitude of the DL. The double layer solutions $\psi(\varphi)$ must additionally satisfy the following boundary conditions:

(i) $\psi(\varphi) = 0, d\psi/d\varphi = 0$ and $d^2\psi/d\varphi^2 < 0$ at $\varphi = 0$, (14a)

(ii) $\psi(\varphi) = 0, d\psi/d\varphi = 0$ and $d^2\psi/d\varphi^2 < 0$ at $\varphi = \varphi_{\max}(\ne 0)$, (14b)

(iii) $\psi(\varphi) < 0,$ for $0 < |\varphi| < |\varphi_m|$ (14c)

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudoparticle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity vanish. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m .

Now we expand the r.h.s of (11) and (12) in power series of φ and taking the terms up to φ^5 we obtain

$$\frac{d^2\phi}{d\eta^2} = A_1\phi + A_2\phi^2 + A_3\phi^3 + A_4\phi^4 + A_5\phi^5 + ... \tag{15}$$

$$A_1 = \left[(1-\beta) + \chi\sigma_p - \frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i p_{io}}{n_{io}}} - \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j p_{jo}}{n_{jo}}} \right] \tag{16a}$$

$$A_2 = \frac{1}{2}\left[\begin{array}{l} 1 - \chi\sigma_p^2 + \frac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-3} - \left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-3} \right\} \\ - \frac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-3} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-3} \right\} \end{array} \right] \tag{16b}$$

$$A_3 = \frac{1}{6}\left[\begin{array}{l} (1+3\beta) + \chi\sigma_p^3 + \frac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-5} - \left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-5} \right\} \\ + \frac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} \right\} \end{array} \right] \tag{16c}$$

$$A_4 = \frac{1}{24}\left[\begin{array}{l} (1+8\beta) + \chi\sigma_p^4 + \frac{15n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-7} - \left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-7} \right\} \\ + \frac{15Z^5 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{7}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-5} \right\} \end{array} \right] \tag{16d}$$

$$A_5 = \frac{1}{120}\left[\begin{array}{l} (1+15\beta) + \chi\sigma_p^5 + \frac{105n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{ \left(V - u_{io} + \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-9} - \left(V - u_{io} - \sqrt{\frac{3\sigma_i p_{io}}{n_{io}}} \right)^{-9} \right\} \\ + \frac{105Z^6 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{9}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-9} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j p_{jo}}{Qn_{jo}}} \right)^{-9} \right\} \end{array} \right] \tag{16e}$$

And

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2 - \frac{1}{3}A_2\varphi^3 - \frac{1}{4}A_3\varphi^4 - \frac{1}{5}A_4\varphi^5 - \frac{1}{6}A_5\varphi^6 - \cdots\cdots \tag{17}$$

Now, for small amplitude ion acoustic wave we take the terms up to φ^3 of Eq.(15). Applying boundary conditions (14a) and (14b) in (17), we find that

$$\varphi_m = 2A_2 / 3A_3 \tag{18}$$

and

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m - \varphi)^2 \tag{19}$$

The small amplitude double solution of (15) satisfying (18) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1 - \tanh[\sqrt{(A_3/8\varphi_m)}\eta]\right) \tag{20}$$

The width of double layers is given by

$$W_d = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{21}$$

Equation (20) relates the amplitude φ_m of the DL to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (14). It is important to note that double layers in the plasma can only exist for positive value of A_3, i.e., for plasma parameters satisfying the following condition:

i) The nonthermal parameter β must be,

$$\beta > \frac{1}{3}\left[\begin{array}{l} 1 + \chi\sigma_p^3 + \dfrac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V - u_{io} + \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5} - \left(V - u_{io} - \sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-5}\right\} \\ + \dfrac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{\left(V - u_{jo} + \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-5} - \left(V - u_{jo} - \sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-5}\right\} \end{array}\right] \tag{22}$$

The critical values of nonthermal parameter β_c for the existence of small amplitude DL in the muticomponent plasma can be numerically estimated from (22), if the values of other parameters are known.

Again, from Eqs. (18) and (20), it follows that the sign of A_2 determines the nature of the DL. If A_2 is positive (negative), a rarefactive (compressive) DL is formed. Since A_2 is independent of β , the nature of DL, whether compressive or rarefactive, remains invariant under changes in β. But A_2 is dependent of positron density χ and positron temperature σ_p , so the nature of DL, whether compressive or rarefactive, remains variable under changes in χ and σ_p .For a cold ion plasma ($\sigma_i = \sigma_j = 0$), A_2 is negative or positive in presence of positrons and so both compressive and rarefative DL will be formed;

but in absence of positrons only compressive DLs can be formed in cold ion plasma.
For compressive DL, A_2 must be positive and the positron density χ should be less than χ_c i.e. $\chi < \chi_c$, χ_c is the critical value of positron density. For rarefactive DL, A_2 must be negative and the positron density χ should be greater than χ_c i.e. $\chi > \chi_c$.

The critical values of positron density is given by

$$\chi_c = \frac{1}{\sigma_p^2}\left[\begin{array}{l} 1+\dfrac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i p_{io}}}\left\{\left(V-u_{io}+\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}-\left(V-u_{io}-\sqrt{\dfrac{3\sigma_i p_{io}}{n_{io}}}\right)^{-3}\right\} \\ -\dfrac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j p_{jo}}}\left\{\left(V-u_{jo}+\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-3}-\left(V-u_{jo}-\sqrt{\dfrac{3\sigma_j p_{jo}}{Qn_{jo}}}\right)^{-3}\right\} \end{array}\right] \tag{23}$$

Knowing the values of $n_{i0}, n_{j0}, \sigma_i, \sigma_j, \sigma_p, V, u_{io}, u_{jo}, Q, Z$ of the plasma the value χ_c can be numerically estimated from (23).

4. RESULTS AND DISCUSSION

To see the nature of DLs in multicomponent plasma composed of warm positive and negative ions, nonthermal electrons and isothermal positron, we consider a model plasma. The solution of DLs (20) shows that DLs is dependent on i) the nonthermal parameter of electrons, ii) the density of negative ions, iii) the density of positrons, iv) the temperature of positrons, v) the temperature of ions and vi) the stream velocity of ions. The role of these parameters on the structures of DLs are shown in Figs. 2-6 and the variation of widths with plasma parameters are shown in Figs.7-8. Let us first see the profiles of DLs for different values of the plasma parameters

4.1 Profiles of Double Layers

4.1.1 Effect of nonthermal parameter of electrons

The effect of nonthermal electrons (β) on the DLs in the plasma having Q= 8.875 (He^+. Cl^-) has been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of DLs in (He^+ , Cl^-) plasma for different values of nonthermal parameter of electrons are shown below in Fig. 1.

It is seen from Fig. 1 that DLs may be compressive or rarefactive depending on the values of nonthermal parameter of electrons (β). For β =0.1 and 0.2, the DLs are compressive but for β =0.3 and 0.4 the DLS are rarefactive in nature. The amplitude of compressive DLs increases with the increase of β , but the amplitude of rarefactive DLs decreases with the increase of β .

4.1.2 Effect of negative ion density

The effects of concentration of negative ions on theDLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of DLs in (Cl^-, He^+) plasma for different negative ion density are shown below in Fig. 2(a) and Fig. 2(b) respectively.

It is observed from Fig. 2(a) that the DLs may be rarefactive depending on the values of negative ion density (n_{j0}) . The amplitude of rarefactive DLs increases with the increase of n_{j0}.

It is observed from Fig. 2(b) that the DLs may be compressive depending on the values of negative ion density. For n_{j0} =0.01- 0.04 the DLs are rarefactive but for n_{j0} =0.10- 0.16 the DLs are comprsseve in nature. The amplitude of rarefactive DLs increases with the increase of n_{j0} .

4.1.3 Effect of positron density

The effects of positron density (χ) on the DLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of IADLs in (He^+, Cl^-) plasma for different values of positron density are shown below in Fig. 3.

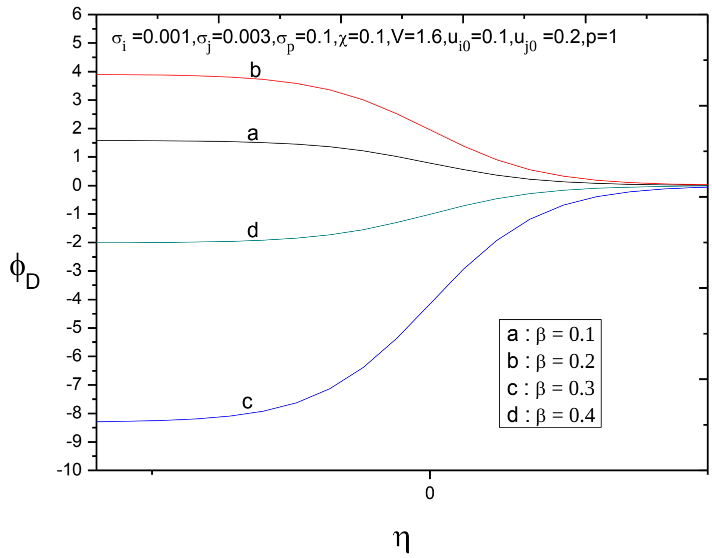

Fig. 1. Profiles of the DLs for different nonthermal parameter (β) in (He^+,Cl^-) with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.1, χ =0.1, u_{i0} =0.1, u_{j0} =0.2, V=1.6,p=1. Curves labeled with a,b,c and d correspond to four values of β, β =0.1,0.2,0.3 and 0.4 respectively

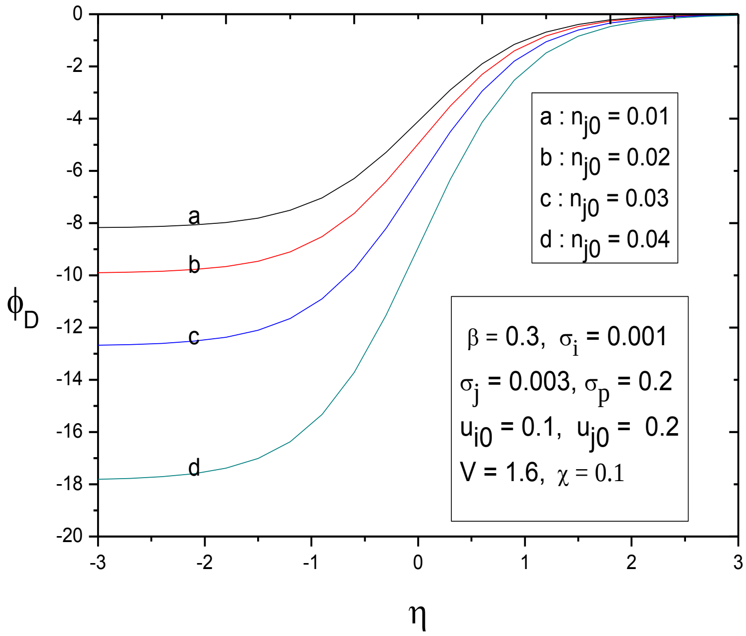

Fig. 2(a). Profiles of the rarefactive DLs for different negative ion density in (He^+, Cl^-) plasma with fixed values of β = 0.3, σ_i = 0.001, σ_j = 0.003, σ_p = 0.2, u_{i0} = 0.1, u_{j0} =0.2, V=1.6, p=1; Curves labeled with a,b,c and d correspond to four values of n_{j0}, n_{j0} =0.01,0.02,0.03 and 0.04 respectively

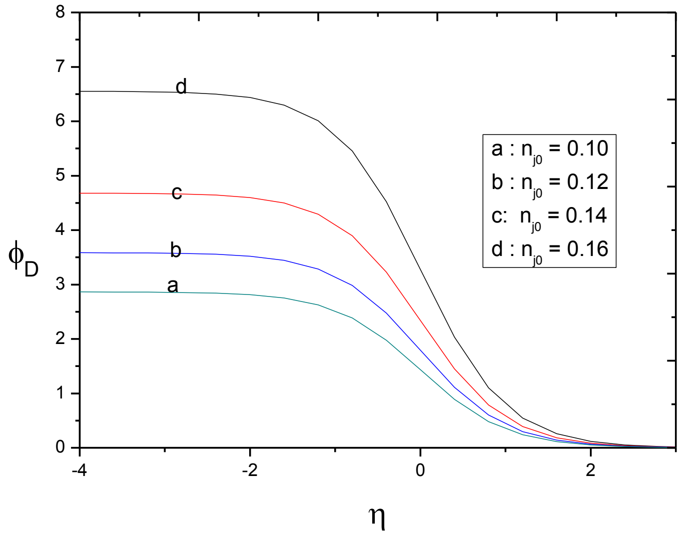

Fig. 2(b). Profiles of the compressive DLs for different negative ion density in (He^+, Cl^-) plasma with fixed values of β = 0.3, σ_i = 0.001, σ_j = 0.003, σ_p = 0.2, u_{i0} =0.1, u_{j0} =0.2, V=1.6, p=1; Curves labeled with a,b,c and d correspond to four values of n_{j0}, n_{j0} =0.10,0.12,0.14 and 0.16 respectively. The amplitude of compressive DLs increases with the increase of n_{j0}.

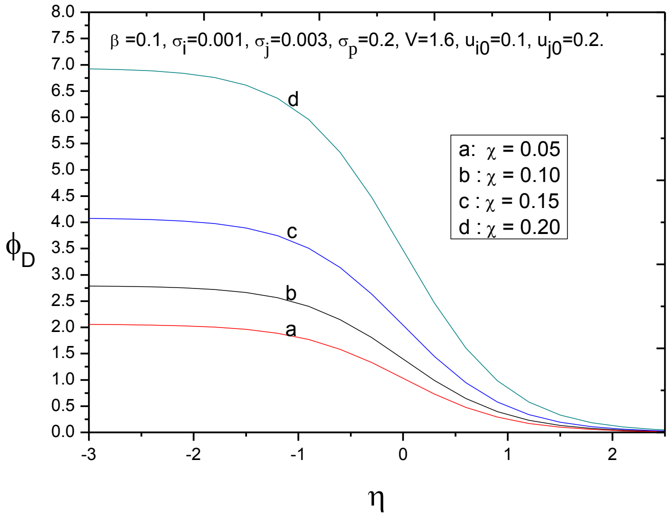

Fig. 3. Profiles of compressive DLs for different values of positron density in (He+, Cl-) plasma with fixed values of β = 0.1, σ_i =0.001, σ_j =0.003, σ_p =0.2, u_{i0} =0.1, u_{j0} =0.2, n_{j0} =0.1, V=1.6, p=1. Curves labeled with a, b, c and d correspond to four values χ, χ =0.05,0.10,0.15 and 0.20. respectively

The Fig. 3 shows that the the DLs are compressive only for different values of positron density (χ). The amplitude of compressive DLs increases with the increase of χ.

4.1.4 Effect of positron temperature

The effects of positron temperature on the DLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of DLs in (He^+, Cl^-) plasma for different values of positron density are shown below in Fig. 4.

From Fig. 4 it is seen that the DLs are rarefactive for different values of σ_p and the amplitude of the DLs decreases with the increase of positron temperature σ_p.

4.1.5 Effect of ion temperature

The effects of ion temperature (σ_i) on the DLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters.The profiles of DLs in (He^+, Cl^-) plasma for different values of ion temperature are shown below in Fig. 5.

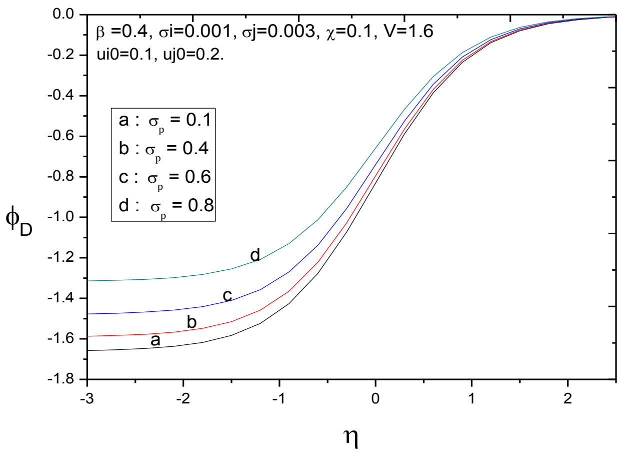

Fig. 4. Profiles of rarefactive DLs for different positron temperature (σ_p) in (He^+,Cl^-) plasma with fixed values of β = 0.3, σ_i =0.001, σ_j =0.003, n_{j0} =0.1, χ =0.1, u_{i0} =0.1, u_{j0} =0.2, V=1.6, p=1. Curves labeled with a, b, c and d correspond to four values σ_p, σ_p 0.1,0.4,0.6 and 0.8 respectively

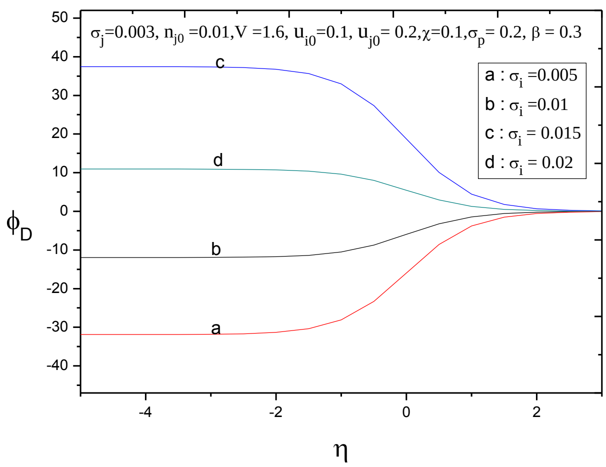

Fig. 5.Profiles of the compressive and rarefactive DLs for different ion temperature (σ_i) in (He^+,Cl^-) plasma with fixed values of n_{j0} =0.1, β = 0.3, σ_j =0.003, χ =0.1, σ_p =0.2, u_{i0} =0.1, u_{j0} =0.2, V=1.6, p=1. Curves labeled with a, b, c and d correspond to four values σ_i, σ_i 0.005,,0.01,0.015. and 0.02 respectively

It is seen in Fig. 5. that the the DLs may be compressive or rarefactive depending on the values of ion temperature (σ_i). The amplitude of rarefactive DLs decreases with the increase of σ_i and the amplitude of compressive DLs also decreases with the increase of σ_i.

4.1.6 Effect of ion stream velocity

The effects of ion stream velocity (u_{i0}) on the DLs in plasma have been numerically estimated and the profiles are drawn keeping fixed values of other plasma parameters. The profiles of DLs in (He^+, Cl^-) plasma for different values of ion stream velocity are shown below in Fig. 6.

It is seen from Fig. 6. that the DLs may be compressive or rarefactive depending on the values of ion stream velocity (u_{i0}). For u_{i0} = 0.1 the DLs are rarefactive, but for u_{i0} = 0.2, 0.3 and 0.4 the DLs are compressive in nature. The the amplitude of compressive DLs decreass with the increase of u_{i0}.

4.2 Amplitude and Width of Double Layers

In experimental observation the amplitude and width of DLs in plasma are generally studied. So, we have numerically estimated the amplitude and width of DLs in (He^+,Cl^-) plasma for different values of the plasma parameters.

The variation of amplitude and width of the DLs with negative ion concentration for the values of nonthermal parameter (β) are shown in Fig. 7(a) and Fig. 7(b) respectively.

It is seen from Fig. 7(a) that the amplitude of DLs is larger for large values of β and it increases up to some critical values of negative ion density n_{j0} then it decreases with the negative ion density.

In Fig. 7(b) it is seen that the width of DLs is larger for large values of β . The width decreases up to some critical values of negative ion density n_{j0} then it increases with the increase negative ion density.

Similarly, in Fig. 8(a) and Fig. 8(b) it is seen the variation of amplitude and width ofDLs s with negative ion concentration in (He^+,Cl^-) plasma for positron density (χ).

It is seen from Fig. 8(a) that the amplitude increase up to certain value of n_{j0} and then it decreases with increase of n_{j0} then it increaees with the increase of negative ion density. The amplitude is large for for large values of χ .

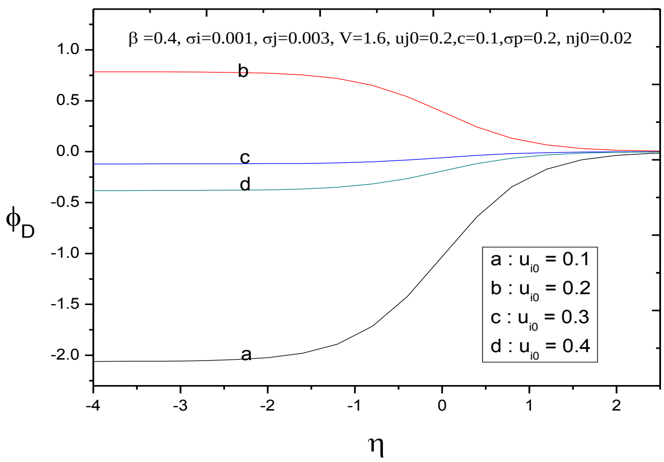

Fig. 6. Profiles of the compressive and rarefactive DLs for different ion stream velocity (u_{i0}) in (He^+, Cl^-) plasma with fixed values of β = 0.4, σ_i =0.001, σ_j =0.003, χ =0.1, n_{i0} =0.02, n_{j0} =0.1, σ_p =0.2, u_{j0} =0.2, V=1.6,p=1. Curves labeled with a, b, c and d correspond to four values u_{i0}, u_{i0} = 0.1,,0.2,0.3. and 0.4 respectively

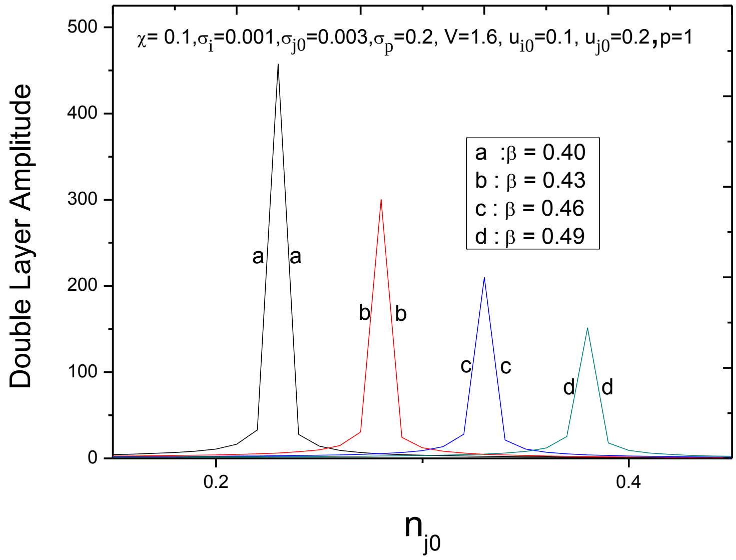

Fig. 7(a). Variation of amplitude of DLs with negative ion concentration for different values of nonthermal parameter (β) in (He^+,Cl^-) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, χ = 0.1, u_{i0} =0.1, u_{j0} =0.2, V=1.6, p=1. Curves labeled with a, b, c and d correspond to four values β , β = 0.40,,0.43,0.46. and 0.49 respectively

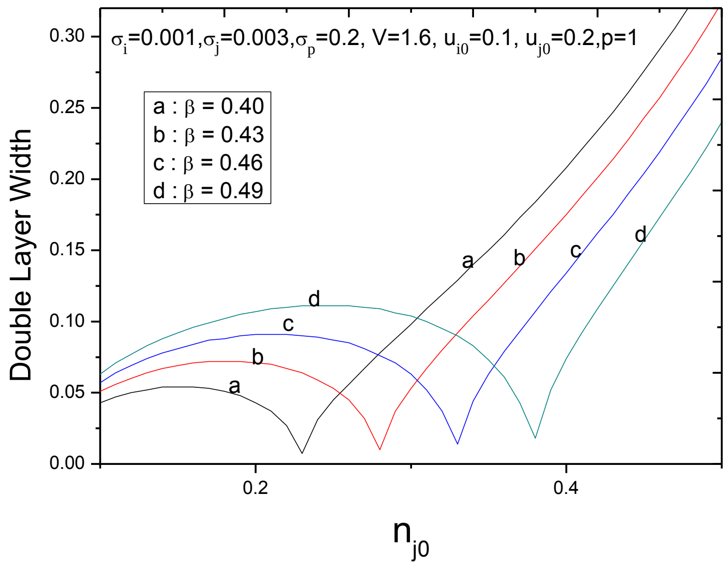

Fig. 7(b). Variation of width of DLs with negative ion concentration (n_{i0}) for different values of nonthermal parameter (β) in (He^+,Cl^-) plasma with fixed values of σ_i =0.001, σ_j =0.003, σ_p =0.2, χ = 0.1, u_{i0} =0.1, u_{j0} =0.2, V=1.6 and p=1. Curves labeled with a, b, c and d correspond to four values β , β = 0.40,,0.43,0.46. and 0.49 respectively

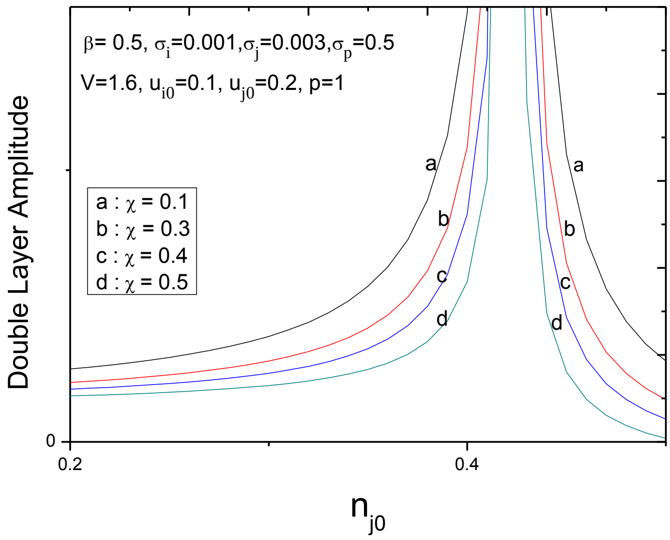

Fig. 8(a). Variation of amplitude of the DLs with negative ion concentration (n_{j0}) for different values of positron density (χ) in (He^+,Cl^-) plasma with fixed values of β =0.5, u_{i0} =0.1, u_{j0} =0.2, σ_i =0.001, σ_j =0.003, σ_p =0.5, V=1.6, p=1. Curves labeled with a, b, c and d correspond to four values χ , χ = 0.1,0.3,0.4. and 0.5 respectively

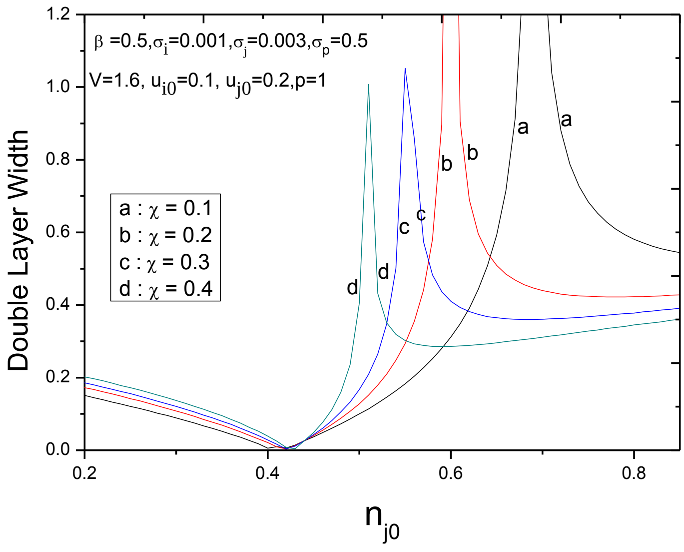

Fig. 8(b). Variation of width of the DLs with negative ion concentration (n_{j0}) for different values of positron density (χ) in (He^+,Cl^-) plasma with fixed values of β =0.5, σ_i =0.001, σ_j =0.003, σ_p =0.5, u_{i0} =0.1, u_{j0} =0.2,V=1.6, , p=1. Curves labeled with a, b, c and d correspond to four values χ , χ = 0.1, 0.3, 0.4. and 0.5 respectively

The width is larger for small values of χ . But Fig. 8(b) shows that the width decreases up certain value n_{j0} and then it increases with the increase n_{j0}. The width is larger for small values of χ. The width is large for small values of χ .

5. CONCLUSION

Considering a multicomponent plasma composed of warm positive ions and negative ions, positrons and nonthermal electrons , we have theoretically studied the excitation of small amplitudeDLs in the plasma. It has been shown that the DLs in such plasma depends on the plasma parameters; i.e. nonthermal electrons, density of negative ions, density and temperature of positrons, ion temperature and ion stream velocity. Numerical estimation is made taking a model plasma and the profiles of DLs and the width of DLs are drawn. It is seen that:

1. The DLs may be compressive or rarefactive depending on the values of nonthermal parameter of electrons (β). For β =0.1 and 0.2, the DLs are compressive but for β =0.3 and 0.4 the DLs are rarefactive in nature. The amplitude of compressive DLs increases with the increase of β , but the amplitude of rarefactive DLs decreases with the increase of β .
2. The DLs may be rarefactive depending on the values of negative ion density (n_{j0}) . The amplitude of rarefactive DLs increases with the increase of n_{j0}.
3. The DLs are compressive only for different values of positron density (χ). The amplitude of compressive DLs increases with the increase of χ.
4. The DLs are rarefactive for different values of σ_p and the amplitude of the DLs decreases with the increase of positron temperature σ_p.
5. The the DLs may be compressive or rarefactive depending on the values of ion temperature (σ_i). The amplitude of rarefactive DLs decreases with the increase of σ_i and the amplitude of compressive DLs also decreases with the increase of σ_i.
6. The DLs may be compressive or rarefactive depending on the values of ion stream velocity (u_{i0}) . For u_{i0} = 0.1 the DLs are rarefactive, but for u_{i0} = 0.2, 0.3 and 0.4 the DLs are compressive in nature. The amplitude of compressive DLs decreass with the increase of u_{i0}.
7. The amplitude of DLs is larger for large values of β and it increases up to some critical values of negative ion density n_{j0} then it decreases with the negative ion density. On the other hand, the amplitude of DLs increases up to certain critical value of n_{j0} and then it decreases with increase of n_{j0} after that it increaees with the increase of negative ion density. The amplitude is large for for large values of χ .
8. The width of DLs is larger for large values of β. The width decreases up to some critical values of negative ion density n_{j0} then it increases with the increase of n_{j0}. On the other hand, the width of DLs is large for small values of χ . The width decreases up certain value n_{j0} and then it increases with the increase n_{j0}.

However, for our present study of small amplitude ion acoustic DLs an unmagnetized plasma has been considered . Considering static magnetic field in the plasma more interesting results on small amplitude DLs may be obtained. For the study of large amplitude DLs the terms φ^4, φ^5 etc. of right hand side of Eq. (15) should be taken into account and the solution of DLs can be obtained using tanh-method [44-47].

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Torven S, Lindberg L, Carpenter R. Spontaneous transfer of magnetically stored energy to kinetic energy by electric double layers. Plasma Phys. 1985;27:143.
2. Raadu MA. Physics of double layers and their role in astrophysics Phys. Rep. 1989;178: 25.
3. Alfven H, Carlqvist P, Solar Phys. 1967;1: 220.

4. Bostrom R, Gustafsson G, Holback B, Holmgren G, Koskinen H, Kintner P. Characteristics of solitary waves and weak double layers in the magnetospheric plasma. Phys. Rev. Letts. 1988;61:82.
5. Temerin M, Cerny K, Lotko W, Mozer FS. Phys. Rev. Letts. 1982;48:1175.
6. Borovsky LE, Geophys J. The production of ion conics by oblique double layers. Res. 1984;89:2251.
7. Carlqvist P. IEEE Trans. Plasma Sci. 1986;PS-14:794.
8. Mishra MK, Arora AK, Chhabra RS. Ion-acoustic compressive and rarefactive double layers in a warm multicomponent plasma with negative ions. Phys. Rev., E. 2002;66:46402.
9. Saeki K, Iizuka S, Sato N. Phys. Rev. Letts. 1980;45:1853.
10. Baldwin DE, Logan BG. Improved tandem mirror fusion reactor. Phys. Rev. Letts. 1979;43:1318.
11. Massey H. Negative ions, 3rd edn. (Cambridge University Press, Cambridge) 1976;663.
12. Gottscho RA, Gaebe CE. IEEE Trans. Plasma Sci. 1986;PS- 14:92.
13. Bacal M, Hamilton GW. Phys. Rev. Letts. 1979;42:1538.
14. Majumdar SR, Chowdhury AR, Paul SN. Higher order nonlinearity and double layer in a multicomponent plasma consisting of cold positive and negative ions and two temperature isothermal electrons. Physica Scripta. 2004;69(4):335.
15. R.t L. Merlino, Loomis JJ. Phys. Fluids B: Plasma Physics. 1990;2:2865.
16. Jain SL, Tiwari RS, Sharma SR.. Canadian J. Phys. 1990;68(6):474.
17. Mishra MK, Arora AK, Chhabra RS. Phys Rev E. 2002;046402.
18. Paul SN, Chattopadhyaya S, Bhattacharya SK, Bera B. On the study of ion-acoustic solitary waves and double-layers in a drift multicomponent plasma with electron-inertia. Pramana. 2003;60(6):1217-33.
19. Baboolal S, Bharuthram R, Hellberg MA. J. Plasma Phys. 2009;46:247.
20. Dovner PO, Eriksson AI, Boström R, Holback B. Freja multiprobe observations of electrostatic solitary structures. Geophysical research letters. 1994 Aug 15;21(17):1827-30.
21. Bostrom R, Gustafsson G, Holback B, Holmgren G, H. Koskinen, P. Kintner, Phys. Rev. Letts. 1988;61:82.
22. Cairns RA, Bingham R, Dendy RO, Nairn CMC, Shukla PK, Mamun AA, J. Ion sound solitary waves with density depressions. Phys. IV France. 1995;5:C6–43.
23. Cairns RA, Mamun AA, Bingham R, Bostrom R, Dendy RO, Nairn CMC, Shukla PK. Geophys. Res. Letts. 1995;22:2709.
24. Hall DS, et al. J. Geophys. Res. 1991;96: 7869.
25. Cairns RA, Mamun AA, Bingham R, Shukla PK. Ion-acoustic solitons in a magnetized plasma with nonthermal electrons. Physica Scripta. 1996;1996(T63):80.
26. Shapirov VD, Soloviev GI, Dawson JM, Bingham R. Phys. Plasmas. 1995;2: 3148.
27. GillTS,P. Bala,H. Kaur,N. S. Saini,S. Bansal,J. Kaur ,The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics
28. Das J, Bandyopadhyay A, Das KP. Ion-acoustic double layers in magnetized plasma consisting of warm adiabatic ions and non-thermal electrons having vortex-like velocity distribution: existence and stability. Journal of plasma physics. 2008;74(2):163-86.
29. Das B, Chatterjee P. Phys. Letts. 2009;A373: 1144.
30. Ghosh B, Paul SN, Das C, Paul I, Banerjee S, Brazilian J. Phys. 2013;43:(1-2).
DOI: 0.1007/s13538-012-0115-February
31. Kaur H, Gill TS, Bala P. Propagation characteristics of ion-acoustic double layer in multicomponent inhomogeneous auroral zone plasma. Pramana. 2017;89(2):1-6.
32. Paul I, Chatterjee A, Paul SN. Effects of Boundary of Cylindrical Waveguide and Nonthermal Electrons on the Ion Acoustic Double Layers in Multicomponent Bounded Plasma. Brazilian J. Phys; 2020.
Available:https://doi.org/10.1007/s13538-020-00741-2
33. Paul I, Chatterjee A, Paul SN. Effects of nonthermal electrons and ion beams on ion-acoustic double layers in warm ion plasma Indian J, Phys; 2020.
Available:https://doi.org/10.1007/s12648-020-01899-w
34. Misner W, Thorne KS, Wheeler JA, Gravitation, Freeman, San Francisco; 1980.
35. Rees MJ, Gibbons GW, Hawkins SW, Siklas S. (editors) The very earlier universe, Cambridge Universal Press; 1983.
36. Zeldovich YB, Novikov I. Relativistic Astrophysics, Universitry of Chicago, Chicago; 1983.
37. Millar HR, Wilte PJ. Active Galactic Nuclei, Springer-Verlag, Berlin. 1987;202.
38. Sturrock PA, Baker KB. Positron production by pulsars. The Astrophysical Journal. 1979;234:612-4.

39. Michel FC, Theory of Neutron Star Magneto-sphere, Chicago Univ. Press, Chicago; 1991.
40. Rozzato FB, J. Plasma Phys. 1988;40:289.
41. Bharuthram R. Astrophys. Sp. Science. 1992;189:213.
42. Mishra MK, Tiwari RS, Jain SK. Small amplitude ion-acoustic double layers in multicomponent plasma with positrons. Physical Review E. 2007;76(3):036401.
43. Jain SK, Mishra MK. J. Plasma Phys. 2013;79:661.
44. Das GC, Sarma J, Talukdar M. Dynamical aspects of various solitary waves and double layers in dusty plasmas. Physics of Plasmas. 1998;5(1):63-9.
45. Bandyopadhyay A, Das KP. Phys. Scripta. 2000;61:92.
46. Sahu B, Roychoudhury R. Electron-acoustic solitary waves and double layers in a relativistic electron-beam plasma system. Physics of Plasmas. 2004;11(5):1947-54.
47. Paul SN, Chatterjee A, Paul I, Ghosh B. International J. Chemical and Physical Sciences, ISSN:2319-6602, IJCPS. 2015; 4(6):38.

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Effects of Relativistically Degenerate Electrons and Positrons on Ion- Acoustic Double Layers in Warm Ion Plasma

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Abstract: *Ion- acoustic double layers are studied in a plasma having relativistically degenerate electrons and positrons, and nondegenerate warm ions using pseudo potential approach. Expression of Sagdeev potential is derived and from the nonlinear equation the solution of double layers are obtained. The profiles of double layers are drawn and discussed for different parameters in degenerate plasma. It is seen that double layers are compressive in nature and the presence of relativistically degenerate electrons and positrons has significant contribution on the excitation of double layers in relativistically degenerate plasma. The results on double layers are new which are not obtained by any author in degenerate (or nondegenerate) plasma consisting of non-Maxwellian (or Maxwellian) electrons and positrons.*

Keywords: Ion acoustic wave . Relativistically degenerate electrons and positrons , Pseudo potential method, Double layers.

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I. INTRODUCTION

Nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently in the plasma consisting of electrons and ions. Due to relevance on the wave processes in galactic nuclei [1], pulsar magnetosphere [2], polar caps of neutron stars [3] etc. many authors have considered e-i-p plasma for their studies on nonlinear propagation of waves [4-8]. The relativistic effects on solitary waves in e-i-p plasma is found to be interesting. Following the works of Das and Paul [9], Mushtaq and Shah [10], Gill et al. [11] other authors [12-18] have shown that both positrons and relativistic factor causes to increase the amplitude of soliton and to shrink the width of soliton. But in the plasmas, where the density is quite high and temperature is very low, the thermal de-Broglie wavelength may become comparable to the interparticle distances. In such situations, quantum effects become important due to the overlapping of wave functions of the neighbouring particles. The quantum effects can significantly modify the linear and nonlinear characteristics of waves in quantum (degenerate) plasma [19-23]. Considering an unmagnetized two-species relativistic quantum plasma system comprised of electrons and ions Sahu [24] have derived the Korteweg–de Vries (dKdV) equation by reductive perturbation method using one-dimensional quantum hydrodynamic model (QHD).

But, in relativistically-degenerate dense plasma, propagation of waves are generally studied following the works of Chandrashekhar [25-26] instead of Das and Paul [9] . The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the non-relativistic limit, and $P_e \propto n_e^{4/3}$ for the ultra-relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Shah et al [27] have undertaken investigation on the effect of trapping on the formation of solitary structures in relativistic degenerate (RD) plasmas. They have used the relativistic Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. The Sagdeev potential approach has been employed to obtain the arbitrary amplitude solitary structures both when the plasma has been considered cold and when small temperature effects have been taken into account. The theoretical results obtained have been analyzed numerically for different parameter values, and the results have been presented graphically. Masood and Eliasson [28] have studied electrostatic solitary waves in a degenerate (quantum) plasma with relativistically degenerate electrons and cold ions. The inertia is given by the ion mass while the restoring force is provided by the relativistic electron degeneracy pressure, and the dispersion is due to the deviation from charge neutrality. A nonlinear Korteweg–de Vries (KdV) equation is derived for small but finite amplitude waves and is used to study the properties of localized ion acoustic solitons for parameters relevant for dense astrophysical objects such as white dwarf stars. Later, Chandra et al. [29] have studied electron-acoustic solitary waves in a relativistically degenerate quantum plasma with two-temperature electrons using the QHD model and degenerate electron pressure $P_e \propto n_e^{4/3}$ .They have shown that degeneracy parameter significantly influences the conditions of formation and properties of solitary structures.

Since double layers in relativistically degenerate plasma has not been studied , we are interested here to study the ion-acoustic double layers (IADLs) in a unmagnetized nonlinear plasma with relativistically degenerate electrons and positrons and nondegenerate warm ions using pseudo potential approach. We have derived the expression of Sagdeev potential and from the nonlinear equation we have obtained the double layer solution of ion acoustic wave in such plasma. The profiles of IADLs are drawn and discussed for different values of the plasma parameters. The effect of degenerate electrons and positrons on the IADLs are shown graphically.

II. FORMULATION

We consider an unmagnetized plasma whose constituents are relativistically degenerate electrons and positrons,and nondegenerate warm ions. The dynamics of such plasma are governed by the following equations,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \frac{1}{2}\frac{\partial u_i^2}{\partial x} + \frac{\partial \varphi}{\partial x} + \frac{\sigma_i}{2}\frac{\partial n_i^2}{\partial x} = 0 \tag{2}$$

$$en_e \frac{\partial \varphi}{\partial x} - \frac{\partial P_e}{\partial x} = 0 \tag{3}$$

$$en_p \frac{\partial \varphi}{\partial x} + \frac{\partial P_e}{\partial x} = 0 \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \beta n_e + n_i + \alpha n_p = 0 \tag{5}$$

Where,

$\alpha = n_{p0} / |Z_i| n_{i0}, \beta = n_{e0} / |Z_i| n_{i0}, \sigma_i = 3T_i / T_{Fe}$, All other physical quantities are dimensionless. The number density of electrons, ions positrons are represented by n_e ,n_i and n_p respectively and rescaled by their equilibrium values. u_i is the ion velocity, T_i and T_{Fe} are the ion temperature and electron Fermi temperature respectively. Other parameters have their usual meanings. The equilibrium condition of charge neutrality is $\beta = 1+\alpha.$

Following Chandrasekhar [25,26] the degeneracy pressure of electron and positron in fully degenerate and relativistic configuration can be expressed in the form:

$$P_j = \left(\pi m_e^4 c^5 / 3h^3\right)\left[R_j(2R_j^2-3)\sqrt{1+R_j^2} + 3\sinh^{-1}(R_j)\right] \tag{6}$$

in which $R_j = p_{Fj}/m_e c = \left[3h^3 n_j / 8\pi m_e^3 c^3\right]^{\frac{1}{3}} = R_{j0} n_j^{\frac{1}{3}}$ (7)

where $R_{j0} = \left(n_{j0}/n_0\right)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3 / 3h^3 \approx 5.9\times10^{29} cm^{-3}$, 'c' being the speed of light in vacuum. p_{Fj} is the electron Fermi relativistic momentum; the index j will denote either electrons (e) or positrons (p) in the plasma. It is to be noted that the parameter R_{j0} is a measure of the relativistic effects and may be called relativistic degeneracy parameter. For ultra- relativistic case $R_{j0} >> 1$ and for weakly relativistic case $R_{j0} << 1$. The parameter R_{j0} can also be related to mass density as $\rho(gr/cm^3) = 1.97\times10^6 \cdot R_{j0}^3$. The density of white dwarfs can be in the range $10^5 < \rho < 10^9$. So in this case the relativity parameter R_{j0} can be in the range $0.37 < R_{j0} < 8$.

In the limits of very small and very large values of relativity parameter R_j, we obtain:

$$P_j = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e} n_j^{\frac{5}{3}} \quad \text{(For } R_j \to 0\text{)}, \tag{8}$$

$$P_j = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} hc n_j^{\frac{4}{3}} \quad \text{(For } R_j \to \infty\text{)} \tag{9}$$

Note that the degenerate electron (positron) pressure depends only on the electron (positron) number density but not on the temperature. Now, from Eqs.(1) - (9) the densities of degenerate electrons and positrons are obtained as

$$n_e = R_{e0}^{-3}[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2]^{3/2} \tag{10}$$

$$n_p = p^{-1}R_{e0}^{-3}[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)]^{3/2} \tag{11}$$

where, $p = n_{p0}/n_{e0} = \alpha/\beta$.

To obtain localized solutions of Eqs. (1) –(5) using the Sagdeev potential formalism [30] we consider a new moving frame of a single variable $\eta = x - Mt$, which is the moving frame with velocity (Mach number) M. Using this transformation and equilibrium quasi-neutrality condition as well as the boundary conditions i) $n_i \to n_{i0}$, ii) $u_i \to 0$, iii) $\varphi \to 0$ at $|x| \to \infty$ we obtain from Eqs.(1) and (2) the non-degenerate ion density as

$$n_i = \frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2} \pm \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2}] \tag{12}$$

Now, using Eqs.(10),(11) and (12) in Eq.(5) we get

$$\frac{d^2\varphi}{d\eta^2} = \beta R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2)\right]^{3/2}$$
$$-\alpha p^{-1}R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)\right]^{3/2}$$
$$-\frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2} - \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2}] = -\frac{d\psi}{d\varphi} \quad (13)$$

(Taking negative branch of ion density).

Where, ψ is the Sagdeev potential and it is given by

$$\psi = -\int \beta R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi\sqrt{1+R_{e0}^2} + \varphi^2\right]^{3/2} d\varphi$$
$$+\int \alpha p^{-1}R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi\sqrt{1+p^{2/3}R_{e0}^2} + \varphi^2)\right]^{3/2} d\varphi \quad (14)$$
$$+\int\left[\frac{[(\sigma_i + M^2)/2 - \varphi)] - \sqrt{[(\sigma_i+M^2)/2-\varphi]^2 - \sigma_i M^2}}{\sigma_i}\right]^{1/2} d\varphi$$

After simplification ψ becomes

$$\psi = \frac{\beta_1}{8}\left[-2(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{3/2} + 3(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2} - 3\ln\{\frac{(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2}}{(R_{eo}+\gamma_1)}\}\right]$$
$$+\frac{\alpha_1}{8}\left[2(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{3/2} - 3(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2} + 3\ln\{\frac{(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2}}{(p^{1/3}R_{eo}-\gamma_2)}\}\right]$$
$$+\beta_2\gamma_1(2R_{eo}^2+3) + \alpha_2\gamma_2(2p^{2/3}R_{eo}^2-3) + (M^2+\frac{\sigma_i}{3}) - \frac{2^{1/2}\sigma_i}{3}(M_1-M_2)^{1/2}(2M_1+M_2)$$

(15)

where

$$\gamma_1 = (1+R_{e0}^2)^{1/2}, \gamma_2 = (1+p^{2/3}R_{e0}^2)^{1/2}, \beta_1 = \beta/\mathrm{R}_{eo}^3, \alpha_1 = \alpha/pR_{eo}^3, \beta_2 = \beta/8\mathrm{R}_{eo}^2, \alpha_2 = \alpha/8p^{2/3}R_{eo}^2,$$
$$M_1 = 1+M^2/\sigma_i - 2\varphi/\sigma_i, M_2 = (M_1^2 - 4M^2/\sigma_i)^{1/2}, \beta = 1+\alpha, p = \alpha/\beta.$$

Now, integrating (14) we obtain an equation (the Energy law) in terms of a single independent variable

$$\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \quad (16)$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi \to 0, d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0.$

Eq. (16) describes the motion of a pseudoparticle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $\psi(\varphi)$. The first term on the right-hand side of Eq. (16) can be regarded as the kinetic energy of the pseudo particle. Since the kinetic energy is always nonnegative, it follows that $\psi(\varphi) \le 0$ for the entire motion, zero being the maximum value of $\psi(\varphi)$. Eq. (16) shows that $d\psi/d\varphi$ is the force on the particle at the position φ . Eq. (16) 8may also be regarded as an anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the double layer solution of Eq. (13), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi = 0$ and φ_m , where φ_m is some extremum value of the potential φ , called the amplitude of the DL. For the double layer solutions, $\psi(\varphi)$ must additionally satisfy the following boundary conditions:

(i) $\psi(\varphi) = 0, d\psi / d\varphi = 0$ and $d^2\psi / d\varphi^2 < 0$ at $\varphi = 0$, (17a)

(ii) $\psi(\varphi) = 0, d\psi / d\varphi = 0$ and $d^2\psi / d\varphi^2 < 0$ at $\varphi = \varphi_{\max}(\neq 0)$, (17b)

(iii) $\psi(\varphi) < 0,$ for $0 < |\varphi| < |\varphi_m|$ (17c)

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudo particle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity are vanished . Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m .

III. SOLUTION FOR THE DOUBLE LAYERS

Now we expand the right hand side of (13) and (14) in power series of ϕ and we obtain

$$\frac{d^2\varphi}{d\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + A_4\varphi^4 + \cdots\cdots \tag{18}$$

Where,

$$A_1 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{3(1+R_{e0}^2) - \frac{3}{8}(1+R_{e0}^2)^{-1} - \frac{3}{16}(1+R_{e0}^2)^{-2} - \frac{15}{128}(1+R_{e0}^2)^{-3}\}] \\ -p^{-1}[\{3(1+p^{2/3}R_{e0}^2) - \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-1} - \frac{3}{16}(1+p^{2/3}R_{e0}^2)^{-2} \\ -\frac{15}{128}(1+p^{2/3}R_{e0}^2)^{-3}\}] \end{bmatrix} \tag{19a}$$

$$-\frac{1}{2\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-1}\} - \{(M-\sqrt{\sigma_i})^{-1}\}\right]$$

$$A_2 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{3(1+R_{e0}^2)^{1/2} + \frac{3}{8}(1+R_{e0}^2)^{-3/2} + \frac{3}{8}(1+R_{e0}^2)^{-5/2} + \frac{45}{128}(1+R_{e0}^2)^{-7/2}\}] \\ -p^{-1}[\{3(1+p^{2/3}R_{e0}^2)^{1/2} + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-3/2} + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-5/2} \\ +\frac{45}{128}(1+p^{2/3}R_{e0}^2)^{-7/2}\}] \end{bmatrix}$$

$$-\frac{1}{4\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-3}\} - \{(M-\sqrt{\sigma_i})^{-3}\}\right] \tag{19b}$$

$$A_3 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{1-\frac{3}{8}(1+R_{e0}^2)^{-2}-\frac{5}{8}(1+R_{e0}^2)^{-3}-\frac{105}{128}(1+R_{e0}^2)^{-4}\}] \\ -p^{-1}[\{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}] \end{bmatrix}$$

$$-\frac{1}{8\sqrt{g_i}}\left[\{(M+\sqrt{g_i})^{-5}\}-\{(M-\sqrt{g_i})^{-5}\}\right] \tag{19c}$$

$$A_4 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{\frac{3}{8}(1+R_{e0}^2)^{-5/2}+\frac{15}{16}(1+R_{e0}^2)^{-7/2}+\frac{105}{64}(1+R_{e0}^2)^{-9/2}\}] \\ -p^{-1}[\{\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-5/2}+\frac{15}{16}(1+p^{2/3}R_{e0}^2)^{-7/2}+\frac{105}{64}(1+p^{2/3}R_{e0}^2)^{-9/2}\}] \end{bmatrix}$$

$$-\frac{5}{32\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-7}\}-\{(M-\sqrt{\sigma_i})^{-7}\}\right] \tag{19d}$$

and

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2-\frac{1}{3}A_2\varphi^3-\frac{1}{4}A_3\varphi^4-\frac{1}{5}A_4\varphi^5-\frac{1}{6}A_5\varphi^6-\cdots\cdots \tag{20}$$

Assuming small amplitude waves $\varphi < 1$ and taking the terms up to φ^3 of Eq.(18) we get the solution of IADLs in plasma. Applying boundary conditions (17a), (17b) and (17c) we obtain

$$\varphi_m = -\frac{2A_2}{3A_3} \tag{21}$$

and

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m-\varphi)^2 \tag{22}$$

The solution of small amplitude IADLs of (18) satisfying (20) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1-\tanh[\sqrt{(A_3/8\varphi_m)}\eta]\right) \tag{23}$$

The width of IADLs is given by

$$W_{dL} = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{24}$$

It is seen that width of IADLs depends on the plasma parameters e.g. relativistically degenerate parameter, density of degenerate electrons, density of degenerate positrons , ion temperature etc. Eq. (21) relates the amplitude φ_m of the IADL to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (18). It is important to be noted that the IADLs in plasma can only exist for positive value of A_3 i.e. $A_3 > 0$. To satisfy the condition , the plasma parameters should have some critical values. The degenerate electron density β must satisfy the condition $\beta < \beta_c$, β_c is the critical value of β. For the existence of IADLs , the parameter β_c obtained from (19c) is

$$\beta_c = (p^{-1}C_2 + D)/C_1 \tag{25}$$

Where,

$$C_1 = \mathrm{R}_{e0}^{-3}[\{1-\frac{3}{8}(1+R_{e0}^2)^{-2}-\frac{5}{8}(1+R_{e0}^2)^{-3}-\frac{105}{128}(1+R_{e0}^2)^{-4}\}$$
$$-p^{-1}\{\{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$
$$C_2 = \{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$
$$D = \frac{1}{8\sqrt{\sigma_i}}[\{(M-\sqrt{\sigma_i})^{-5}\}-\{(M+\sqrt{\sigma_i})^{-5}\}]$$

Now, if IADLs are excited in plasma for $A_3 > 0$, we see that the sign of A_2 will determine the nature of IADLs. If A_2 is positive IADL will be rarefactive and for negative value of A_2 the IADL will be compressive. It is seen that A_2 depends on the parameters R_{e0}, α, β, p, σ_i and M.

IV. RESULTS AND DISCUSSIONS

From the solution of ion-acoustic double layers (IADLs) (21) in a relativistically degenerate electron-ion-positron plasma it seen that excitation of IADLs depends on i) relativistic parameter (R_{e0}), ii) degenerate positron density(α), iii) degenerate electron density(β) and iv) ion temperature (σ_i). The role of these parameters on the IADLs are shown in Figs. 1-4.

A) **Profiles of Ion-Acoustic Double Layers**

i) **Effect of relativistic degeneracy parameter**

To see the effects of relativistic degeneracy parameter (R_{e0}) on the IADLs in degenerate plasma having fixed values of p = 0.1, σ_i =0.01, α =0.4 , M =2; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.1. It is seen from Fig.1 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having R_{e0} = 10.5, 12, 13.5 and 15 with fixed values of p = 0.1, σ_i =0.01, α =0.4 , M =2. It is also observed that the amplitudes of compressive IADLs are decreased with increase of R_{e0}.

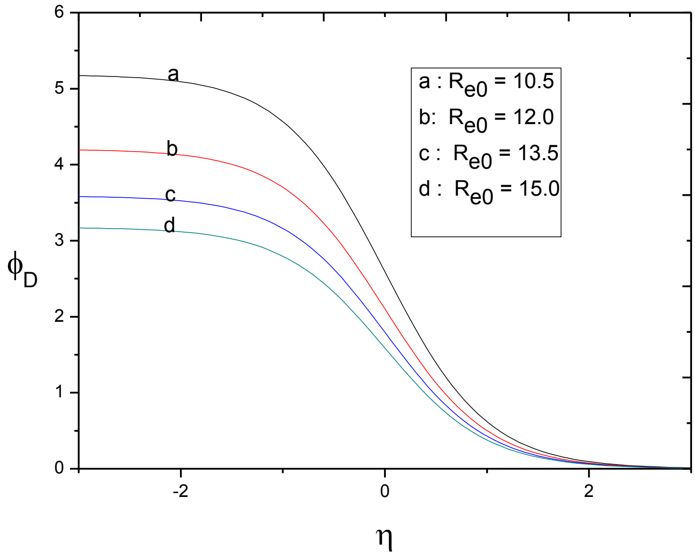

Fig. 1. Profiles of IADLs for different values of R_{e0} in degenerate plasma with fixed values of p =0.1, σ_i =0.01, α =0.4 , M =2; Curves a, b, c and d represent four values of R_{e0} = 10.5, 12, 13.5 and 15 respectively.

ii) **Effect of degenerate positrons**

Similarly, to find the effects of relativistically degenerate positron density (p) on the IADLs in degenerate plasma having fixed values of α =0.3, σ_i =0.01, M =2 , R_{e0} =10; we have followed the conditions for the existence of IADLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs, the profiles of DLs are drawn as shown in Fig.2. It is observed from Fig.2 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having p = 0.017, 0.021, 0.025, 0.029 with fixed values of α = 0.3, σ_i =0.01, M =2 , R_{e0} =10. It is also observed that the amplitudes of compressive IADLs are decreased with increase of p .

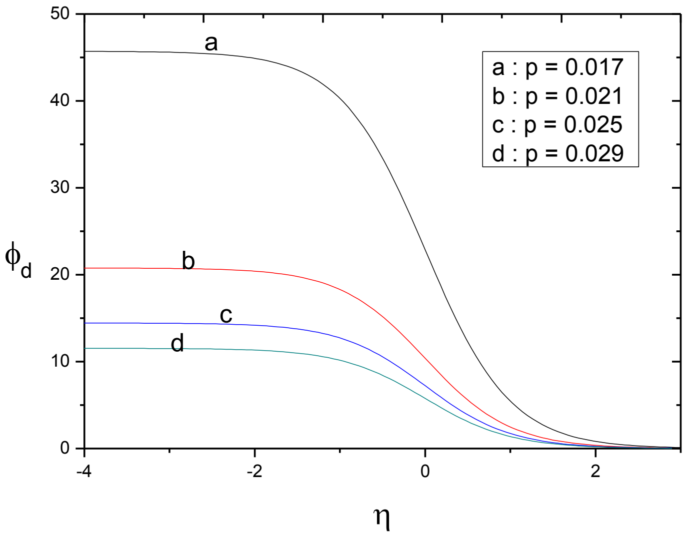

Fig. 2. Profiles of IADLs for different values of p in degenerate plasma with fixed values of α = 0.3, σ_i =0.01, M =2 , R_{e0} =10; Curves a, b, c and represent four values of p = 0.017, 0.021, 0.025, 0.029 respectively.

iii) **Effect of degenerate electrons**

In similar way , to see the effects of relativistically degenerate positron density (α) on the IADLs in relativistically degenerate plasma having fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.3. It is observed from Fig.3 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having α = 0.15, 0.3, 0.45 and 0.6 with fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1. It is seen that the amplitudes of compressive DLs are decreased with increase of α .

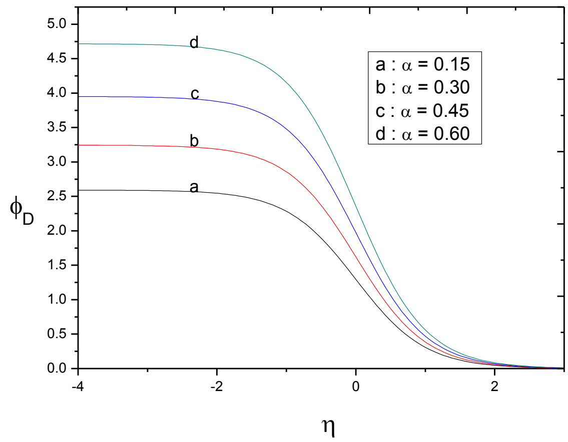

Fig. 3. Profiles of IADLs for different values of α in degenerate plasma with fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1; Curves a, b, c and d represent four values of α = 0.15, 0.3, 0.45 and 0.6 respectively.

iv) **Effect of ion temperature**

Similarly , to see the effects of ion temperature (σ_i) on the IADLs in relativistically degenerate plasma having fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.4. It is observed from Fig.4 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having σ_i = 0.3, 0.4, 0.5 and 0.6 with fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10. It is seen that the amplitudes of compressive DLs are decreased with increase of σ_i .

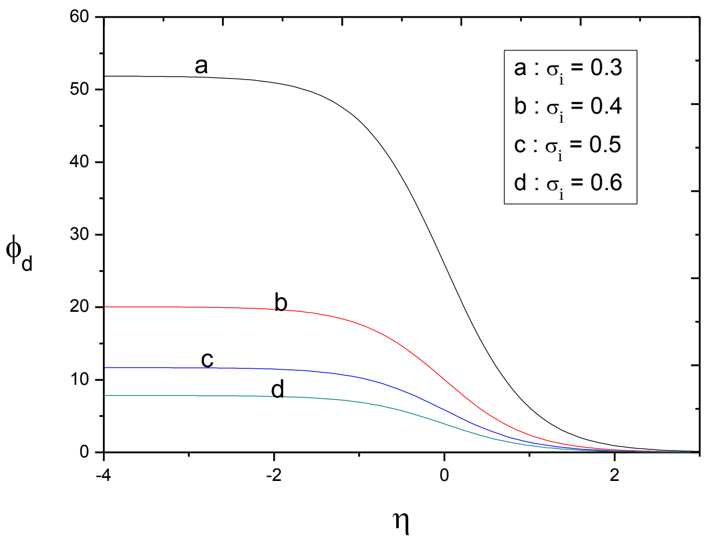

Fig. 4. Profiles of IADLs for different values of σ_i in degenerate plasma with fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10. Curves a, b, c and d represent four values of σ_i = 0.3, 0.4, 0.5 and 0.6 respectively.

B). **Width of Ion-Acoustic Double Layers**

It is known that the width (or thickness) of DLs is important for the study of DLs in plasma because it is important for experimental studies in laboratory and in space plasma. We have numerically estimated the width of IADLs using (24) and its variation for different parameters of an ion beam plasma is graphically discussed. It is to be remembered that for the numerical estimation we have followed the conditions for the existence of IADLs (i.e. A_3 must be positive).

Now, we have plotted the thickness of IADLs with the relativistic degenerate parameter (R_{e0}) in plasma for different values of relativistically degenerate positron density (p) shown in Fig.5. It is seen that the width of IADLs increases with increase of R_{e0} for p = 0.01, 0.014, 0.016, 0.018 for the fixed values of σ_i =0.01, α =0.12, M =2. More over, the thickness of Dl increases with increase of p.

To see the effect of relativistically degenerate electrons (α) and relativistic degenerate parameter (R_{e0}) we have plotted Fig.(6). It is seen that the width of IADLs increases with increase of α for R_{e0} =9, 11, 13, 15 for the fixed values of σ_i =0.01, p =0.1, M =2. More over, the thickness of DL increases with increase of R_{e0}.

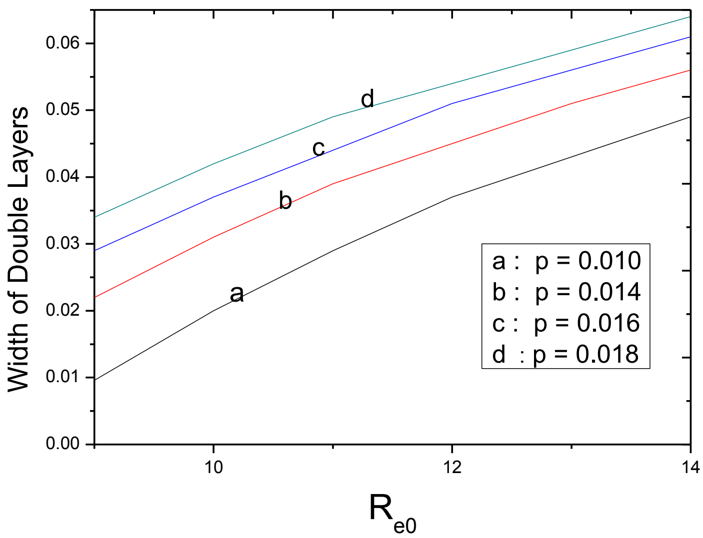

Fig.5. Variation of Width of IADL with R_{e0} for different values of p in degenerate plasma with fixed values of σ_i = 0.01, α =0.12, M =2; Curves a, b, c and d represent four values of p = 0.01, 0.014, 0.016 and 0.018 respectively.

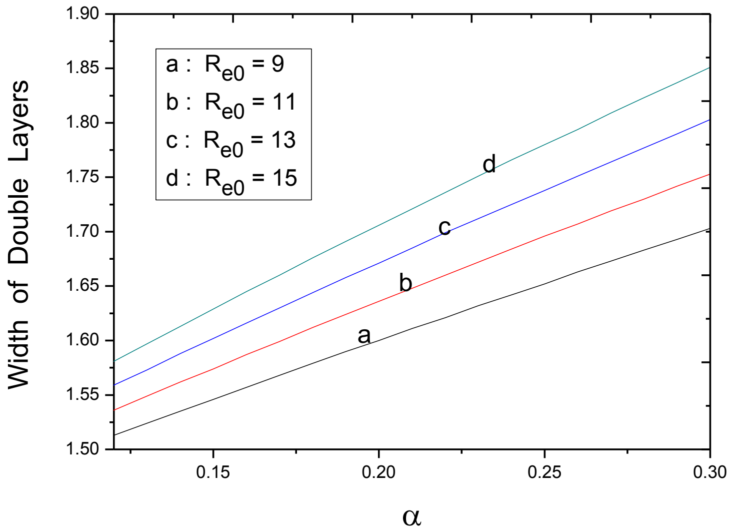

Fig.6. Variation of width of IADLs with α for different R_{e0} in degenerate plasma with fixed values of σ_i =0.01, p =0.1, M =2; Curves a, b, c and d correspond to four values of R_{e0} =9, 11, 13 and 15 respectively.

V. CONCLUSIONS

Considering a relativistically degenerate plasma composed of degenerate electrons and positrons with nondegenerate warm ions we have theoretically studied the excitation of ion- acoustic double layers (IADLs) in the plasma using the pseudo-potential method. It has been shown that the IADLs in such plasma depends on the plasma parameters. From numerical estimation the profiles of IADLs are drawn. The IADLs in relativistically plasma are compressive and the amplitudes depend on the values of the plasma parameters.

Our main findings are :

i) The IADLs in relativistically degenerate e-i-p plasma are compressive in nature for different values of plasma parameters e.g. relativistic degenerate parameter, degenerate positron, degenerate electrons and ion temperature.

ii) The amplitudes of IADLs s are small for large values the plasma parameters.

iii) The width of IADLs increases with increase of R_{e0} for p = 0.01, 0.014, 0.016, 0.018 with the fixed values of σ_i =0.01, α =0.12, M =2. More over, the width of IADL increases with increase of p .

iv) The width of IADLs increases with increase of α for R_{e0} =9, 11, 13, 15 with the fixed values of σ_i =0.01, p =0.1, M =2. More over, the thickness of DL increases with increase of R_{e0} .

However, the effect of ion drift has not been considered in the present investigation of IADLs. It is known that ion acoustic wave will propagate through the plasma with two modes , fast-mode and slow- mode, in presence of drifting (streaming) ions. Considering the ion drift some new results on nonlinear propagation of waves in degenerate plasma may be obtained from which the causes of some phenomena in astrophysical objects may be understood.

REFERENCES

[1]. H.R.Millar and P.J. Wilte, *Active Galactic Nuclei*, Springer-Verlag, Berlin, 202 (1987).
[2]. P.A.Sturrock and K B Baker, *Astrophys. J*. **234,** 612 (1979).
[3]. F.C.Michel, *Theory of Neutron Star Magneto-sphere*, Chicago Univ. Press, Chicago, (1991).
[4]. R. Pillay and R. Bharuthram, *Astrophys. Space Sci*. **188,** 592 (1992).
[5]. S.I.Popel, S.V.Vladimirov and P.K.Shukla, *Phys. Plasmas* **2,** 716 (1995).
[6]. S.Ghosh and R.Bharuthram,*Astrophys.Space Sci*. **189 ,** 121 (2008).
[7]. H.Pakzad, *Indian J.Phys.* **83**, 1605 (2009).
[8]. S. N. Paul, Chandra Das, Indrani Paul, B. Bandyopadhyay, S.Chattopadhya and S .S. De , *Indian J. Phys*. **86 (6)**, 545 (2012).
[9]. G.C.Das and S.N.Paul, *Phys. Fluids*,**28** 823 (1985).
[10]. A.Mushtaq and H. A. Shah, *Phys. Plasmas* **12**, 072306 (2005).
[11]. T. S. Gill, A. Singh, H.Kaur, N. S.Saini, P.Bala, *Phys. Letts.* A **361**(4,5), 364 (2007).
[12]. R. Saeed, A. Shah, and M. N.Haq ,*Phys. Letts.* A **373** (45), 4164 (2009).
[13]. A. Shah and R.Saeed, *Phys. Plasmas* **17,** 102301 (2010).
[14]. A. Shah, S. Mahmood, and Q. Haque , *Phys. Plasmas* **18,** 114501 (2011).
[15]. K. Javidan and D. Saadatmand, *Astrophys. Space Sci.* **333,** 471 (2011).
[16]. A. Shah, Q. Haque and S. Mahmood, *Astrophys. Space Sci*. **335 (2)**, 529 (2011)
[17]. E. Saberian, A. Esfandyari-Kalejahi and M. Akbari-Moghanjoughi, *Canadian J. Phys*. **89** (3), 299 (2011).
[18]. R. Das and R. Sarma, *International J. Sci & Engineering Res*. **6 (7)** , 1115 (2015) .
[19]. F Hass, *Phys. Plasmas* **12**, 062117 (2005).
[20]. P. K. Shukla and B. Eliasson, *Phys. Rev. Letts.* **96**, 245001 (2006).
[21]. A. P. Misra and A. Roy Chowdhury, *Phys. Plasmas* **13**, 072305 (2006).
[22]. A. P. Misra and C. Bhowmik, *Phys. Plasmas* **14**, 012309 (2007).
[23]. B. Ghosh, S. Chandra and S. N. Paul, *Phys. Plasmas* **18**, 012106 (2011).
[24]. B. Sahu, *Pramana* , **76 (6),** 933 **(** 2011).
[25]. S. Chandrasekhar , *Philosophical Magazine* **11 (7),** 592**(** 1931).
[26]. S. Chandrasekhar , *Mon. Not. R. Astron. Soc.* **170,** 405**(** 1935).
[27]. H A Shah, W Masood , M N S Qureshi & N L Tsintsadze *Phys. Plasmas* **18,** 102306 **(** 2011).
[28]. W Masood and B.Eliasson, *Phys. Plasmas* **18,** 034503**(**2011).
[29]. S. Chandra , B.Ghosh and S.N.Paul, *Astrophys. Space Sci***. 343 ,** 21**3 (**2013).
[30]. R. Z .Sagdeev, *Rev. Plasma Phys*. **4** (ed. M A Leontovich) (New York: Consultant Bureau) (1966)

PUBLISHED PAPERS OF INDRANI PAUL

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1). B. Ghosh, S. N.Paul, C. Das, A. Sinhamahapatra and **Indrani Paul**, "Resonant interaction of a magneto-hydrodynamic wave with its second harmonic in a planar plasma filled waveguide", Indian Journal of Physics , Vol. 85, pp.745–753(2011).

2). **Indrani Paul** , G. Pakira. S K Chattopadhyay, S N Paul and B Ghosh, "Analytical study on the existence of ion-acoustic solitary waves in a plasma consisting of warm streaming ions and nonthermal electrons", Indian Journal of Physics, 86(5), pp.395–400 (2012).

3). B.Ghosh**,** S.N.Paul,Chandra Das and **Indrani Paul**, "Modulational instability of high frequency surface waves on warm plasma half-space", Canadian Journal of Physics, Vol. 90, pp. 291-297 (2012).

4). S. N. Paul, Chandra Das, **Indrani Paul**, B. Bandyopadhyay, S.Chattopadhya and S .S. De , "Ion -acoustic solitary waves in an electron-ion-positron plasma ", Indian Journal of Physics, Vol. 86 (6), pp. 545-553 (2012).

5). B.Ghosh, S.N.Paul, C.Das, **Indrani Paul** and S.Banerjee, "Electrostatic double layers in multicomponent drifting plasma having nonthermal electrons", Brazilian Journal of Physics, Vol. 43(1-2) , pp. 28-33 (2013).

6). S.N.Paul, A.Chatterjee, **Indrani Paul** and B.Ghosh, " Ion-acoustic double layers in dusty plasma consisting of two- temperature nonisothermal electrons and positrons" , International J. Chemical and Physical Sciences, ISSN:2319-6602 ,IJCPS **Vol. 4** Special Issue-6 , APST–2015, pp.38-52 (2015).

7). **Indrani Paul** , S. Chandra ,S. Chattopadhyay and S. N. Paul, "W-type ion-acoustic solitary waves in plasma consisting of cold ions and nonthermal electrons", Indian Journal of Physics, Vol. 90, Issue 10, pp. 1195–1205 (2016).

8). **Indrani Paul**, S.K.Bhattacharya, S. N. Paul and A.Chatterjee, "Instability of ion-acoustic wave in a quantum dusty plasma bounded in finite geometry", Journal of Physical and Chemical Sciences, Vol-4(3), pp. 1-5 (2016).

9). S. N. Paul, A .Chatterjee, G.Pakira and **Indrani Paul**, "Ion -acoustic solitary waves in a self-gravitating dusty plasma consisting of weak non-isothermal two-temperature electrons" , Journal of Applied Physical Science International (JAPSI), Vol. 8(2), pp. 77-90 (2017).

10). S. N. Paul, A. Chatterjee and **Indrani Paul** , "Nonlinear propagation of ion- acoustic waves in self-Gravitating dusty plasma consisting of non-isothermal two-temperature electrons" , Indian Journal of Physics, Vol. 91, pp 101–107 (2017).

11). **Indrani Paul** , S.Chandra, J. Goswami , S.N.Paul and J. Sarkar , "Effect of q-nonextensive electrons on the ion -acoustic double layers in plasma having warm ions" , Proceedings of National Conference on Plasma Physics and Nonlinear Dynamics held at JIS University on 23-24 March 2017.

12). **Indrani Paul,** A.Chatterjee and S.N.Paul, "Nonlinear propagation of ion-acoustic waves in quantum plasma in presence of an ion beam", Laser and Particle Beams , Vol. 37(4) , pp. 370 – 380(2019)

13). S. N. Paul , A Roy Chowdhury and I**ndrani Paul**, **"**Modulational instability of bright envelope soliton and rogue waves in ultra-relativistic degenerate dense electron-ion-positron plasma", Plasma Physics Reports, Vol. 45(11), pp. 1011–1025 (2019).

14). **Indrani Paul**, A.Chatterjee and S. N. Paul, "Effects of nonthermal electrons and ion beams on ion-acoustic double layers in warm ion plasma", Indian Journal of Physics, https://doi.org/10.1007/s12648-020-01899-w (2020).

15). **Indrani Pau**l, A. Chatterjee and S. N. Paul, "Effects of boundary of cylindrical waveguide and nonthermal, electrons on the ion-acoustic double layers in multicomponent bounded plasma", Brazilian Journal of Physics, Vol. 50, pp.272–281(2020).

16). **Indrani Pau**l, A. Chatterjee and S. N. Paul, "Slow and fast modulation instability and envelope soliton of ion- acoustic waves in fully relativistic plasma having nonthermal electrons", e-Proceedings of 8th.PSSI-PLASMA SCHOLARS COLLOQUIUM (PSC-2020)October 8-9, 2020, KIIT University, Bhubaneswar-751024, Odisha, India, PSC-4. pp-75-79 (2020).

17). **Indrani Paul,** A. Chatterjee and S. N. Paul , "Effects of positrons and nonthermal electrons on the ion-acoustic double layers in drifting negative ion plasma"- Journal of Applied Physical Science International (JAPSI) Vol.13(3): pp.44-60 (2021).

18). **Indrani Paul,** A. Chatterjee and S. N. Paul, "Dressed solitons and double layers of ion- acoustic waves in a dusty plasma having ultra-relativistic degenerate two-temperature electrons", Indian Journal of Physics; DOI :10.1007/s12648-021-02059-4 (2021).

19). S.N.Paul, A.Roychowdury , **Indrani Paul** and S.Choudhury, "Slow and fast modulation instability and envelope soliton in plasma having ion stream and superthermal electrons"- Radiation Effect and Defects in Solid, https://doi.org/10.1080/10420150.2021.1972112 (2021).

20). **Indrani Paul** , S. Chattopadhyaya and S. N. Paul, "Effects of relativistically degenerate electrons and positrons on the ion-acoustic double layers in warm ion plasma"- International Journal of Research in Engineering and Science) (IJRES) Vol. 10(2), pp. 01-11 (2022).

21). **Indrani Paul**, A. Chatterjee, S. N. Paul, "Spiky and explosive ion-acoustic solitons in a weakly relativistic plasma having non‑isothermal electrons, Brazilian Journal of Physics, 52:133 (2022).